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$d\sigma/dE_t$ in heavy ion collisions and nucleon-nucleon cross section fluctuations

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The prediction of transverse energy distributions, $d\sigma/dE_t$, in heavy ion collisions is examined and extended and the evidence for the effect of anomalous cross section fluctuations on the cross section at high transverse energy is critically examined. We calculate the contribution to the transverse energy of rescattering of target nucleons on their unstruck neighbors.

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The prediction of $d\sigma/dE_t$ in heavy ion collisions is at present not well understood because there are secondary contributions to the E_t from two sources that are difficult to calculate reliably. They are (a) the rescattering of struck nucleons on nucleons in the same nucleus and (b) the scattering of the produced final state particles (pions, etc.) themselves within the nucleus. (This latter effect is especially difficult to estimate since the time evolution to on-shell final states is not well understood.) There are also uncertainties in the experimental energy scales in E_t calorimeters as well as uncertainties arising from the lack of p-p data in the same calorimeters. Nevertheless, a great deal has been learned from the ISR measurements comparing light nuclei and p-p data studying both multiplicity and E_t distributions. One goal of this paper is to review and extend the calculations of $d\sigma/dE_t$ to include an effect of rescattering on heavy ion collisions.

Examination of 200 GeV p-Pb collisions has led Baym *et al.* [1] to claim there are anomalous fluctuation effects showing up in the high E_t part of the spectrum. More recently Heiselberg *et al.* [2] claim that there is a discrepancy between conventional theoretical expectations of the observed E_t distributions in nucleus-nucleus interactions which can be explained by assuming that the nucleon size, and hence the nucleon-nucleon cross section, fluctuates about its mean value. If this were true, large nucleons would make more scatters in passing through a nucleus than an average nucleon and this effect might dominate over the cases where small nucleons make fewer scatters. This would distort the distribution of the coefficients, b_n, which are the relative probabilities that there are n scatters of nucleons in projectile or target. In their calculations they assume that the expected nuclear E_t distribution is the n-fold convolution of the E_t distribution measured in a free nucleon-nucleon collision, weighted with the b_n's.

In this Rapid Communication we examine these claims by making detailed calculations of the E_t spectra. First, we discuss what is known about E_t spectra from collisions of very light nuclei. We then discuss the rescattering effects that are expected to become important in heavy nuclei. Specifically we calculate the contribution to E_t in a p-Pb collision from the rescattering of struck nucleons in the Pb on spectator nucleons, those that were not struck by the incoming proton. We then show the effect on the full E_t distribution of including fluctuations in the nucleon-nucleon cross section. By calculating the scaled invariance from the spectra we demonstrate that the scaled invariance is a poor measure of fluctuation effects in observed transverse energy distributions. Finally, we suggest heavy ion experiments more likely to determine whether fluctuation effects are indeed real.

A fundamental new idea of relativistic nuclear physics, first proposed in 1977 [3], was that the number of scatter*ers* [4], and not the number of scatters, determines the multiplicity of particles produced in nuclear collisions. It is based on the idea that, at extreme relativistic energies, a proton making n collisions in a nucleus cannot produce n contributions to the event structure because the very strong time dilation prevents the incoming nucleon from hadronizing n times. Thus it was not the number of binary scatters but the number of scatter*ers*, or "wounded nucleons" (to use the nomenclature of Bialas *et al.* [3]), that determined the mean multiplicity in nuclear collisions. This idea was further extended [5] and tested with high precision in interactions of light nuclei at the CERN ISR by examining multiplicity *distributions*. The calculation involves deconvoluting the p-p $d\sigma/dE_t$ (or alternately multiplicity) distributions to get the contribution from a *single* scatter*er* and weighting n convolutions of this function with a_n, the probability of obtaining n scatter*ers* in the nuclear collision.

These analyses demonstrated that the multiplicity [5] and E_t *distributions* [6,7] were well fit by this requirement, verifying the importance of the role of the time dilation at high energies. The binary scattering model was tested and failed badly, seriously overestimating the multiplicity or transverse energy. These models were tested in light nuclei since in heavy nuclei two other sources of multiplicity or E_t enter, the rescattering of struck nucleons on nucleons in their own nucleus, producing more particles, and the scattering of the hadronization products before leaving the nuclear volume. It was no surprise [8] then that the convolution model underpredicted the available E_t in O-Pb collisions. The maximum observed transverse energy deposition exceeded the theoretical prediction [8] without rescatterings, by about 25% [9].

