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Electromagnetic scattering from relativistic bound states

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The quasipotential formalism for elastic scattering from relativistic bound states is formulated based on the instant constraint in the Breit frame. The quasipotential electromagnetic current is derived from Mandelstam's five-point kernel and obeys a two-body Ward identity. Breit-frame wave functions are obtained directly by solving integral equations with nonzero total three-momentum, thus accomplishing a dynamical boost. Calculations of electron-deuteron elastic form factors illustrate the importance of the dynamical boost versus kinematic boosts of the rest frame wave functions.

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In the study of relativistic bound states and scattering processes for two particles, it is possible to perform a reduction from four dimensions to three dimensions to obtain a quasipotential formalism [1–4]. The quasipotential is the kernel of the three-dimensional equation, which in general may be defined covariantly. A quasipotential reduction is commonly used in studies of the nucleon-nucleon (*NN*) interaction because it provides a covariant dynamics which is similar to the Schrödinger dynamics. For *NN* scattering, one generally assumes a quasipotential in the form of meson-exchange interactions with coupling constants selected to provide a realistic description of the *NN* scattering data and deuteron properties [5].

In this work, a quasipotential reduction procedure is applied to the Mandelstam five-point function to derive an electromagnetic current operator consistent with quasipotential wave functions for relativistic bound states. The quasipotential reductions for the five-point kernel and initial and final states must be consistent with each other in order to reduce the current to a simple form. We find a consistent reduction to three dimensions in the Breit frame, where the time component of photon momentum vanishes for elastic scattering, i.e., $q^0 = 0$, based on the instant constraints, $p^0 = p'^0 = 0$, where p is the relative momentum in the initial state and p' is the relative momentum in the final state. Moreover, the instant constraints lead to a gauge invariant formalism which is symmetric in its treatment of the particles.

With the instant constraints in the Breit frame for electromagnetic matrix elements, the initial and final wave functions must be calculated with total three-momentum $\mathbf{P} = -\frac{1}{2}\mathbf{q}$ for the initial state and $\mathbf{P} = \frac{1}{2}\mathbf{q}$ for the final state. Only for $\mathbf{q} = 0$ are the usual rest frame wave functions used. For $\mathbf{q} \neq 0$, the required wave functions may be thought of in terms of a boost of the rest frame wave function. In the instant form of relativistic quantum mechanics one encounters a similar form of boost, and it must be dynamical in the sense that the generator of boosts depends on the interaction. In this paper, we formulate the dynamical boost within the instant quasipotential formalism. We present calculations of deuteron electromagnetic form factors using a simple approximation of the boost to demonstrate the feasibility of calculating the required wave functions in the Breit frame and we illustrate the importance of the dynamical boost.

Formally, the quasipotential is defined such that the results for two-body scattering based on the Bethe-Salpeter equation are reproduced by use of the quasipotential and a three-dimensional propagator. Consider the Bethe-Salpeter t matrix,

$$T(p,q;P) = K^{BS}(p,q;P) + \int \frac{d^4p'}{(2\pi)^4} K^{BS}(p,p';P) \times G_0^{BS}(p';P)T(p',q;P), \quad (1)$$

where $G_0^{BS} = iS_{F(1)}(\frac{1}{2}P+p')iS_{F(2)}(\frac{1}{2}P-p')$ is the propagator for two free particles, and K^{BS} is the kernel consisting of irreducible graphs. This equation may be abbreviated as $T^{BS} = K^{BS} + K^{BS}G^{BS}T^{BS}$, with implied four-dimensional integration. Note that $P = p_1 + p_2$ is the total momentum and $p = \frac{1}{2}(p_1 - p_2)$ is the relative momentum.

A bound state with mass M gives rise to a pole at $P^0 = \hat{P}^0 \equiv \sqrt{M^2 + \mathbf{P}^2}$:

$$T(p,q;P) = -\frac{i}{2\hat{P}^0}\frac{\Gamma(p;\hat{P})\overline{\Gamma}(q;\hat{P})}{P^0 - \hat{P}^0}, \quad (2)$$

where $\overline{\Gamma} = \Gamma^\dagger\gamma_1^0\gamma_2^0$ and terms regular when $P^0 = \hat{P}^0$ are omitted.

