Double-doorway model for pion-nucleon elastic scattering in the S_{11} channel

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Resonance energy pion-nucleon elastic scattering in the S_{11} channel is modeled by assuming that the pion plus nucleon couple to two resonances, the resonances couple to inelastic channels, but there is no direct coupling of the pion-nucleon channel to the inelastic channels. The model is solved by matrix N/D methods. The coupling of the inelastic channels to the elastic channel is taken directly from data. Using form factors from the constituent quark model, we find the model is able to reproduce the experimentally determined pion-nucleon phases in the S_{11} channel over the resonance region and that the resonance part of the amplitude is negligible at low energies.

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The use of pions to study properties of nuclei has necessitated the construction of dynamic models of the pion-nucleon interaction which must reproduce the measured two-body data accurately, must be simple enough to be used in the many-body problem, and yet must also be based on the physical processes that we believe drive the interaction. The first such model was the separable potential model of Ref. [1]. It was soon realized [2] that the coupling to the inelastic, pion-production channels was an important piece of physics to incorporate into these models. Furthermore, the potential models do not have a pole at the nucleon mass in the P-wave two-body amplitudes. It has been shown [3] that this produces an artificially long-range (in coordinate space) interaction which has a geometrical [4] effect in pion-nucleus scattering. This can be rectified by utilizing the Chew-Low [5] model if it is extended [3,4,6] to also include the coupling to inelastic channels. Only with this coupling included can the model fit, in detail, the experimentally measured P-wave phase shifts. The model has been generalized [7] to include the Δ_{33} as an explicit degree of freedom. A more recent model [8], however, contradicts these models in that it does not include the affects of the coupling to the inelastic channels and fits the pion-nucleon data, albeit only up to 400 MeV.

Recently, data for pion-nucleus reactions have been taken [9–12] at energies above the delta resonance. Work underway at KEK and the AGS at Brookhaven, as well as proposals for PILAC at LAMPF or KAON at TRIUMPF, would extend this work. Theoretical work at these higher energies requires a dynamical model of the N^*'s and Δ^*'s which led to a "doorway-resonance" model [13]. The model is essentially an extended Lee model [14] in which the inelastic, pion-production channels are included but can only be reached by first going through the resonance channel. The model was able to reproduce the measured phase shifts in the resonating channels for pion laboratory kinectic energies up to 1 GeV. In the S_{11} channel, however, a single resonance was used although it is well known that there are two resonances in this channel. The form factor used had an interesting momentum dependence. The good fit resulted from the momentum dependence of the form factor and the energy dependence produced by the inelastic cut which was also included in the model.

Recently, a two-resonance model [15] has been used very successfully to understand why the $S_{11}(1535)$ couples strongly to the eta-nucleon channel while the $S_{11}(1650)$ does not. The double-doorway model proposed here is very similar to the model of [15]. There, the inelasticities were modeled using the eta-nucleon channel. The model proposed here includes all the inelastic channels by taking their contribution to the elastic channel from data. This allows us a detailed fit to the elastic channel over the resonance region and, hence, a model which we can use in studying pion-nucleus reactions.

The basic assumption of the model is that the pion plus nucleon interact by the pion being absorbed and leaving the nucleon in one of several excited hadronic states. These excited hadrons may either decay back into the pion-nucleon channel or into inelastic channels. The inclusion of the inelastic channels is necessary as the experimentally measured amplitudes are highly inelastic. In constructing the model, it is assumed that (1) there is no direct pion-nucleon to pion-nucleon coupling and (2) the elastic pion-nucleon channel does not couple directly to any of the inelastic channels; the reaction must proceed through a resonance on its way to the other channels. This last assumption is the origin of the term "doorway." We are motivated to make these assumptions by several qualitative features of the pion-nucleon data. The data show very small phase shifts until the resonance energy is reached. At the same energy as the phase begins a quick rise through 90°, the inelasticity parameter falls steeply from its low-energy value of 1.

