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Gamow-Teller beta decay and isospin impurity in nuclei near the proton drip line

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It is shown that "superallowed" Gamow-Teller (GT) beta decays are possible for $N=Z$ nuclei heavier than $^{56}_{28}\mathrm{Ni}_{28}$, performing Hartree-Fock plus random phase approximation (Tamm-Dancoff approximation) calculations. Since Fermi-type beta decays are forbidden in $N=Z$ even-even nuclei, it is pointed out that the main decay mode of the nuclei such as $^{100}_{50}\mathrm{Sn}_{50}$ may be "superallowed" GT beta decay, which is a beta decay to the GT giant resonance state. The amplitude of isospin $T=1$ admixed to the $T=0$ ground state in $N=Z$ nuclei is also calculated and discussed.

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The experimental and theoretical study of drip-line nuclei is currently very popular, especially in connection with developing facilities of radioactive heavy ion beams [1]. In the present Rapid Communication we give a report of the structure study of proton-drip-line nuclei, specifically Gamow-Teller (GT) (T_+) beta decays as well as isospin-mixing amplitudes in even-even nuclei with $N=Z$.

In Ref. [2] we show that the "superallowed" GT (T_-) beta decays with $\Delta Z=+1$ could occur as a beta decay to the Gamow-Teller giant resonance (GTGR) state in some very light neutron-drip-line nuclei, estimating the GT collective modes using the spherical Hartree-Fock (HF) plus the random phase approximation (RPA) or Tamm-Dancoff approximation (TDA) as well as using a schematic model. We apply approximately the same model here to proton-rich nuclei. Thus, first we perform spherical HF calculations using Skyrme-type interactions. Since we are interested in GT beta decays of light-medium nuclei, in numerical examples we choose the SG2 interaction [3] and the SIII interaction [4], which reasonably reproduce $M1$ properties as well as the level order of known one-particle orbitals.

As a numerical example, in Fig. 1(a) we show the HF potential of ^{100}Sn, which is calculated by using the SG2 interaction. The lowest occupied orbit ($=1s_{1/2}$ orbit) and the highest occupied orbit ($=1g_{9/2}$ orbit) are shown for both neutrons and protons. It is seen that the binding energies of one-particle orbitals with the same quantum numbers, (nlj), as well as the depth of the total one-particle potential (expressed by solid lines in Fig. 1) are about 15 MeV different for protons and neutrons. The difference comes from the Coulomb interactions between $Z=50$ protons. For comparison, in Fig. 1(b) the HF potential of a light neutron-drip-line nucleus, $^{22}_{6}\mathrm{C}_{16}$, is shown, which is calculated by using the BKN interaction [5] in Ref. [2]. All occupied orbitals are shown at respec-

