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Full three-body calculation for $\vec{d}+p \rightarrow {}^3\text{He}+\gamma$ with a realistic NN interaction

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A full three-body calculation for the tensor analyzing powers of ${}^1\text{H}(\vec{d},\gamma){}^3\text{He}$ over the energy range of available data is presented. The Paris two-nucleon interaction is used in the three-body equations and the electromagnetic transition is calculated with the Siegert $E1$ operator. Reasonable agreement with all the data is obtained only when full three-body dynamics and the nucleon-nucleon P-wave interactions are present in the initial state.

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Radiative-capture or photonuclear reactions are very useful for the purpose of learning about the D-state properties of bound few-body systems [1]. This aspect of radiative-capture reactions derives from the fact that the electromagnetic operator for a given multipole is selective in connecting specific channels (partial waves) from the continuum to specific components of the ground-state wave function. The nature of the ground state emanates mainly from the nature of the nucleon-nucleon (NN) interaction; for example, the spin dependence of the NN interaction leads to the spatially mixed-symmetry component in the ^{3}He wave function and the NN tensor force is the source of the D-state component in the wave functions of ^{2}H, ^{3}H, ^{3}He, and ^{4}He. It is possible that even three-nucleon forces are required to properly account for the binding energies of ^{3}H, ^{3}He, and ^{4}He [2,3] and will modify their wave functions accordingly. Of course, the same underlying interactions are also present in the continuum. However, given the large number of partial waves that are needed to calculate observables, their impact on the physics of the continuum is more diffuse compared to the ground state; thereby, photonuclear reactions allow us to be selective in learning about ground-state properties.

The observables that provide information on the D-state structure of ^{3}He are the tensor analyzing powers (TAPs), T_{2q} and A_{yy}, for example [1]. Measurement of these quantities permits us to ascertain the actual presence of the D-state component of the wave function. Beyond its presence, however, we aim to learn abut the structure of the three-body D state, or equivalently, the role of the NN tensor force in the structure of the D state. Besides this latter central goal, we also strive to understand the role of three-nucleon dynamics in the initial state of the capture process and to understand the nature of the electromagnetic transition, i.e., are there dominant multipoles? Now is an excellent time to pursue these objectives because data are available for the T_{pq} at 10 MeV incident deuteron energy [4], T_{20} at 19.8 MeV [5], A_{yy} at 29.2 [6], 45.3 [6], and 95 Mev [7]. Some theoretical work involving complete $3N$ solutions for both the continuum initial state and the bound final state have been carried out. Torre completed a full configuration-space $3N$ calculation with the Reid soft-core (RSC) NN interaction [8] specifically to compare with the data of Ref. [6]. The result of this comparison at $E_d = 29.2$ MeV indicated that the RSC-calculated D-state component of the triton wave function is too large by about 20%, perhaps implying that one should use NN interactions that predict smaller D-state probabilities. Ishikawa and Sasakawa [9] have also considered A_{yy} at $E_d = 29.2$ MeV with full three-nucleon wave functions derived from various realistic NN potentials and the two-pion-exchange $3N$ potential. They found their results to be sensitive to the particular choice of NN interaction and argue that the comparison with the one high-precision data point at $E_d = 29.2$ MeV requires the presence of the three-nucleon force. The present authors also presented a numerically exact three-body calculation of ${}^1\text{H}(\vec{d},\gamma){}^3\text{He}$ using simple rank-1 NN interactions chosen to represent well the low-energy data [10]. Our focus was mainly on T_{20} at $E_d = 19.8$ MeV in the first stage of the work in order to understand the dynamical mechanism of the process. It is clear from this work that good agreement with the data for T_{20} requires full treatment of the initial state. Also, the critical role of the NN P waves was emphasized in the second report on the calculations [11].

The purpose of this paper is to present the results of a full Faddeev calculation for radiative capture of polarized deuterons on protons over the whole energy range of available data for one representative "realistic" NN interaction (Paris [12]). The Paris interaction is handled by the Ernst-Shakin-Thaler (EST) separable-expansion method [13] in order to reduce the two-variable integral equations to a single variable. The electromagnetic operator is limited to $E1$, but this is not a serious limitation in that the measurements fall in the domain where the $E1$ operator dominates for the TAPs.