But, convoluting *scatters* alone (even neglecting rescattering) overpredicts the E_t distributions by a huge margin. To illustrate this effect for heavy nuclei we show, in Fig. 1(a), the central region NA35 O-Pb data and our calculation of $d\sigma/dE_t$, reproduced from our 1988 paper [8], based on convoluting scatter*ers* (wounded nucleons) using the p-p $d\sigma/dE_t$ taken from ISAJET minimum bias calculations. To this plot we have now added the calculation based on convoluting the number of binary scatters, b_n, with the p-p distribution, giving direct evidence for the large overprediction in the maximum E_t obtained by convoluting binary scatters [10].

Since convoluting scatters vastly overpredicts the energy deposition it cannot be a reliable starting point for the study of possible fluctuations in E_t, as in the model of Heiselberg *et al.* [2].

It is worth noticing two other effects in Fig. 1. As seen in Fig. 1(a) the *slope* of the central region NA35 data ($-0.82<\eta_{\mathrm{c.m.s.}}<0.75$) in the region of E_t where $d\sigma/dE_t$ falls rapidly is not so different than the normal scatter*er* prediction. On the other hand, as seen in Fig. 1(b), comparison of these data and the HELIOS data, which cover the Pb fragmentation region ($-3.1<\eta_{\mathrm{c.m.s.}}<-0.1$), shows that the slope of the HELIOS data falls off more slowly than the prediction. This effect could come about because of the rescattering of struck protons in the lead. Those excited recoiling hadrons in the Pb target moving with about only 20 GeV in the laboratory produce an extra contribution, mainly in the fragmentation region, as contrasted with the central rapidity region. And because the particles produced in these secondary low energy interactions are soft they easily rescatter in the nuclear volume, further building up the E_t.

To illustrate the effects of adding in the recoil contribution we have calculated the first effect and compare our calculation with the p-Pb data of the HELIOS Collaboration. Figure 2 shows the data and the prediction of $d\sigma/dE_t$ taken from Ref. [8]. In that work $d\sigma/dE_t$ was calculated using the ISAJET beam jets to separate the projectile and target contributions [11]. ISAJET also gives the energy imparted to the recoiling struck nucleons so it is possible to iterate the ISAJET calculation and add in the E_t produced when a struck nucleon recoils and produces E_t in a collision with one of its unstruck neighbors. The typical average recoil energies are about 20 GeV so that ISAJET gives a rough idea of the magnitude. Figure 2 shows the results of this new calculation, which, on an event by event basis, convolutes the primary and secondary contributions to E_t. We note that the E_t is increased, but not by a sufficient amount to account for the full E_t. This is to be expected since we have not included the buildup of E_t due to the scattering of those pions materializing within the nucleus from either the primary interaction or the secondary lower energy interaction.

The calculations also illustrate an important point that follows from convolutions. Adding the rescattering energy to very low energy primary events depletes the low energy region and causes a turnover in the low energy region which was not present without rescattering and which is also seen in the data.

We cannot know whether the hadronization process would provide full agreement with the shape of the high energy portion of the measured spectra, since those additional contributions are much too complicated to treat reliably.

However, it is the slow falloff of the data as compared with some model calculations which may have led to the interpretation of the slow falloff as an anomalous fluctuation effect showing up at high E_t in heavy ion reactions [11].