The same T matrix and bound state vertex function can be produced with a quasipotential propagator, $G_0^{QP}(p,P) = ig_0 2\pi\delta(C(p))$, where $C(p) = 0$ is a constraint which reduces integrations from four to three dimensions:

$$T = K^{QP} + K^{QP}G_0^{QP}T. \quad (3)$$

The quasipotential kernel is defined by

$$K^{QP} = K^{BS} + K^{BS}(G_0^{BS} - G_0^{QP})K^{QP}. \quad (4)$$

For the bound state vertex, one has a homogeneous equation, found by substituting Eq. (2) into Eq. (3) and retaining pole terms:

$$\Gamma(p;\hat{P}) = \int \frac{d^4p'}{(2\pi)^4} K^{QP}(p,p';\hat{P})G_0^{QP}(p';\hat{P})\Gamma(p';\hat{P}). \quad (5)$$

Note that if the quasipotential constraint is not applied to momentum p in this expression, in principle one obtains the full four-dimensional vertex function on the left-hand side in terms of the quasipotential vertex function on the right-hand side.

The two most frequently used quasipotential constraints are the one-particle-on-shell formalism developed by Gross and collaborators [4,5], based on $C(p) \equiv (p_2^0 - \epsilon_2) = 0$ ($\epsilon_2 \equiv \sqrt{m_2^2 + \mathbf{p}_2^2}$), and the equally off-shell formalism in the center-of-mass frame developed by Blankenbecler and Sugar and Logunov and Tavkhelidze [1,2], based on $C(p) \equiv (p_1^2 - p_2^2)/2\sqrt{P^2} = p \cdot P/\sqrt{P^2} = 0$. When one particle is on mass shell, there is an inherent and inconvenient asymmetry of the formalism. This can be overcome [5] by symmetrizing the formalism but then there arise unphysical singularities in the one-boson-exchange quasipotential which must be circumvented. An alternative, manifestly symmetric treatment that has no singularities is afforded by the equally off-shell formalism. In the center-of-mass frame, the equally off-shell constraint is equivalent to an instant formalism because the constraint causes interactions to have zero time component of momentum transfer. An extension of the equally off-shell formalism to the full Dirac space for two fermions has been developed by Wallace and Mandelzweig [3] by incorporating the iterative parts of crossed-box Feynman graphs, using a form of the eikonal approximation.

Before analyzing the five-point kernel, we sketch the derivation of the Dirac two-body propagator of Ref. [3], modified to incorporate the instant constraint in the Breit frame instead of the equally off-shell constraint. This propagator is derived by consideration of the four-point box and crossed-box Feynman diagrams, such that $V^{OBE} G_0^{QP} V^{OBE}$ is a good approximation to $K^{(\text{box})} + K^{(\text{c-box})}$. The instant constraint is applied to the relative momenta in the boson-exchange interactions, and the crossed box is approximated in an iterative form yielding

$$K^{(\text{box})} \approx \int \frac{d^4p}{(2\pi)^4} V^{OBE}(\mathbf{k}', \mathbf{p})\, iS_{F(1)}(p_1) iS_{F(2)}(p_2) \times V^{OBE}(\mathbf{p}, \mathbf{k}) \tag{6}$$

and

$$K^{(\text{c-box})} \approx \int \frac{d^4p}{(2\pi)^4} V^{OBE}(\mathbf{k}', \mathbf{p})\, iS_{F(1)}(p_1) iS_{F(2)}(q_2) \times V^{OBE}(\mathbf{p}, \mathbf{k}), \tag{7}$$

where $q_2 = p_1 + k_2 - k_1' \approx (p_1^0, \mathbf{p}_2)$, which is the eikonal approximation. The nucleon propagator is expanded as

$$iS_{F(i)}(p_i) = i \sum_{\rho_i} \frac{\Lambda_i^{\rho_i}(\mathbf{p}_i)}{\rho_i p_i^0 - \epsilon_i + i\eta}, \tag{8}$$

where $\rho_i = \pm$ and $\rho_i \Lambda_i^{\rho_i}(\mathbf{p}_i)\gamma_i^0$ are projection operators for Dirac spinors with Hermitian normalization, $\epsilon_i \equiv \sqrt{m_N^2 + \mathbf{p}_i^2}$ ($\mathbf{p}_{1,2} = \frac{1}{2}\mathbf{P} \pm \mathbf{p}$), and m_N is the nucleon mass. The deuteron mass and total energy are m_D and $p_1^0 + p_2^0 = \epsilon_D = \sqrt{m_D^2 + \mathbf{P}^2}$. Integration of p^0 yields