The separable expansion method proposed by EST for representing a given NN interaction is very reliable in both three-nucleon bound [14] and continuum calculations [15]. Calculation of the ^{3}H binding energy from the

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$^1S_0$ and $^3S_1$-$^3D_1$ *NN* partial waves of the Paris potential by means of the EST expansion gives precisely the value obtained directly from the Paris potential and, at the same time, the numerical quality of the ^{3}H wave function is excellent as measured by the values obtained for the wave-function component percentages plus the *S*- and *D*-wave asymptotic normalization constants [14]. Furthermore, predictions of the Paris interaction of three-nucleon continuum observables by the EST approach and the direct use of the Paris potential yield practically identical results to within an accuracy of 1–2% [15]. These outcomes form the basis for our use of the EST expansion in the present work, where, as a result, our concern need only be the issue of convergence with the number of terms in the expansion. Separable expansions of local interactions by methods like those of EST appear to carry all the essential features of the underlying interaction that are required in a three-nucleon calculation.

We use the Siegert operator in the long-wavelength limit for the $E1$ transition which gives the dominant part of the $E1$ contribution without explicit knowledge of the nuclear current density. The choice of the Siegert operator corresponds, in general, to a gauge transformation of the electric multipole fields, but in the long-wavelength limit the issue of a gauge choice does not enter. Siegert's hypothesis is that the two-body-exchange charge density vanishes in the nonrelativistic limit (which is the case for explicit meson-exchange models in the static limit), so explicit meson-exchange currents must be considered only if the two-nucleon interaction does not commute with the one-body charge density. Two-body exchange currents are determined uniquely only through a specific dynamical model that links them to the underlying two-nucleon interaction. However, Siegert's hypothesis constrains only the longitudinal part of the two-body current; arbitrary transverse currents can be added without destroying the gauge condition. Nevertheless, the Siegert operator includes the exchange-current contribution since it is obtained through the current conservation equation for the full current which involves the commutator of the Hamiltonian with the charge density; i.e., the interaction part of the commutator is included.

In this work, the *NN* interaction is given by the Paris potential present in the $^1S_0$, $^3S_1$-$^3D_1$, $^1P_1$, $^3P_0$, $^3P_1$, and $^3P_2$ partial waves. The ground state of ^{3}He has all possible L-S configurations: the spatially symmetric and spatially mixed-symmetric $^2S_{1/2}$ components, the $^2P_{1/2}$ and $^4P_{1/2}$ components, and the $^4D_{1/2}$ component. All these components are present even in the absence of the *NN P* waves. The nucleon-deuteron (*Nd*) initial state is connected to these ground-state components through the $E1$ operator and as a consequence the continuum states have all possible orbital angular momentum values consistent with total angular momentum and parity of $J=\frac{1}{2}^-$ and $\frac{3}{2}^-$. Though the role of the *NN P* waves in the ^{3}He ground state is minor, their presence in the continuum dynamics is important [11]. In the present nonrelativistic framework, the deuteron contains no *NN P*-wave interactions, and the *P*- and *F*-wave relative motions between the nucleon and deuteron are present even in the absence of the *NN P*-wave interactions. However, the *NN P* waves play the important role of permitting the *n*-*d* system to make off-shell transitions to states of a nucleon plus correlated *NN P*-wave pair which then connect through the $E1$ operator (odd parity) to the dominant wave-function components of the ^{3}He ground state whose existence is not associated with the presence or absence of the *NN P* waves [11]. Therefore, the information contained in the ^{3}He wave function can only be extracted through the isospin breaking $E1$ operator if both even and odd parity *NN* partial waves are present in the continuum.