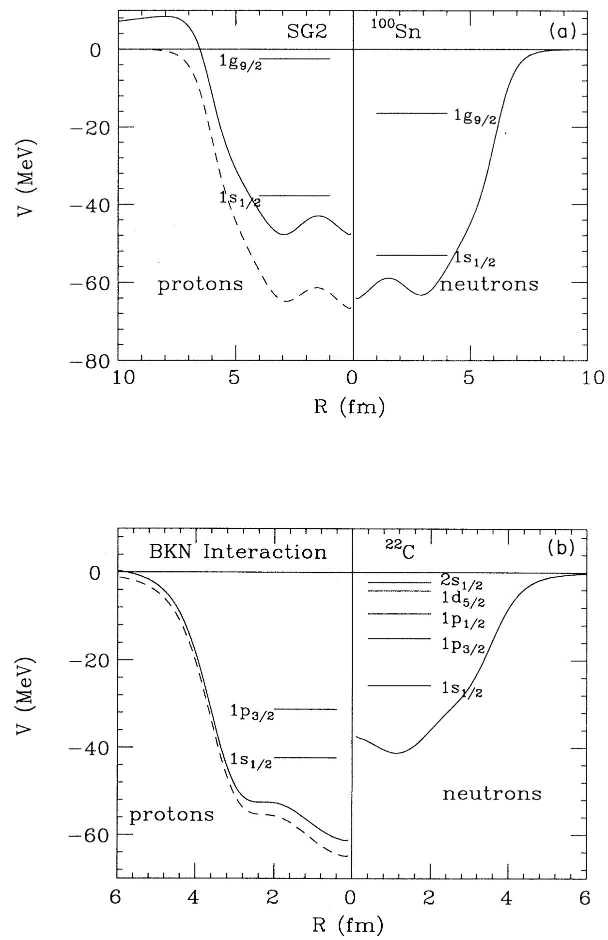

FIG. 1. HF potentials of ^{100}Sn (a) and ^{22}C (b). The Skyrme interactions, SG2 and BKN, are used for ^{100}Sn and ^{22}C, respectively. The dashed (solid) lines for protons show the HF potential without (with) the Coulomb potential. Since the HF calculation is performed with the Coulomb potential, there is a small difference between the dashed line for protons and the solid line for neutrons in ^{100}Sn. In ^{100}Sn only the lowest-lying one-particle level ($=1s_{1/2}$) and the highest occupied level ($=1g_{9/2}$) are shown, while in ^{22}C all occupied one-particle levels are denoted.

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tive calculated energies. It is seen that, in contrast to the case of ^{100}Sn in Fig. 1(a), in ^{22}C the binding energies of proton one-particle orbitals are about 16 MeV larger than those of neutron one-particle orbitals with the same quantum number, (nlj). The largeness comes from the large neutron excess together with the stronger p-n interaction than the p-p and n-n interactions.

When the neutron number decreases in a given isotope, there occurs a striking change in the proton one-particle energies, while the change in the neutron one-particle energies is minor. For example, the HF one-particle energy of the proton $1g_{9/2}$ orbital, which is calculated by using SG2 interaction, changes from -13.4 to -2.5 MeV when we go from $^{132}_{50}\mathrm{Sn}_{82}$ (which is a nucleus on the neutron-rich side relative to the beta-stability line) to $^{100}_{50}\mathrm{Sn}_{50}$. In contrast, the corresponding HF one-particle energy of the neutron $1g_{9/2}$ orbital changes from -17.6 to -16.4 MeV. The strong increase in the energies of given proton one-particle orbitals with decreasing neutron number towards the proton drip line [6] suggests that the "superallowed" GT beta decay with $\Delta Z=-1$ could occur as a beta decay to the GTGR state in heavier proton-drip-line nuclei. The decrease of the Coulomb energy in daughter nuclei helps further to make the GT transitions to GTGR states as beta decays, especially as the proton number increases.

In Table I we show the GT (with $\Delta Z=-1$) strengths of $Z\approx N$ even-even nuclei from ^{56}Ni to ^{106}Sn, which are calculated in HF+TDA using the SG2 interaction. Only those peaks which carry more than 1% of the total sum of the transition strength are shown. For the nucleus ^{56}Ni we show also the calculated result for HF+RPA, for reference. In order to obtain nonzero GT strength in our present calculation, only those nuclei with the configurations of unsaturated spin for the transitions of $p \rightarrow n$ are selected. The total GT strength and the estimated energy of the GTGR state depend sensitively on the orbitals of unsaturated spin. We note that some of the nuclei tabulated in Table I (for example, ^{76}Sr) are supposed to be nonspherical and the pair correlation may play a role in those nuclei, while our calculation is carried out assuming spherical configurations without including possible pair correlations. In the present beta decay from even-even to odd-odd nuclei the neglect of possible pair correlation may have an unfavorable shift in energy by the magnitude of $\Delta_p+\Delta_n$. Nevertheless, from Table I it is clearly seen that as $N=Z$ increases beyond that of ^{56}Ni the major strength carried by the GTGR state would begin to lie energetically increasingly lower than the ground state and, consequently, the "superallowed" beta decay may become possible. If the proton number is as large as 50, the "superallowed" beta decay can be expected also for some $N>Z$ nuclei such as ^{106}Sn. For the nuclei with $Z<50$ the $N=Z$ nuclei lie well inside the proton drip line and, thus, the "superallowed" beta decay may be observed also for some $N<Z$ nuclei.

TABLE I. Properties of GT (with $\Delta Z=-1$) strength calculated in HF+TDA using SG2 interaction. Only the peaks, which carry more than 1% of the non-energy-weighted sum rule (NEWSR), are shown. A negative energy in the second column means that the mother nucleus can beta decay to the corresponding state of the daughter nucleus. Using the tabulated values we obtain, for example, the half-life of ^{100}Sn is calculated to be 0.6 sec.