$$K^{(\text{box})} + K^{(\text{c-box})} \approx \int \frac{d^3p}{(2\pi)^3} V^{OBE}(\mathbf{k}', \mathbf{p})\, ig_0(p; P)\, V^{OBE}(\mathbf{p}, \mathbf{k}), \tag{9}$$

where the three-dimensional propagator is

$$g_0(\mathbf{p}, P) = \sum_{\rho_1, \rho_2} \frac{\Lambda_1^{\rho_1}(\mathbf{p}_1)\Lambda_2^{\rho_2}(\mathbf{p}_2)}{(\rho_1 + \rho_2)(\epsilon_D/2) - \epsilon_1 - \epsilon_2}, \tag{10}$$

and its inverse is

$$g_0^{-1}(\mathbf{p}, P) = \gamma_1^0 \gamma_2^0 \left[(\hat{\rho}_1 + \hat{\rho}_2)\epsilon_D/2 - \hat{\rho}_1 \hat{\rho}_2 (\epsilon_1 + \epsilon_2)\right], \tag{11}$$

where $\hat{\rho}_i u^{\pm}(\pm \mathbf{p}_i) = \pm u^{\pm}(\pm \mathbf{p}_i)$. Using the instant constraint we write

$$G_0^{QP}(p; P) = ig_0(\mathbf{p}, P)(2\pi)\delta(p^0). \tag{12}$$

The corresponding wave equation for the relativistic bound states is given further on in Eq. (26).

To formulate the quasipotential reduction for electromagnetic interactions, we start from the Mandelstam formalism [6] for a five-point function, T_5, which has a photon of momentum q coupled in all possible ways to the two particles and exchanged mesons. The five-point function, T_5, may be expressed as follows:

$$T_5 = (1 + TG_0^{BS}) K_5^{BS} (1 + G_0^{BS} T), \tag{13}$$

where four-dimensional integrations are implied. The irreducible five-point kernel, K_5^{BS}, is given by coupling the photon to particles one and two (lowest order impulse contributions) plus coupling the photon to all possible internal lines of the two-body kernel K^{BS}. To extract the electromagnetic matrix element for elastic scattering from the bound state, one substitutes Eq. (2) into Eq. (13). The electromagnetic matrix element is proportional to the residue of the double pole term in the resulting expression,

$$\langle J^\mu \rangle = N\, \overline{\Gamma} G_0^{BS} K_5^{BS} G_0^{BS} \Gamma, \tag{14}$$

with implied four-dimensional integrations. A normalization factor $N = -1/\sqrt{4P'^0 P^0}$ arises because of the factors in Eq. (2).

To write the matrix element with quasipotential vertex functions we use Eq. (5) in Eq. (14) to obtain

$$\langle J^\mu \rangle = N\, \overline{\Gamma} G_0^{QP} K_5^{QP} G_0^{QP} \Gamma, \tag{15}$$

with

$$K_5^{QP} = K^{QP} G_0^{BS} K_5^{BS} G_0^{BS} K^{QP}. \tag{16}$$

Equations (15) and (16) are rather general and they can be used with any choice of initial and final quasipotential propagators [7–9]. We have found that the formalism reduces to a simple and appealing form when instant constraints are employed in the Breit frame.

We proceed to analyze the five-point kernel, K_5^{QP}, in the same spirit as was used to derive the Dirac two-body propagator. For photon coupling to particle one, we develop the quasipotential current $J^\mu_{IA}(1)$, corresponding to the impulse approximation. The current $J^\mu_{IA}(2)$ describing the photon coupling to particle two may be obtained by relabeling $1 \leftrightarrow 2$. Meson-exchange currents can be treated by a straightforward extension of the analysis.