In Fig. 1, we show the calculated vector and tensor analyzing powers at $E_d=10$ MeV compared to the data of Ref. [4]. This is the only energy where all the T_{pq}'s have been measured. The good agreement with the T_{2q} data emphasizes the dominance of the $E1$ process in the TAPs which substantiates the conclusions reached in the analysis of the data [4]. However, the comparison for iT_{11} makes clear the need for small contributions from other multipoles, for example, most likely $M1$, while the asymmetrical nature of $\sigma(\theta)$ (not shown) about 90° argues for the presence of the $E2$ multipole as well. The most striking outcome of this comparison is that the *NN P* waves play a significant role even at this low energy. This can be seen by comparing the solid curve labeled 342222 for the number of terms in the EST expansion of each *NN* partial wave, i.e., $^1S_0$ (3), $^3S_1$-$^3D_1$ (4), $^1P_1$ (2), $^3P_0$ (2), $^3P_1$(2), and $^3P_2$ (2), respectively, with the short-dashed curve labeled 34 which results from the calculation

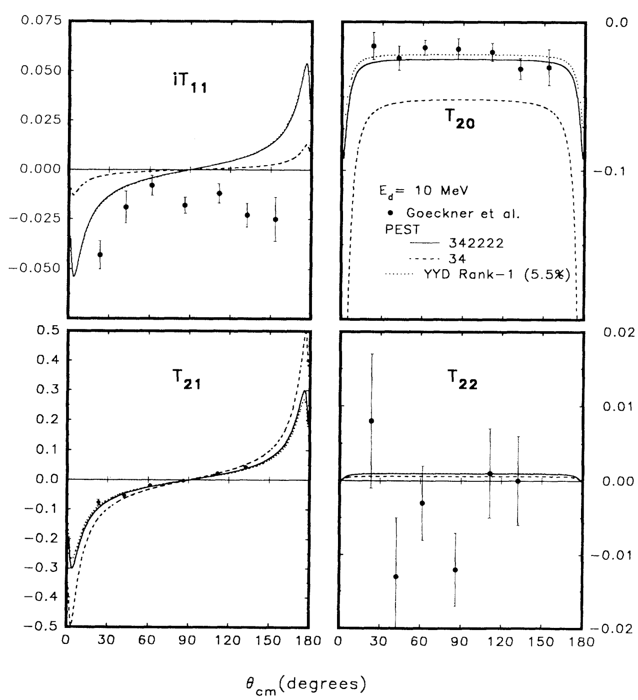

FIG. 1. Vector and tensor analyzing powers for ^{1}H($\vec{d},\gamma$)^{3}He at $E_d=10$ MeV. All theoretical curves correspond to full three-body calculations: (1) Paris *NN* interaction by means of the EST expansion (PEST) with (342222) and without (34) the *NN P* waves present; (2) Simple rank-1 separable *NN* interactions including *NN P* waves. The experimental data are from Ref. [4].

without the NN P waves. The presence or absence of the NN P waves for T_{20} and T_{21} means the difference between agreement or disagreement with the experimental data. The reason for this sensitivity to the NN P waves is that without the P waves in the three-body continuum state, the ground state does not yield its full content through the $E1$ operator. We have already determined that the role of the NN P waves in the ground state itself is much less significant, almost negligible [11]. Finally, we point out that simple rank-1 forms for the NN interaction, for example, Yamaguchi for ${}^1S_0$, Yamaguchi-Yamaguchi for ${}^3S_1$-${}^3D_1$ (5.5% D state in d) [16], and Doleschall for ${}^1P_1$, ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$ [17], give results very close to those of the Paris interaction, thus justifying the use of simple rank-1 interactions to extract the key dynamical aspects as we did in our earlier work [10,11].

Figure 2 displays the data at $E_d=19.8$ MeV [5] with our results at 19.6 MeV. Unlike at 10 MeV, the effect of the NN P waves is much smaller. If one sets up a graph of T_{20} (or A_{yy}) as a function of deuteron energy, with and without NN P waves present, one finds that both curves intersect at about 21 MeV, just a few MeV beyond the giant-dipole resonance peak of $\sigma(\theta=90°)$. From this behavior, it is apparent that the NN P waves play a more significant role from threshold up to about 15 MeV and beyond 25 MeV where the NN P waves gain importance as the energy increases. Also from Fig. 2, we see the convergence of the calculation with the rank of the EST expansion for the ${}^1S_0$ and ${}^3S_1$-${}^3D_1$ NN partial waves. In addition, the calculation is in good agreement with the data over the bulk of the angular range, once again emphasizing the dominance of the $E1$ radiation at this energy. At the extreme forward and backward angles, we expect the $M1$ multipole to play a role.