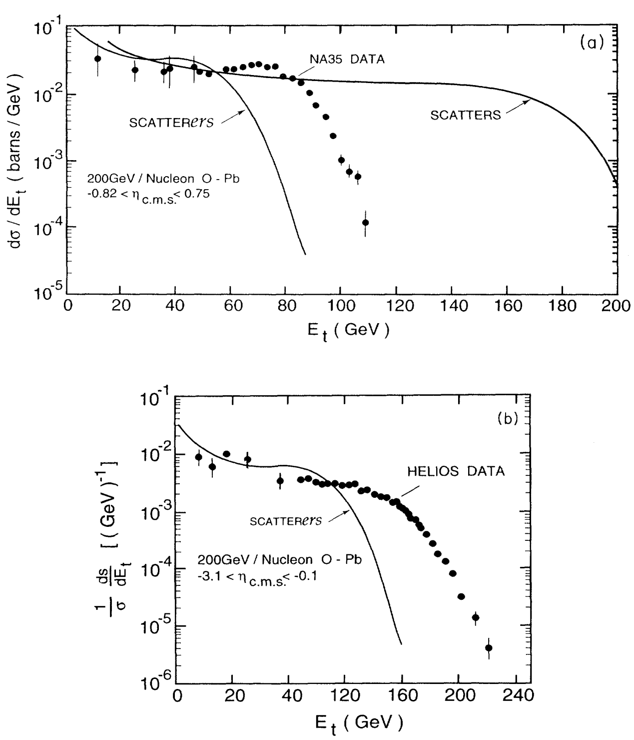

FIG. 1. (a) The NA35 central rapidity data for O-Pb collisions. Superimposed are the predicted cross section, neglecting rescattering and hadronization effects, for both models: scatter*ers* and scatters. (b) The NA34 data in the Pb fragmentation region and the theoretical prediction, without rescattering contributions. Note slow falloff of data at high E_t.

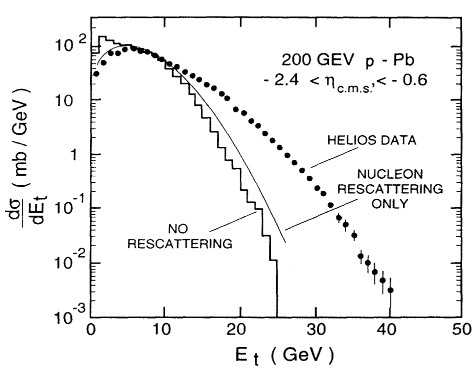

FIG. 2. $d\sigma/dE_t$ vs E_t for p-Pb collisions. The smooth curve (HELIOS) shows our calculation of the effect of adding rescattering to the basic calculation.

One can examine the effects of the fluctuations to see whether they contribute appreciably to the energy distribution using the correct (a_n) counting rule. To do this we have chosen a simple distribution of nucleon radii, namely, $P(r)=\alpha^2 re^{-\alpha r}$. This long tailed distribution was chosen to accentuate large radii. α is fixed by the nucleon-nucleon cross section and the radii of the projectile and target nucleons via the averaging: $\sigma_{p\text{-}p}=\pi\langle(r_p+r_t)^2\rangle$. For p-p inelastic interactions $\alpha=4.47$ for a 31.4 mb cross section. The nucleons of the incoming nucleus are chosen from the distribution and, as in Ref. [2], are frozen in size as they pass through the target, in keeping with the time dilation requirement. The nucleons in the target nucleus are also chosen from the same fluctuation distribution.

It is not necessary to actually carry out the convolutions to understand the physics, since the pertinent effects are easily seen *directly* from a calculation of the coefficients a_n and b_n which represent the number of scatterers and scatters, respectively. Our calculation of these coefficients is carried out in the usual way [5] by generating nucleons in the nucleus according to the Woods-Saxon spatial distributions, passing the incoming proton through the nucleus and noting the numbers of collisions determined by the measured p-p cross section.

Figure 3 shows plots of the a_n and b_n vs n, showing the comparison between fluctuations and no fluctuations. As seen directly from the coefficient distributions, the fluctuations will in fact make little change in the magnitude of the energy disposition but do change the rate of falloff at the largest values of n, slightly broadening the falloff in the coefficients. However, the effect is much smaller in the established scatter*er* counting scheme than in counting binary scatters. (This is easy to understand qualitatively: A large entering proton increases the number of scatters but affects the number of wounded nucleons less, since many nucleons would have already been struck by normal protons and being struck, in addition, by a larger proton would not affect the fact that it was already wounded.)