Starting with the Bethe-Salpeter impulse approximation current, $K_5^{BS} \approx \Gamma_1^\mu S^{-1}_{F(2)}(p_2)(2\pi)^4\delta^4(p_2'-p_2)$, the quasipotential impulse approximation current takes the form

$$K_5^{QP(\mathrm{box})}(k',P';k,P) = -i\int\frac{d^4p'd^4p}{(2\pi)^4}\delta^4\left(p'-p-\frac{1}{2}q\right)$$
$$\times \sum_{\rho_1'\rho_1\rho_2} K^{QP}(k',p',P')\,\frac{\Lambda_1^{\rho_1'}(\mathbf{p}_1')}{\rho_1'p_1'^0-\epsilon_1'+i\eta}\,\Gamma_1^\mu(p_1',p_1)$$
$$\times\frac{\Lambda_1^{\rho_1}(\mathbf{p}_1)}{\rho_1p_1^0-\epsilon_1+i\eta}\,\frac{\Lambda_2^{\rho_2}(\mathbf{p}_2)}{\rho_2p_2^0-\epsilon_2+i\eta}\,K^{QP}(p,k,P), \tag{17}$$

where Γ_1^μ is the one-body electromagnetic operator for a particle of charge e_1,

$$\Gamma_1^\mu = e_1\left(\gamma_1^\mu F_1(q^2) + i\sigma_1^{\mu\nu}q_\nu F_2(q^2)/(2m_1)\right). \tag{18}$$

Again we neglect the dependence of the quasipotential factors on the time component of loop momentum, p^0. We choose to work in the Breit frame, where $q^0 = p'^0 - p^0 = 0$ for elastic scattering because the instant constraints on the quasipotential, i.e., $p^0 = p'^0 = 0$, are consistent with $q^0 = 0$. Neglected contributions to the current from singularities of K^{QP} are expected to yield small effects. Because the quasipotential singularities are neglected, only the fermion poles contribute.

Because $K^{QP}G_0^{QP}K^{QP}$ contains an eikonal approximation of crossed graphs, we include the five-point crossed graphs for the current with similar approximations,

$$K_5^{QP(\text{c-box})} \approx K^{QP}G_0^{QP}K_5^{BS(\text{c-box})}G_0^{QP}K^{QP}. \tag{19}$$

To be consistent with the treatment of crossed graphs in the derivation of the Dirac two-body propagator, in the foregoing expression for the crossed-box contribution to the current we have replaced G_0^{BS} by G_0^{QP}. Moreover the boson exchanges of $K_5^{BS(\text{c-box})}$ are replaced by K^{QP}. The reduction $G_0^{QP}K^{QP}G_0^{QP}\Gamma = G_0^{QP}\Gamma$ (discussed below) is used to remove factors of $G_0^{QP}K^{QP}$ from Eq. (19). This yields an expression for the $K_5^{QP(\text{c-box})}$ identical to $K_5^{QP(\text{box})}$ except that p_2 is replaced by q_2, which is then evaluated as in Eq. (7) using the eikonal approximation.

The sum of $K_5^{QP(\text{box})}$ and $K_5^{QP(\text{c-box})}$ may be expressed in three dimensions as

$$K_5^{QP}(\mathbf{k}',P';\mathbf{k},P) = \int\frac{d^3p}{(2\pi)^3}K^{QP}(\mathbf{k}',\mathbf{p}')$$
$$\times ig_0(\mathbf{p}';P')\,\hat{J}_{IA}(1)\,ig_0(\mathbf{p};P)\,K^{QP}(\mathbf{p},\mathbf{k}) \tag{20}$$

where the particle one impulse-approximation current operator is

$$\hat{J}^\mu_{IA}(1) \equiv \Gamma_1^\mu\gamma_2^0\hat{\rho}_2 - [\Gamma_1^\mu - \gamma_1^0\hat{\rho}_1'\gamma_1^0\Gamma_1^\mu\hat{\rho}_1]\frac{\gamma_2\cdot p_2 - m_2}{\epsilon_1'+\epsilon_1}. \tag{21}$$

Equation (21) summarizes in a relatively simple form twelve contributions which follow from evaluating the fermion pole contributions to Eq. (17) and the similar crossed-box term. This result involves particle two operators even though $\hat{J}_{IA}(1)$ describes the coupling to particle one. A corresponding term is obtained for photon absorption by particle two.