Mathematically, the model can be defined in terms of projection operators. We define [16] P to be the projection operator onto the channel in which there is one pion and one nucleon. Q_R is the projector onto the channels where any one, but only one, of the resonances is present, and Q_I is a projector onto the physical inelastic channels. These inelastic channels can be a nucleon plus several pions, an excited nucleon or delta plus a pion or several pions, etc. The only nonzero pieces of the interaction are then

$$P\,V\,Q_R = \sum_i \lambda_i\,|\,v_i\rangle \tag{1}$$

and

$$Q_I\,V\,Q_R = \sum_i \lambda_i^I\,|\,w_i\rangle\ , \tag{2}$$

plus the Hermitian conjugates. Here λ_i and λ_i^I are coupling constants and $|\,v_i\rangle$ and $|\,w_i\rangle$ are form factors for pion-nucleon to resonance or resonance to inelastic channel couplings respectively. This model is a solvable model. We follow [6] and [13] and introduce an effective interaction $\langle q'\,|\,V_{\rm eff}\,|\,q\rangle$. This interaction, when inserted into a Lippmann-Schwinger equation, will produce the exact two-body scattering amplitude for the model. We here work with invariant amplitudes which are free of kinematical singularities. This produces [13,17] a Lippmann-Schwinger equation of the form

$$\langle q'\,|\,T(\omega)\,|\,q\rangle = \langle q'\,|\,V_{\rm eff}(\omega)\,|\,q\rangle \int \frac{q''^2\,dq''}{4\,\omega''_\pi\,\omega''_N}\,\langle q'\,|\,V_{\rm eff}(\omega)\,|\,q''\rangle\,\frac{1}{\omega-\omega''+i\eta}\,\langle q''\,|\,T(\omega)\,|\,q\rangle\ , \tag{3}$$

where $\omega_\pi = \sqrt{q^2+m_\pi^2}$, $\omega_N = \sqrt{q^2+m_N^2}$, and $\omega = \omega_\pi + \omega_N$. The phase space factor that enters the integral results from using the invariant amplitude. This equation was first proposed [18] by Kadyshevski and was first applied [19] to the pion-nucleon problem by Mathelitsch and Garcilazo. The solution for $V_{\rm eff}$ is

$$\langle q'\,|\,V_{\rm eff}(\omega)\,|\,q\rangle = \sum_{i,j} \lambda_i\,\lambda_j\,\langle q'\,|\,v_i\rangle\,G_{ij}(\omega)\,\langle v_j\,|\,q\rangle\ \ , \tag{4}$$

where $G_{ij}(\omega)$ satisfies

$$\sum_k \left[\delta_{ik}(\omega - m_i^0) - I_{ik}(\omega)\right] G_{kj}(\omega) = \delta_{ij}\ \ , \tag{5}$$

with

$$I_{ik}(\omega) = \lambda_i^I\,\lambda_k^I\,\langle w_i\,|\,Q_I\,g(\omega)\,Q_I\,|\,w_k\rangle\ \ , \tag{6}$$

and $g(\omega) = (\omega - H_i)^{-1}$. Here H_i is the Hamiltonian in the inelastic channels and m_i^0 is the bare mass of the excited hadronic state, i.e., the mass of the resonance in the absence of coupling to decay channels. This result leads directly to a solution for the T matrix.

We write the T matrix as

$$\langle q'\,|\,T(\omega)\,|\,q\rangle = \sum_{i,j} \langle q'\,|\,v_i\rangle\,\tau_{ij}(\omega)\,\langle v_j\,|\,q\rangle\ . \tag{7}$$

Insert Eqs. (4) and (7) into the Lippmann-Schwinger equation, Eq. (3), and utilize Eq. (5) for the inverse of $G_{ij}(\omega)$ to get

$$\sum_k \{\,\delta_{ik}(\omega - m_i^0) - I_{ik}(\omega) - \lambda_i\,\lambda_k\,\mathcal{E}_{ik}(\omega)\}\,\tau_{kj}(\omega)$$
$$= \sum_k D_{ik}(\omega)\,\tau_{kj}(\omega) = \delta_{ij}\ \ , \tag{8}$$

where $\mathcal{E}_{ij}(\omega)$ is defined by

$$\mathcal{E}_{ij}(\omega) = \int \frac{q'^2\,dq'}{4\,\omega'_\pi\omega'_N}\,\frac{v_i(q')\,v_j(q')}{\omega-\omega'+i\eta}\ . \tag{9}$$

