PHYSICAL REVIEW C

NUCLEAR PHYSICS

THIRD SERIES, VOLUME 48, NUMBER 6 DECEMBER 1993

RAPID COMMUNICATIONS

The Rapid Communications section is intended for the accelerated publication of important new results. Manuscripts submitted to this section are given priority in handling in the editorial office and in production. A Rapid Communication in **Physical Review C** *may be no longer than five printed pages and must be accompanied by an abstract. Page proofs are sent to authors.*

Statistical multistep direct reaction

H. Feshbach

Center for Theoretical Physics, Laboratory for Nuclear Science, and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 23 July 1993)

A difficulty in the derivation of the cross section for statistical multistep direct reactions is resolved.

PACS number(s): 24.60.Gv, 24.50.+g, 24.10.−i

For many years nuclear reactions were analyzed using two extreme models. One involved a long interaction time that led to the formation of the compound nucleus. The other involved short interaction times such as the single interaction model DWBA which was used to describe the direct reactions. Eventually these were found to be inadequate. Emission into the final state before the formation of the compound nucleus was found to occur. The single step process was found in many cases not to be adequate for the description of direct reactions. A semiclassical theory to describe the precompound formation of the final state was developed [1] while coupled channels were introduced to describe additional interactions for the direct process [2]. A statistical approach [3] to the precompound situation was presented to the Munich conference in 1973. In 1980 a paper was published on a statistical approach to both types of reactions by Feshbach, Kerman, and Koonin [4] (FKK). The analysis was called the statistical multistep theory. It had two components, the statistical multistep compound theory which described the precompound process as well as the formation of the compound nucleus. The second, the statistical multistep direct theory, is the subject of this Rapid Communication. Both of these theories have been successfully used in the analysis of experiment. A review of the first is given by Bonetti *et al.* [5]. A review of the second can be found in the *Workshop on Multistep Direct Reactions*, edited by Lemmer [6].

The theory for statistical multistep direct reactions developed by FKK had an error which was pointed out by Kawai [7] and by Tamura *et al.* [8]. The error was remedied in 1985 [9] with the result that the final FKK expression for this process was unchanged so that the analysis of several experiments where it had been successfully used remained valid. However, for reasons which are not clear to this writer, the derivation of Ref. [9] was not accepted by many, although no valid criticism of the derivation was ever presented to me. In this paper, I shall develop another proof which is simpler and hopefully more acceptable.

The issue is the spectral decomposition of a Green's function $1/E^{(+)} - H$ where H is not Hermitian. The derivation in FKK takes into account only the δ function component. We are therefore concerned with the expression

$$M = v\,\delta(E-H)\,v \tag{1}$$

where v is the transition interaction. Instead of expanding $\delta(E-H)$ in the eigenfunctions of H, we introduce the eigenfunctions ψ_α and $\tilde{\psi}_\alpha$ of the energy average of H, $\bar{H}$, as follows:

$$M = \sum_{\alpha,\beta} v\,\psi_\alpha^{(+)}\rangle\langle\tilde{\psi}_\alpha^{(+)}\,\delta(E-H)\,\tilde{\psi}_\beta^{(-)}\rangle\langle\psi_\beta^{(-)}\,v. \tag{2}$$

The function $\tilde{\psi}_\beta^{(-)}$ satisfies

$$\tilde{\psi}_\beta^{(-)} = \phi_\beta + \frac{1}{E-\bar{H}-i\epsilon}V\phi_\beta, \tag{3}$$

where $\psi_\beta^{(+)}$ satisfies

$$\psi_\beta^{(+)} = \phi_\beta + \frac{1}{E-\bar{H}+i\epsilon}V\phi_\beta. \tag{4}$$

We note that $\sum_\alpha \tilde{\psi}^{(-)}\rangle\langle\psi_\alpha^{(-)}$ and $\sum_\alpha \psi_\alpha^{(+)}\rangle\langle\tilde{\psi}_\alpha^{(+)}$ are unit operators. From Eqs. (3) and (4) one obtains

$$\tilde{\psi}_\beta^{(-)} = \psi_\beta^{(+)} + 2\pi i\delta(E-\bar{H})V\phi_\beta. \tag{5}$$

Since the energy spectra of H and $\bar{H}$ differ, because H and $\bar{H}$ are complex and $\bar{H}$ is the energy average of H, $\delta(E-H)\,\delta(E-\bar{H})$ is zero, and M becomes

$$M = \sum_{\alpha,\beta} v\psi_\alpha^{(+)}\rangle\langle\tilde{\psi}_\alpha^{(+)}\,\delta(E-H)\,\psi_\beta^{(+)}\rangle\langle\psi_\beta^{(-)}v. \tag{6}$$

Performing an energy average replaces H by $\bar{H}$ so that

$$\bar{M} = \sum_{\alpha,\beta} v\psi_\alpha^{(+)}\rangle\langle\tilde{\psi}_\alpha^{(+)}\,\delta(E-\bar{H})\,\psi_\beta^{(+)}\rangle\langle\psi_\beta^{(-)}v. \tag{7}$$

But

$$\langle\tilde{\psi}_\alpha^{(+)}, \psi_\beta^{(+)}\rangle = \delta_{\alpha\beta}$$

so that

$$\bar{M} = \sum_\alpha v\,\psi_\alpha^{(+)}\rangle\langle\tilde{\psi}_\alpha\,\delta(E-\bar{H})\,\psi_\alpha\rangle\langle\psi_\alpha^{(-)}v$$

or

$$\bar{M} = \sum_\alpha v\,\psi_\alpha^{(+)}\rangle\,\delta(E-E_\alpha)\,\langle\psi_\alpha^{(-)}\,v \tag{8}$$

where E_α is the eigenvalue of $\bar{H}$. This form is the one used by FKK, thus completing the derivation of the desired final result. The expression for the cross section obtained by FKK is thereby justified.

We note that the full Green's function could have been inserted in place of $\delta(E-H)$ in Eq. (1). The analysis which leads to Eq. (8) can still be carried out with the result that $\delta(E-E_\alpha)$ is replaced by $(1/E-E_\alpha+i\epsilon)$.

This work was supported in part by funds provided by the U. S. Department of Energy (DOE) under Contract No. DE-AC02-76ER03069.

[1] For example, see *Intermediate Processes in Nuclear Reactions*, edited by N. Cindro, P. Kalešić, and T. Mayer-Kuckuk (Springer-Verlag, Heidelberg, 1972).
[2] For example, see G. R. Satchler, *Direct Nuclear Reactions* (Oxford University Press, New York, 1983).
[3] H. Feshbach, in *Proceedings of the International Conference in Nuclear Physics, Munich, 1973*, edited by J. de Boer and H. J. Mang (North-Holland, Amsterdam, 1973).
[4] H. Feshbach, A. K. Kerman, and S. Koonin, Ann. Phys. (N.Y.) **125**, 429 (1980).
[5] R. Bonetti, M. B. Chadwick, P. E. Hodgson, B. V. Carlson, and M. S. Hussein, Phys. Rep. **202**, 171 (1991).
[6] *Workshop on Multistep Direct Reactions*, edited by R. H. Lemmer (World Scientific, Singapore, 1992).
[7] M. Kawai, private communication.
[8] T. Tamura, T. Udagawa, and H. Lenske, Phys. Rev. C **26**, 379 (1982).
[9] H. Feshbach, Ann. Phys. (N.Y.) **159**, 1650 (1985).