Because the calculation of the current in the Breit frame is performed with the quasipotentials K^{QP} in Eq. (17) consistently evaluated at $k'^0 = p'^0 = p^0 = k^0 = 0$, it follows that the quasipotential factors in the current may be absorbed into the initial and final state vertex functions when matrix elements are evaluated,

$$\bar{\Gamma}G_0^{QP}\left[K^{QP}g_0\hat{J}_{IA}(1)g_0K^{QP}\right]G_0^{QP}\Gamma = \bar{\Gamma}g_0\hat{J}_{IA}(1)g_0\Gamma, \tag{22}$$

owing to the relation $K^{QP}G_0^{QP}\Gamma = \Gamma$, which is a shorthand for Eq. (5) evaluated with constrained relative momentum, $p^0 = 0$. This reduction is exact only if the constraint in K_5^{QP} is the same as the constraint in the initial and final vertex functions. If the equally off-shell constraints are used in the vertex functions, then the reduction should not be made and unphysical singularities in K^{QP} must be circumvented [8]. (In general, constraining the left and right sides of a quasipotential with inconsistent constraints always leads to unphysical singularities.) Almost all formalisms based on the equally off-shell constraint, including the "equal time" and "BSLT" approximations of Hummel and Tjon [9,10], assume the $K^{QP}G_0^{QP}\Gamma \to \Gamma$ reduction while ignoring the inconsistency of the constraints on the left and right sides of K^{QP}. This leads to an inconsistency with conservation of the relative energy, which dictates $p'\cdot P' = 0$ (and $P' = P + q$), because $p\cdot P = 0$ and $p'\cdot P' = 0$ are incompatible with it. It also leads to an ambiguity as to whether the constraint used in K_5^{QP} or the equally off-shell constraint is used when the rest frame vertex functions are boosted to the Breit frame. It will be seen later that this boost ambiguity produces significant differences in the form factors in comparison with the required dynamical boost.

Using Eqs. (20), (15), and (22) for K_5^{QP}, $\langle J^\mu\rangle$, and $K^{QP}G_0^{QP}\Gamma = \Gamma$, and defining $\psi \equiv g_0\Gamma$, the current matrix element for elastic scattering takes the simple form

$$\langle J^{\mu}_{IA}(1)\rangle = \frac{1}{2\epsilon_D}\int\frac{d^3p}{(2\pi)^3}\overline{\psi}\left(\mathbf{p}+\frac{1}{2}\mathbf{q};P'\right)\hat{J}^{\mu}_{IA}(1)\psi(\mathbf{p};P). \quad (23)$$

The current, $\hat{J}_{IA}(1)$, is called the box-cross current because it derives from the corresponding terms of K_5. Remarkably, it obeys an exact isoscalar, Ward-Takahashi identity with respect to the two-body Dirac propagator of Eqs. (10) and (11):

$$q\cdot\hat{J}_{IA}(1) = e_1F_1(q^2)\left(g_0^{-1}(\mathbf{p}';P') - g_0^{-1}(\mathbf{p};P)\right). \quad (24)$$

Thus working in the Breit frame and retaining the five-point crossed-box contributions leads to a gauge invariant quasipotential analysis of electromagnetic scattering. The formalism of Ref. [9] based on the equally off-shell constraint satisfies current conservation only at the level of positive-energy states.

The required wave functions cannot in general be obtained by a kinematical boost of rest frame wave functions because one does not have a four-dimensional relative momentum. Instead they must be calculated directly in the Breit frame. Thus the boost problem within the instant quasipotential formalism is nontrivial. The complete relation of the rest frame wave function to the Breit frame one involves a change of quasipotential constraint and a corresponding change of the quasipotential used to solve for the wave function. It is a straightforward matter to prove that the change of the quasipotential is governed by

$$K^{QP2} = K^{QP1} + K^{QP1}(G_0^{QP1} - G_0^{QP2})K^{QP2}, \quad (25)$$

where, for example, $QP1$ refers to the instant constraint in the rest frame, $C_1(p) = p\cdot P/\sqrt{P^2}$, and $QP2$ refers to the Breit frame instant constraint, $C_2(p) = p\cdot\widetilde{P}$, where $\widetilde{P} = \frac{1}{2}(P+P')$. Equation (25) defines formally how the interaction kernel changes when a boost from the rest frame to the Breit frame is performed, such that the same underlying covariant kernel applies in the four-dimensional formalism.