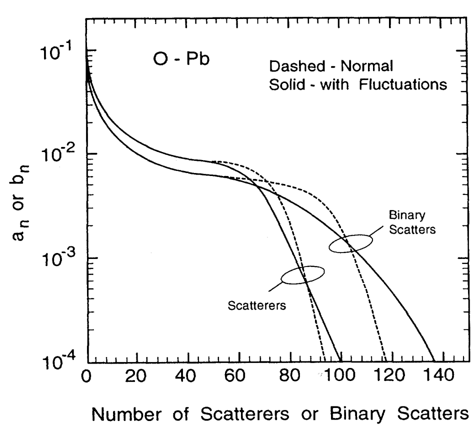

FIG. 3. a_n or b_n vs n, showing the effect of nucleon fluctuations on the coefficients a_n and b_n, the probabilities that there are n scatter*ers* or n scatters, in the case of O-Pb collisions. The solid lines (NA35) show the effect of introducing fluctuations, as described in the text.

Note that we have plotted the so-called "Glauber" coefficients in Fig. 3 to explain the physics in the simplest way. (One can get a good estimate of the E_t distributions by simply plotting $n\, a_n$ multiplied by the factor $\langle E_t\rangle$, the mean E_t per wounded nucleon. For the scatter theory the plot is $n\, b_n$ multiplied by $2\langle E_t\rangle$. We will discuss the $d\sigma/dE_t$ spectra below.)

There are other serious problems with computing effects of fluctuations on multiplicity or E_t distributions.

(1) The authors of Ref. [2] implicitly assume that large and small nucleons produce the same multiplicity, E_t, and rapidity distributions of the produced particles. Yet, we know that the color screening which would result from small configurations would change even the relative elastic or inelastic cross sections and might change the multiplicities as well. Thus there is a basic difficulty in implementing the fluctuation hypotheses. We need to know the event structure as a function of the radius (i.e., the p-p cross section) but have no clear *a priori* knowledge of it.

(2) If nucleons fluctuate in size, the generation of the nuclear spatial distributions should include the spatial correlations between nucleons. Unfortunately, we know nothing about spatial correlations between large and small nucleons. Note that by discriminating against close neighbors in the region of a large nucleon it is possible that the number of scatter*ers* could actually be *reduced* in central collisions which populate the high E_t region.

As we have pointed out earlier, the *shape* of a nuclear $d\sigma/dE_t$ spectrum contains the convolutions of many contributions to the transverse energy: the basic energy production in the nucleon-nucleon interaction, the reinteraction of struck nucleons with their neighbors, and the scattering of the hadronization products as they evolve over time. Such processes are not well understood and it is a challenge to attempt to understand the shapes of the spectra with models that simultaneously must account for other features of the interactions such as rapidity distributions and multiplicity distributions.

Reference [2] suggests that the presence of nucleon fluctuations might be revealed by studying a simple parameter of the data, namely the "scaled variance," $V_{\rm scal}$, which they define as proportional to the usual variance, $\sigma^2=\langle E_t^2\rangle-\langle E_t\rangle^2$ divided by the mean energy squared, $\langle E_t\rangle^2$. Thus $V_{\rm scal}=\sigma^2/\langle E_t\rangle^2$.

The scaled variance has the virtue of being independent of the energy scale, so it could be useful if the calorimeter energy scales were not precisely known.

It is well known that the variance of the convolution of two independent distributions is the sum of the variances and that the mean of the variable (here E_t) is the sum of the means of each of the distributions [12]. Thus convoluting an E_t contribution (1) with any other function (2) will produce the scaled invariance $V_{\rm scal}=(\sigma_1^2+\sigma_2^2)/[\langle E_t(1)\rangle+\langle E_t(2)\rangle]^2$. This will increase both the numerator and denominator of $V_{\rm scal}$ so that the scaled variance may not be sensitive to convolutions which increase the mean energy. To calculate the scaled invariance we must know $d\sigma/dE_t$.

Figure 2 shows the large discrepancy between the *shapes* of the p-Pb data and the simple no-rescattering calculation. Although the shapes are quite different the value of V_{scal} for the data (0.42) is indistinguishable from the V_{scal} (0.43) of the no-rescattering calculation. In fact the V_{scal} for the calculation which includes the rescattering is 0.33. It is not difficult to understand this behavior: Convoluting one energy contribution with another always pushes events to higher E_t. The effect near $E_t=0$ is to depress the number of events of the lowest energy where most of the events occur. (See the log plot in Fig. 2.) This raises the average energy and therefore lowers V_{scal}. Thus we conclude that a crude parameter like the scaled invariance is quite sensitive to the low energy region of $d\sigma/dE_t$, not the high energy region. In fact it might be a useful parameter in the study of rescattering effects because the low energy region is sensitive to convolutions.