	E (MeV)	B(GT) $(g_A^2/4\pi)$	NEWSR
$^{56}_{28}\mathrm{Ni}_{28}\rightarrow^{56}_{27}\mathrm{Co}_{29}$	+0.47	13.5	(97.6%)
(HF+RPA)	(+0.24)	(11.1)	
$^{64}_{32}\mathrm{Ge}_{32}\rightarrow^{64}_{31}\mathrm{Ga}_{33}$	−7.46	0.4	(2.2%)
	−7.10	2.4	(12.6%)
	−0.78	16.1	(84.1%)
$^{76}_{38}\mathrm{Sr}_{38}\rightarrow^{76}_{37}\mathrm{Rb}_{39}$	−8.50	5.3	(95.5%)
$^{78}_{40}\mathrm{Zr}_{38}\rightarrow^{78}_{39}\mathrm{Y}_{39}$	−12.38	0.5	(8.0%)
	−10.31	5.5	(86.5%)
$^{100}_{50}\mathrm{Sn}_{50}\rightarrow^{100}_{49}\mathrm{In}_{51}$	−5.18	17.5	(95.8%)
$^{106}_{50}\mathrm{Sn}_{56}\rightarrow^{106}_{49}\mathrm{In}_{57}$	−3.71	17.5	(96.6%)

In order to obtain a simple qualitative picture of the possible occurrence of the beta decay of $N=Z$ nuclei to the GTGR state of respective daughter nuclei, next we illustrate the situation using a schematic harmonic oscillator model combined with a spin-orbit splitting. Denoting the principal oscillator quantum number of the last filled major shell by N_F, we consider the mother configuration in which the $j_2=l_F+\frac{1}{2}$ shell is fully occupied and the $j_1=l_F-\frac{1}{2}$ shell is unoccupied, where $l_F=N_F$. Among the nuclei tabulated in Table I ^{56}Ni (in which $l_F=3$) and ^{100}Sn (in which $l_F=4$) belong to the type of the configuration considered here. Using $N=Z=A/2$ and $N_F \gg 1$ we obtain

$$l_F=N_F\approx(\tfrac{3}{2}A)^{1/3}-2\ . \tag{1}$$

Writing the spin-orbit splitting of the orbital l_F of like nucleons as ϵ_i, the TDA equation for the collective $\sigma\tau_+$ state is written as

$$\frac{\langle (j_1 j_2^{-1})1^+|\sigma_0\tau_{+1}|0\rangle^2}{\epsilon_i-\epsilon}=-\frac{1}{\kappa_{\sigma\tau}} \tag{2}$$

where

$$\langle (j_1 j_2^{-1})1^+|\sigma_0\tau_{+1}|0\rangle^2=\frac{16}{3}\frac{l_F(l_F+1)}{2l_F+1}\ . \tag{3}$$

In Eq. (2) the energy of GT state, ϵ, is measured in the mother (Z,N) system. Thus, when ϵ is smaller than the Coulomb energy difference (minus some energy correction expressed by α),

$$\begin{aligned} D &\equiv E_{\mathrm{Coul}}(Z,N)-E_{\mathrm{Coul}}(Z-1,N+1)-\alpha \\ &\approx(0.70)A^{-1/3}\{2Z-1 \\ &\qquad -0.76[Z^{4/3}-(Z-1)^{4/3}]\}-\alpha\ , \end{aligned} \tag{4}$$

the beta decay to the collective $\sigma\tau_+$ state (i.e., the GTGR state) becomes energetically possible. The quantity α in (4), which is expected to be a few MeV in the present examples, could come partly from the difference of the isospins as well as the configurations of the (Z,N) and the $(Z-1,N+1)$ nuclei and partly from a correction due to the fact that the properties of the spin-orbit partners with

$l_F = N_F$ are approximated by the A dependence obtained from the harmonic oscillator model. For $N_F \gg 1$ we obtain

$$\epsilon \sim \epsilon_i + cA^{-2/3}, \quad \text{where } c>0 ,$$

$$D \sim A^{2/3} .$$

Thus, it is seen that the relation $\epsilon < D$ can be obtained for sufficiently large A values. In Fig. 2 the quantities ϵ in (2) (expressed by a dash-dotted line) and D in (4) (by a solid line) are plotted as a function of the mass number $A=2Z=2N$, where $\epsilon_i = 5$ MeV and $\alpha = 2.5$ MeV are used. The magnitude of α was chosen so that the result for ^{56}Ni and ^{100}Sn was approximately in agreement with our HF+TDA calculations.