Calculations of the instant wave functions in the Breit frame have been performed based on solving

$$g_0^{-1}(\mathbf{p}';P)\psi(\mathbf{p}';P) = \int\frac{d^3p}{(2\pi)^3}\hat{V}(\mathbf{p}',\mathbf{p};P)\psi(\mathbf{p};P), \quad (26)$$

for $\mathbf{P} = \pm\mathbf{q}/\mathbf{2}$ and $\hat{V} = iK$. A one-boson-exchange potential is used with scalar, pseudovector, and vector meson exchanges $(\sigma,\delta,\eta,\pi,\omega,\rho)$ based on modified Bonn B parameters [8,11]. The normalization condition is

$$1 = \frac{1}{2\epsilon_D}\int\frac{d^3p}{(2\pi)^3}\overline{\psi}(\mathbf{p};P)\gamma_1^0\gamma_2^0\frac{1}{2}(\hat{\rho}_1+\hat{\rho}_2)\psi(\mathbf{p};P). \quad (27)$$

The Breit frame total angular momentum operator is $\mathbf{J} = \mathbf{J}_1+\mathbf{J}_2 = \boldsymbol{\mathcal{L}}+\mathbf{S}$, where $\boldsymbol{\mathcal{L}} = \mathbf{l}+\mathbf{L}$, $\mathbf{l} = \mathbf{r}\times\mathbf{p}$, $\mathbf{L} = \mathbf{R}\times\mathbf{P}$, and $\mathbf{S} = \frac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)$. Because $\mathbf{J}$ and J^z commute with g_0^{-1} and V, solutions of the homogeneous equation (26) are eigenfunctions of $\mathbf{J}$ and J^z. However, $\mathbf{l}$, $\mathbf{L}$, and $\mathbf{S}$ separately do not commute with g_0^{-1}. To proceed, we define sixteen Dirac plane-wave basis functions,

$$\chi^{\rho_1,\rho_2}_{s_1,s_2;M_J}(\mathbf{p},\mathbf{P}) \equiv u_1^{\rho_1}(\rho_1\mathbf{p}_1)u_2^{\rho_2}(\rho_2\mathbf{p}_2)\mathcal{Y}^{M_J}_{s_1,s_2}(\phi), \quad (28)$$

where

$$\mathcal{Y}^{M_J}_{s_1,s_2}(\phi) = e^{i(M_J-s_1-s_2)\phi}|s_1\rangle|s_2\rangle, \quad (29)$$

and $s_i = \pm\frac{1}{2}$ and $J^z\mathcal{Y}^{M_J} = M_J\mathcal{Y}^{M_J}$.

The usual partial-wave analysis is inapplicable due to the nonzero total angular momentum $\mathbf{L}$. The homogeneous equation is solved in three dimensions using the basis functions (28) with only ϕ integrations carried out analytically. Radial and polar angle integrations are performed numerically. The homogeneous equation is solved for $M_J = 0$ at fixed values of total momentum using the Malfliet-Tjon iteration procedure [12]. Wave functions with polarization states $M_J = \pm 1$ are obtained from the $M_J = 0$ state by using the raising and lowering operator, $\psi^{M_J\pm 1} = \sqrt{(J+M_J)(J-M_J+1)}J^{\pm}\psi^{M_J}$.

In principle, the change of the quasipotential must be determined by solving Eq. (25), but it is of interest to study simpler approximations in these initial calculations. Variation of the quasipotential with total momentum is approximated in a very simple manner in this work, $\hat{V}(\mathbf{p}'-\mathbf{p},\mathbf{P} = \pm\frac{1}{2}\mathbf{q}) = \hat{V}(\mathbf{p}'-\mathbf{p})/\lambda(\mathbf{q}^2)$ where $\lambda(\mathbf{q}^2)$ is fit to produce the correct deuteron total energy, $\epsilon_D = (M_D^2+\mathbf{q}^2/4)^{1/2}$. Figure 1 shows that the required change of the potential is modest, with λ varying linearly over a wide range of q^2 values. When $\lambda(\mathbf{q}^2) = 1$ is used, the potential is too attractive and the binding energy of the deuteron increases from $2m_N - M_D \approx 2.2$ MeV at $\mathbf{q}^2 = 0$ to $2m_N - M_D \approx 4.2$ MeV at $\mathbf{q}^2 = 200$ fm^{-2}. The difference in the deuteron form factors produced by setting $\lambda(\mathbf{q}^2) = 1$ is minor in comparison with the ambiguity in the boost of rest frame wave functions. Note that with the instant constraint, $\hat{V}^{(OBE)}$ as well as g_0 and J_{IA} are nonsingular.