Figure 4 shows $d\sigma/dE_t$ calculated from the coefficients plotted in Fig. 3. Because of the mathematical effects of smearing the coefficients, the difference between the shapes of the cross sections, with and without fluctuations, is even less than appears in the plots of the a_n and b_n.

Since fluctuations are effects on the high energy tails of $d\sigma/dE_t$, they should not be expected to change the scaled invariances. In fact the effects of the fluctuations increase the scaled invariances for the cross sections in Fig. 4 by less than 4%.

Thus we do not find the scaled variance to be a useful parameter to study fluctuations that might be seen in typical E_t spectra.

Our discussion of E_t distributions suggests that one might better search for possible fluctuation effects in the *multiplicity* distribution measurements confined to the *central* region. There are several reasons for this suggestion. (a) If there are cross section fluctuations, they will affect any region in rapidity because of the changes in the a_n coefficients. (b) The central region is least sensitive to the ill determined rescattering effects which contribute mainly in the fragmentation regions. (c) The convolutions for determining the distributions in regions *symmetric* about zero rapidity in the c.m. system do not need to use any model like ISAJET or VENUS but can be determined directly from the p-p data. (d) Since multiple scattering of pions tends to increase the E_t but not necessarily to add new particles in the same rapidity region, materialization of the produced particles within the nuclear volume affects multiplicity distributions less than E_t distributions.

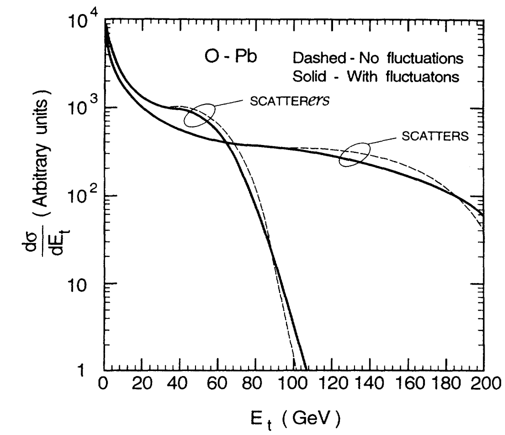

FIG. 4. Effect of fluctuations on $d\sigma/dE_t$. $d\sigma/dE_t$ vs E_t for O-Pb (NA35). The scaled invariance values (see text) are as follows: scatters: 1.01 (no fluctuations), 1.04 (with fluctuations); scatter*ers*: 0.85 (no fluctuations), 0.87 (with fluctuations).

Actually, the transverse energy is not a fundamental variable like multiplicity, since it depends on both the energy and direction of the observed particles. It is also measured with calorimeters that are sensitive to the particle type, proton, pion, kaon, etc. and are difficult to calibrate. In fact, E_t is defined in several manners, the theoretical expression $\sqrt{p_t^2+m^2}$ being quite different from the experimental definition, $E\sin\theta$, with θ the polar angle. Thus E_t distributions are, at present, poor testing grounds for "new physics."

In any case, we find that even large fluctuation effects produce only small changes in the spectra for either of the models studied.

Finally, we conclude that there is no basis for the assertion that there is evidence from E_t distributions that can validate the hypothesis of fluctuating nucleon-nucleon cross sections or that supports the suggestion [2] that fluctuations be incorporated into experimentalists' event simulations.

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[9] Because of both the uncertainties in the calorimeter energy scales and the lack of data on p-p interactions in the same calorimeter, these are only qualitative estimates of the underprediction.
[10] The effect of energy loss in successive collisions has been studied and appears to make only small changes in the predicted E_t spectra.
[11] See Ref. [8] for the general convolution method applicable to all rapidity positions and intervals.
[12] See, for example, B. V. Gnedenko, *Theory of Probability* (Chelsa, New York, 1962).