As mentioned above, convergence of our calculations would be checked to assure stability of the results with

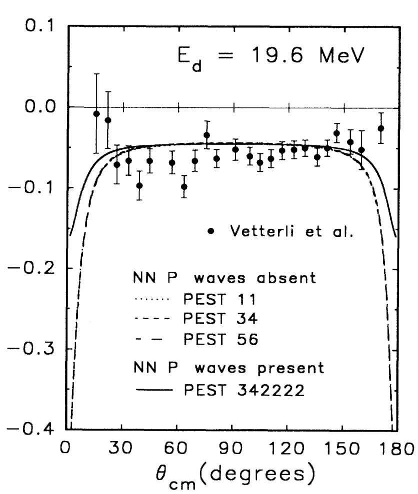

FIG. 2. T_{20} tensor analyzing power for ${}^1\mathrm{H}(\vec{d},\gamma)^3\mathrm{He}$ at $E_d=19.6$ MeV. Theoretical curves correspond to full three-body calculations with the Paris NN interaction by means of the EST expansion. Convergence of the calculations with the number of terms in the EST expansion for the ${}^1S_0/{}^3S_1$-${}^3D_1$ NN interactions are shown. The experimental data are from Ref. [5] at $E_d=19.8$ MeV.

the rank of the EST expansion. In Table I, we show various calculations for A_{yy} at $E_d=29.2$ MeV. The calculations with increasing rank in the NN P waves are given in the rows, and the increasing rank of the ${}^1S_0/{}^3S_1$-${}^3D_1$ NN interactions is given in the columns. What is clearly evident is the convergence of the P waves with rank and the convergence or stability of the results in general, thus once again showing the power of the EST method. At this specific energy, there is a high precision data point at $\theta=96.75°$: $A_{yy}=0.0282\pm0.0016$ [6]. Our theoretical value is $A_{yy}=0.0294$ for rank 563333. This excellent agreement is in contrast to the results of Ishikawa and Sasakawa [9] where they found all the NN interactions considered to yield values for A_{yy} that were too small by at least 15%. Both Ishikawa [9] and Torre [6] have considered the Reid soft-core interaction, but do not agree on the results. Ishikawa obtains $A_{yy}=0.0216$, a value that is $\sim$25% lower than the experimental result, while Torre's calculation is $\sim$20% too large. At $E_d=45.3$ MeV, there is also a high precision measurement of A_{yy} at $\theta=98.4°$ [6]: 0.0113 ± 0.0014. We obtain $A_{yy}=0.0188$ for rank 342222, perhaps the first indication in a TAP of the need for more than just $E1$ radiation. However, it should be noted that $A_{yy}(\theta\sim 90°)$ as a function of incident deuteron energy goes through zero in the region around 50 MeV, thus making this energy region a particularly sensitive test of theoretical predictions.

Finally, in Fig. 3, we show our results compared to the data at the highest energy available, $E_d=95$ MeV [7]. Here, we see that A_{yy} has changed sign relative to the results at 29.2 and 45.3 MeV, emphasizing the zero in $A_{yy}(\theta=90°)$ in the region of 50 MeV. In this graph, we compare several theoretical results. First, we consider a calculation where the interaction of the deuteron and nucleon in the initial state is neglected, i.e., no initial-state rescattering (Born). The Born results have the wrong sign and are far removed from the experimental data. Moreover, since the deuteron gets no contribution from the NN P waves, one can see from the Born curves the small role played by the NN P waves in the three-body ground state (the Born 34 and 342222 curves are essentially identical). When the initial-state rescattering is present, the theoretical A_{yy} turns negative in the region of the experimental data and the presence of the NN P waves becomes important for the reasons mentioned above. Initial-state rescattering is very important in A_{yy} even though it is a negligible in $\sigma(\theta)$ [10]. The full 342222 result is somewhat more positive than the data, indicating the importance of the $E2$ multipole as the excitation energy rises in the initial-state continuum. This is supported by the asymmetrical form of $\sigma(\theta)$ at this en-

TABLE I. Convergence with NN rank for $10^2A_{yy}(90°)$ at $E_d=29.2$ MeV.