Next, we present the amplitudes of the $T=1$ component admixed into the $T=0$ ground state of $N=Z$ even-even nuclei, which are estimated in our HF+TDA (or HF+RPA) calculations using Skyrme-type interactions. In the literature the isospin-mixing amplitude has been discussed a lot for years. In Fig. 3 we plot our calculated probabilities of the isospin mixture. The probabilities are obtained from the calculated sum of reduced probabilities of the Fermi transitions of $N=Z$ even-even nuclei, assuming that the admixture of the $T>1$ component into the ground state of the $N=Z$ nuclei can be neglected. Since the main $T=0$ component cannot make a contribution to any Fermi transitions to the neighboring $(Z \mp 1, N \pm 1)$ nuclei which have $|T_Z| = 1$, the whole Fermi strength has to come from small isospin-impurity components with $T>0$. When RPA is used instead of TDA in our calculation, the calculated probabilities of the isospin mixing increase by 15–20% for the nuclei $^{56}_{28}$Ni$_{28}$-$^{100}_{50}$Sn$_{50}$. It is seen that our calculated isospin-mixed probabilities are a factor of 2–3 larger than the

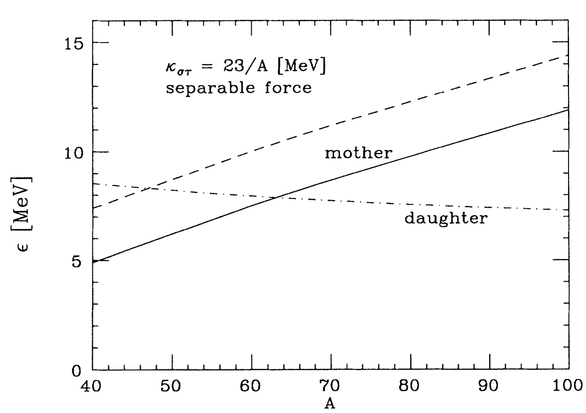

FIG. 2. Energies of GT giant resonance of $N=Z=A/2$ nuclei which are calculated using a schematic model [7,8]. The dashed (solid) line expresses the energy of mother nucleus without (with) the correction term α in Eq. (4), while the dash-dotted line corresponds to the GTGR energy of daughter nucleus calculated by the schematic model. When the dash-dotted line lies below the solid line, the GTGR state of daughter nucleus appears energetically lower than the ground state of mother nucleus. Namely, in the present model the $N=Z$ nuclei with $A \geq 64$ are found to decay to the GTGR state of the $(N+1, Z-1)$ nuclei by "superallowed" beta transition.

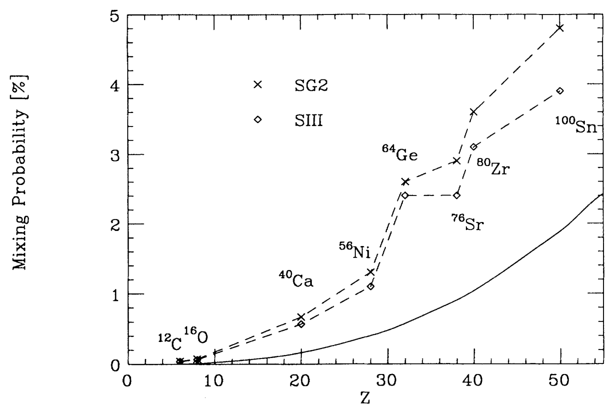

FIG. 3. The isospin-mixing probabilities calculated by the HF+TDA approximation with the Skyrme interactions, SIII (open diamonds) and SG2 (crosses), for $N=Z$ even-even nuclei. The solid curve is taken from Ref. [9].

values, which are estimated by Bohr and Mottelson [9] taking into account the polarization by isovector monopole mode.

We have also examined the effect of isospin mixing on the $0^+ \to 0^+$ Fermi transition between isobaric analog states (IAS) having two nucleons in addition to a "core" with $N=Z$ and $T=0$ (see Ref. [9]). An example is the Fermi decay from $^{78}_{40}$Zr$_{38}$ $(T=1)$ to $^{78}_{39}$Y$_{39}$ (IAS with $T=1$). The idea is that the possible isospin impurity would reduce the Fermi transition matrix element. Now, there is a sum rule on the difference between the T_- and the T_+ transition strengths,

$$S_+ - S_- = \langle N,Z | [T_-, T_+] | N,Z \rangle = Z - N . \quad (5)$$

A small isospin impurity does not only reduce slightly the T_+ transition matrix element to the IAS but also makes nonzero contributions to S_-, which vanishes for the case of a good isospin. Thus, since the magnitude of S_+ becomes slightly larger than $(Z-N)$ due to the isospin impurity, the T_+ transition strength to IAS could end up with a value approximately equal to $(Z-N)$, after including a small reduction from the S_+ value due to the isospin impurity. Therefore, it seems difficult to obtain isospin-mixing amplitudes from absolute magnitudes of measured strength of the Fermi transition between isobaric analog states.