Results for the deuteron magnetic form factor are shown in Fig. 2. The solid line result includes the impulse approximation plus $\rho\pi\gamma$, $\omega\sigma\gamma$, and $\omega\eta\gamma$ meson-exchange currents, calculated with the instant wave functions and current operators. We use the same meson-

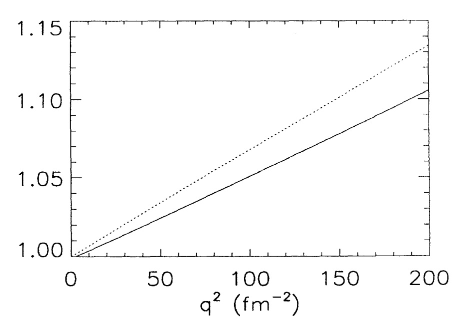

FIG. 1. The scaling of the potential, $\hat{V}(p'-p,\mathbf{P} = \pm\frac{1}{2}\mathbf{q}) = \hat{V}(p'-p)/\lambda(\mathbf{q}^2)$, that produces constant deuteron mass, $M_D = (\epsilon_D^2 - \mathbf{P}^2)^{1/2} = 2m_N - 2.22464$ MeV: full propagator (solid), ++ states only (dotted).

exchange-current operators as Hummel and Tjon [8–10] with $g_{\rho\pi\gamma} = 0.563$, $g_{\omega\sigma\gamma} = -0.4$, and $g_{\omega\eta\gamma} = -0.206$. We find that use of the conserved impulse approximation current of Eq. (21) provides electron-deuteron scattering results which differ little from use of just $\Gamma_1^\mu \gamma_2^0 \hat{\rho}_2$. More important is the dynamical boost. To illustrate this, we compare the impulse approximation contributions based on solving Eq. (26) (dotted line) and two approximations based on using the rest frame wave functions with a kinematical boost as follows:

$$\psi(p,P) = \Lambda_1(\mathcal{L})\Lambda_2(\mathcal{L})\psi(\mathbf{p}_{\text{rest}}, P_{\text{rest}}), \tag{30}$$

where $P_{\text{rest}} = (m_D, \mathbf{0})$, $P = (\epsilon_D, \pm\mathbf{q}/2)$, and $\mathcal{L}P_{\text{rest}} = P$. The Lorentz transform of the relative momenta, $\mathcal{L}p_{\text{rest}} = p$, is ambiguous since it is not possible to simultaneously satisfy both constraints: $(p^0 = 0)$ and $(p^0_{\text{rest}} = 0)$. In Fig. 2, the dashed line is the result of satisfying $(p^0 = 0)$, while the dash-dotted line is the result of satisfying $(p^0_{\text{rest}} = 0)$. The difference of the two kinematical boost prescriptions shown in Fig. 2 provides a measure of the ambiguity of kinematical boosts of equally off-shell wave functions. The dynamical boost does not suffer from this ambiguity and it is seen to provide results which differ significantly from those of both of the kinematical boosts.

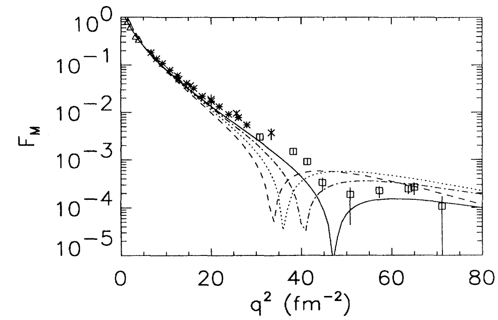

FIG. 2. Elastic *e-d* magnetic form factor: consistent calculation with IA+MEC (solid). Impulse approximation only: consistent calculation (dotted), boost approximations with p^0(Breit)=0 (dashed) and $p^0_{\text{rest}} = 0$ (dash-dotted). See text.

The instant quasipotential formalism in the Breit frame which is developed in this work provides an attractive formalism for analysis of electromagnetic scattering from relativistic bound states. A conserved current and a two-body Ward-Takahashi identity are realized. The formalism is symmetric with respect to particle labels and it is free of singularities, at least at the one-boson-exchange level. The formalism provides in principal a solution to the long-standing problem of how to boost three-dimensional wave functions with the instant constraint. The boost problem is nontrivial but it has been found to be soluble in practical calculations, although the complete boost requires further work. The significant differences in the results based on instant wave functions calculated directly in the Breit frame and the two approximations to them based on kinematical boosts suggest the importance of the formalism of this paper.

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