		${}^1P_0\,{}^3P_0\,{}^3P_1\,{}^3P_2$ rank		
${}^1S_0/{}^3S_1$-${}^3D_1$ rank	0000	1111	2222	3333
11	1.874	3.492	2.990	2.971
34	1.866	3.418	2.897	2.879
56	1.947	3.468	2.947	2.929

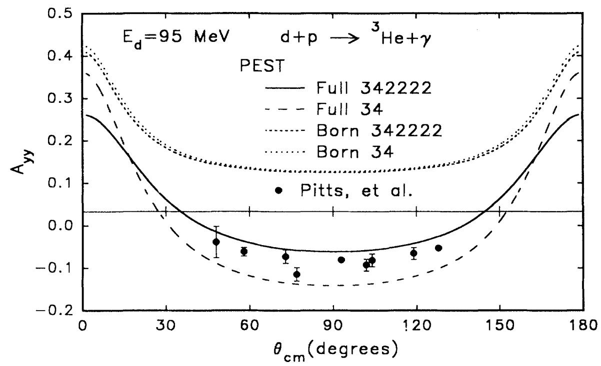

FIG. 3. A_{yy} tensor analyzing power for $^1H(\vec{d},\gamma)^3He$ at $E_d = 95$ MeV. Theoretical curves correspond to full three-body calculations with the Paris NN interaction by means of the EST expansion, except for the Born curves where the initial-state rescattering between the incident deuteron and proton target is absent. The experimental data are from Ref. [7].

ergy (not shown) [7]. Inclusion of the $E2$ multipole most likely will lead to the correct asymmetry in $\sigma(\theta)$ which in effect enters in the denominators of the TAPs. Thus, this effect combined with any new interference in the numerator of the expressions for the TAPs from the presence of both $E1$ and $E2$ radiation may account for a somewhat more negative A_{yy} as seen in the data.

What becomes clear from this overview of radiative capture data over a fairly wide energy range is that precision data, both statistically and systematically, will be needed for discrimination of theory versus experiment. This is easily seen by comparing the data at $E_d = 10$, 19.8, and 95 MeV with those from 29.2 and 45.3 MeV. Furthermore, when we compare the simple rank-1 separable-interaction results with the Paris interaction results, it appears that it may be difficult to distinguish between different NN interactions. This raises the issue as to whether there really is sensitivity to the different NN interactions in the TAPs? However, one should keep in mind the conclusions of Ishikawa and Sasakawa.

In conclusion, we can say that up to $E_d \sim 30$ MeV, the T_{2q} and A_{yy} data can be well explained with a theory that involves only $E1$ radiation, NN partial waves through P waves, and the NN interaction obtained from the Paris potential. However, above ~ 30 MeV, it appears that higher multipoles (at least $E2$) must be added to the theory. Moreover, it is evident that the NN P waves have different significance in different energy regions. Though the Paris interaction does a reasonable job of describing the TAPs over the full energy range, the simple rank-1 separable models do reasonably well also. However, we must keep in mind that $\sigma(\theta)$ and iT_{11} generally are not reproduced by the present $E1$ theory. At this stage, we take this latter disagreement to originate from the limitation to $E1$ radiation as opposed to a shortcoming of the Paris interaction. Most importantly, the question naturally arises as to whether there is a lack of sensitivity to the NN interaction, e.g., does the fact that the TAPs are ratios somehow dampen any differences? Clearly, these latter issues indicate the importance of extending the present calculations to include at least the $E2$ multipole and possibly the $M1$ multipole.

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