When the isospin impurity is estimated in the literature [10], the overlap between the neutron- and the proton-radial wave functions $R^{p,n}_{nlj}(r)$ of the orbits with the same quantum number is often discussed as a key quantity of the impurity. Therefore, in Table II we tabulate values of the quantities,

$$C = 1 - \int R^p_{nlj}(r) R^n_{nlj}(r) r^2 dr , \quad (6)$$

for $^{14}_{8}$O$_6$ and $^{10}_{50}$Sn$_{50}$ which are obtained from the overlap estimated by using the HF wave functions with SG2 and SIII interactions. We notice that the values C in ^{14}O and ^{100}Sn are comparable for the one-particle orbitals around the Fermi surface, while the calculated isospin-mixing probabilities are almost two orders of magnitude

TABLE II. Values of C in Eq. (6) for ${}^{14}_{8}O_6$ and ${}^{100}_{50}Sn_{50}$, which are obtained from the overlap estimated by using HF wave functions with SG2 and SIII interactions. We note that the contribution from the exchange part of the Coulomb potential reduces the C values about 15% in all single-particle orbitals in ^{14}O, while in the case of ^{100}Sn it depends very much on orbitals, namely, from 2% for the $1s_{1/2}$ orbital to 15% for the $1g_{9/2}$ orbital.

^{14}O	SG2	SIII
$1s_{1/2}$	2.5×10^{-4}	2.5×10^{-4}
$1p_{3/2}$	1.5×10^{-3}	1.8×10^{-3}
$1p_{1/2}$	3.3×10^{-3}	4.6×10^{-3}
^{100}Sn	SG2	SIII
$1s_{1/2}$	6.6×10^{-4}	6.0×10^{-4}
$1p_{3/2}$	6.5×10^{-4}	5.5×10^{-4}
$1p_{1/2}$	6.0×10^{-4}	5.1×10^{-4}
$1d_{5/2}$	6.7×10^{-4}	5.5×10^{-4}
$1d_{3/2}$	6.6×10^{-4}	5.3×10^{-4}
$2s_{1/2}$	1.9×10^{-3}	1.6×10^{-3}
$1f_{7/2}$	7.9×10^{-4}	6.2×10^{-4}
$1f_{5/2}$	9.3×10^{-4}	7.4×10^{-4}
$2p_{3/2}$	3.0×10^{-3}	2.5×10^{-3}
$2p_{1/2}$	3.2×10^{-3}	2.7×10^{-3}
$1g_{9/2}$	1.2×10^{-3}	9.1×10^{-4}

different. The values of C, which are essentially determined by the difference of the potentials for neutrons and protons, are larger for one-particle orbitals with more nodes in the radial wave functions. In contrast, the magnitude of isospin mixture is governed by the proton number as well as the magnitude of neutron excess.

Since electric dipole ($E1$) gamma-ray emission is pure isovector, in $N=Z$ nuclei $E1$ decays from $T=0$ initial states must go to $T=1$ final states and vice versa. Thus, for example, statistical giant dipole resonance (GDR) decay is expected to be a sensitive probe of isospin purity in $N=Z$ nuclei [11]. An analysis of the gamma-ray spectra from ${}^{28}_{14}Si_{14}$ suggests a relatively large isospin impurity. Though the temperature dependence of the isospin impurity has to be properly taken into account in interpreting such data, this is an example which shows that actual magnitude of isospin mixing in $N=Z$ nuclei is an interesting issue to understand various phenomena in nuclear physics.

In conclusion, we have shown the following: (i) The $N=Z$ nuclei heavier than ^{56}Ni may beta decay to the GTGR state of the daughter $(Z-1,N+1)$ nuclei. Such "superallowed" GT beta decay may be possible also for some $N<Z$ nuclei if those nuclei lie inside the proton drip line, or for some $N>Z$ proton-drip-line nuclei if the proton number is as large as 50. (ii) Our estimated values of the $T=1$ component admixed into the ground $T=0$ state of $N=Z$ even-even nuclei are a factor of 2–3 larger than the values obtained in an estimate based on the hydrodynamical model by Bohr and Mottelson [9]. We point out that it may not be easy to extract the magnitude of isospin impurity from the absolute magnitude of the measured Fermi transition strength between isobaric analog states.

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