We see that the matrix $D_{ij}(\omega)$, which is the inverse of $\tau_{ij}(\omega)$, is quite simple. If we knew a complete set of inelastic channel states, we could insert them into Eq. (6) for $I_{ij}(\omega)$ and Eq. (9) would be a solution to the problem. At this point, however, we follow Refs. [2–4,6,13] and utilize matrix N/D dispersion theory to circumvent the necessity of constructing a model of the inelastic channels.

Equation (9) demonstrates that $D_{ij}(\omega)$ has a simple analytic structure in the complex ω plane. It has only an elastic and an inelastic cut and satisfies the Schwartz reflection principle. These properties and the behavior of each term as $\omega \to \infty$ allows us to write a simple dispersion relation for $D_{ij}(\omega)$. Following Refs. [3,6] or [13], we arrive at

$$D_{ii}(\omega) = \omega - m_i^0 - \lambda_i^2 \int \frac{q'^2\,dq'}{4\,\omega'_\pi\omega'_N}\,\frac{1}{\hat\eta_i(q')}\,\frac{v_i^2(q')}{\omega-\omega'+i\eta} \tag{10}$$

for $i = 1, 2$. For the off-diagonal terms we find

$$D_{12}(\omega) = D_{21}(\omega)$$
$$= -\lambda_1\lambda_2 \int \frac{q'^2\,dq'}{4\,\omega'_\pi\omega'_N}\,\frac{1}{\hat\eta_{ij}(q')}\,\frac{v_i(q')\,v_j(q')}{\omega-\omega'+i\eta}\ . \tag{11}$$

The function $\hat\eta_i(q)$ is the ratio of the elastic cross section to the total cross section in the S_{11} channel if there were only one resonance channel. The function $\hat\eta_{ij}(q)$ is simply a convenient way of parametrizing the discontinuity of $D_{ij}(\omega)$ across the right hand cut. The simplest model [6] consistent with the measured total inelastic cross sections results from taking each of the $\hat\eta$'s to be equal to $\hat\eta(q)$, the experimentally measured ratio of the elastic to the inelastic cross section in this channel. It is defined in terms of the on-shell T matrix by

$$\langle q\,|\,T(\omega)\,|\,q\rangle = T(q) = \frac{-4\,\omega}{\pi\,q}\,\hat\eta(q)\,e^{i\hat\delta(q)}\sin\hat\delta(q)\ . \tag{12}$$

By taking $\hat\eta(q)$ directly from the experimentally measured ampliudes and using the model to predict $\hat\delta(q)$, we are able to avoid constructing an explicit model of the inelastic channels while incorporating the effect of the coupling of these channels to the elastic channel.

We use the form factors for the $\pi + N \rightarrow N^*$ vertex derived [15] from the constituent quark model [20],

$$\langle q \,|\, v_i \rangle = v_i(q) = -3\,C(q) + \sqrt{\frac{\omega_\pi(q)}{2m_N}}\,[C(q) + D(q)] \;, \tag{13}$$

where

$$C(q) = \sqrt{\pi}\,\frac{2}{27}\,\frac{q^2}{\beta^2}\,\exp(-q^2/6\beta^2)\;, \tag{14}$$

$$D(q) = \sqrt{\pi}\,\frac{4}{9}\left(3 - \frac{q^2}{3\beta^2}\right)\exp(-q^2/6\beta^2)\;. \tag{15}$$

The model differs from that of [15] in two ways. First, the use of invariant amplitudes and phase space factors has led to an extra nucleon energy factor ω_N in the phase space for the integrals in Eqs. (10) and (11). Second, $\hat{\eta}$ appears in these integrals in place of an explicit model of the inelastic channels.

The model contains six parameters, two coupling constants λ_i, two bare masses m_i^0, and two ranges β_i. The constituent quark model sets $\lambda_2 = 2\,\lambda_1\,\beta_2/\beta_1$ and $\beta_2 = \beta_1$. In order to achieve a quantitative fit to the data, we must relax one of these constraints. We keep the relation between the λ's ($\lambda_2 = 2\,\lambda_1\,\beta_2/\beta_1$) but allow the β's to vary independently. We adjust the five parameters of the model to reproduce the phase $\hat{\delta}(q)$ from [21] utilizing the parameter $\hat{\eta}(q)$ from the same reference. The fit is quantitative for momenta $q \geq 250$ MeV/c. We find $\lambda_1^2 = 6.883\,m_\pi$, $\beta_1 = 214.4$ MeV, $\beta_2 = 124.5$ MeV, $m_1^0 = 1718$ MeV, and $m_2^0 = 1833$ MeV. The resulting phases $\hat{\delta}$ are pictured in Fig. 1. It is satisfying that the range parameters β_i remain reasonably equal to each other. The renormalization of the masses (the difference between the bare mass m_i^0 and the observed resonance energy) is also reasonable. We find 273 MeV and 283 MeV for the $S_{11}(1535)$ and $S_{11}(1650)$ respectively. This is somewhat larger than others have found. It is clear that we have a larger renormalization compared to Ref. [15] because we include all of the inelastic channels while they include only the eta-nucleon channel. Similarly we expect a larger renormalization than models such as those of [22] where only the elastic, pion-nucleon renormalization is included. The values for β_i are smaller than found in other models. This is necessary in order to fit the region near $T_\pi \sim 300$ MeV. In order to fit this region a part of the interaction [3,13] must have a cutoff of several hundred MeV. We find a range of $\sqrt{6}\beta_1 = 305$ MeV for our lowest cutoff.

We find that the resonating part of the amplitude contributes very little at low energies. This is encouraging in that the considerable physics [23] that has been learned by working at low energies and neglecting the resonances is justified. The physics does appear to decouple the low-energy region from the resonance energy region. The model developed here could be further expanded to include a purely phenomenological piece, such as a separable potential, that could be adjusted to fit the low energies. A more sophisticated approach [8] would be to add rho exchange.

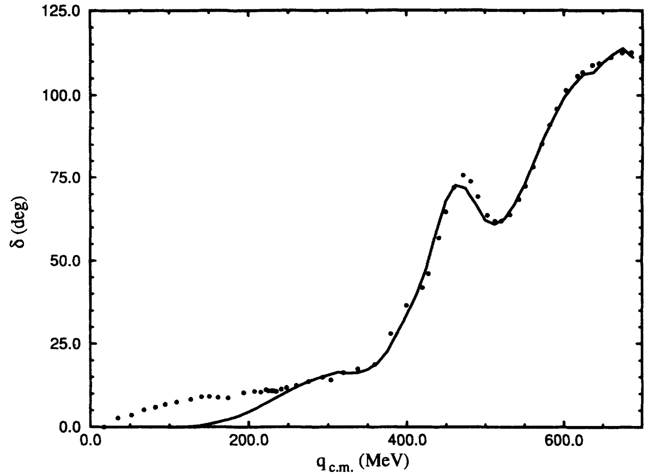

FIG. 1. The phase shift $\hat{\delta}(q)$ versus the center-of-mass momentum q in the S_{11} channel. The data are from Ref. [21] and the curve is the result of the model developed here.

We have found a model of the pion-nucleon interaction in the S_{11} channel which can be used in pion-nucleus calculations in the resonance region. The model has a more physical off-shell behavior than an earlier model [13] which had only a single resonance and used a form factor with structure in its momentum dependence. The low momentum cutoff which we find could have interesting implications for the survival of S_{11} resonances in the nuclear medium. If these resonances are physically as large as this model might indicate, one would expect them to have a sufficiently large cross section with the nucleon that they would not survive as resonances in nuclear matter. Such a phenomenon appears to have been seen in photoabsorption [24] cross sections. Before taking the range parameters from this model too seriously, however, an extended model in which an additional low-energy term is added should be investigated. Recoil corrections to the form factors might also be significant at these energies.

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