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Band structure systematics and symmetries in even-even nuclei

D. Bucurescu, Gh. Cata-Danil, M. Ivascu, and C. A. Ur
Institute of Atomic Physics, P.O. Box MG-6, Bucharest, Romania
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It is shown that the experimental in-band energy ratios for the even-even nuclei obey universal systematics similar to those observed by Mallmann for the quasiground band. Systematic correlations between energy ratios belonging to different bands are also found in certain cases. Finally, correlations between mixed energy ratios are shown to be useful in characterizing the evolution of the nuclear collectivity.

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The experimental data show that in the even-even nuclei there are level structures which evolve rather regularly with A, Z, or N, the best known such structures being the so-called quasiground, quasibeta, and quasigamma bands (denoted in the following by qgb, qβb, and qγb, respectively). A valuable collection of such bands is the compilation of Sakai [1].

An important observation concerning the regularities of the qgb is that of Mallmann [2], who showed that the ratios of the excitation energies of the form $E(J)/E(2)$ fall on universal, smooth curves, when plotted as a function of the first such ratio, $E(4)/E(2)$; here $E(J)$ denotes the excitation energy of the state of spin J. This emphasizes the fact that the energy of the first excited state (2^+) is a valuable scale factor for the whole band. The first and most general explanation of these universal curves was given by the VMI model [3–5].

In this Rapid Communication we point out that other quasiband structures obey universal systematics similar to those of the qgb as well, and also that different energy ratios related to quasiband structures can be used in a simple way to display the evolution of the nuclear collectivity over the whole nuclide chart.

The main point is the recognition of the importance of the energy difference between any two levels in a certain band as a scale factor for the whole band, and perhaps for other bands as well. We therefore deal with energy difference ratios and study the correlations between two such quantities. A recent use of the energy ratios of the type $E(J+2)/E(J)$ within bands has been demonstrated in Ref. [6].

In order to select the most useful energy ratios, and also to be able to recognize meaningful patterns in the experimental correlations studied, we use as a tool the interacting boson model-1 (IBA-1) [7], which has a wide applicability, and examine simple predictions based on its general symmetry properties. It has three limiting cases, the so-called dynamical symmetries U(5), O(6), and SU(3), in which the excitation energies can be written analytically. The energy ratios can then be expressed in the form of a numerical constant to which one adds a term depending on the model parameters. In order to deal with parameter-independent quantities, we keep only the dominant, numerical constants, which we shall refer to as symmetry limits (or, simply, limits). In this extreme simplification, the energy ratio values coincide with those of the limiting geometrical models which are known to correspond to the three dynamical symmetries [harmonic oscillator for U(5), completely γ-unstable rotor for O(6), and axially symmetric rigid rotor for SU(3), respectively]. This adopted approximation might be thought too severe in certain particular cases, as it results mainly from the neglect of the contribution of the Casimir operator of O(3) (namely, $I^{(2)}$) to the excitation energies in the U(5) and O(6) cases. On the other hand, as it will be seen below from comparisons with the available experimental data, the use of these symmetry limits proves rather valuable. In plots representing an energy ratio as a function of another, the three symmetry limits are marked by three points which define, in general, a triangle; this is just a particular, "physical" representation of the abstract symmetry triangle of Casten [8]. Since the three symmetry limits are known to encompass the full range of observed collective nuclear structures, we expect that these triangles will provide a natural limitation on the location of most of the nuclei in such energy ratio plots. The aspect of this triangle will dictate what we expect about the correlation between the two energy ratios considered. Thus, an interesting situation occurs when the symmetry triangle has a very elongated shape (or even reduces to a segment of straight line), in which case we expect that the experimental points will be confined within a rather narrow region, close to a smooth curve ("universal systematics").

Let us present the case of in-band correlations (between two energy ratios within the same band structure), which leads to such a situation. Consider, for a given band, ratios of the type

$$R(I/J) = \frac{E(I) - E(K)}{E(J) - E(K)} \quad (1)$$

where $E(I)$ denotes the energy of the state of spin I. We may choose any three states I, J, K; however, for practical purposes (for many bands only the first few members are known) in what follows we shall mean by K the bandhead, and study the ratios with $J = K+2$. Since in each dynamical symmetry case the excitation energies of any IBA-predicted band can be expressed, as a function of the spin I, as [7]

$$R(I/K+2) = aR(J/K+2) + b \quad (2)$$

with

$$a = \frac{(I-K)(I-K-2)}{(J-K)(J-K-2)} \quad \text{and} \quad b = \frac{(I-K)(J-I)}{2(J-K-2)}, \tag{3}$$

this is a generalization of the result obtained by Ejiri on the basis of a phenomenological formula for the excitation energies of the qgb [9]. Thus, not only the three limit values but also the exact values (for each dynamical symmetry) lie on a straight line as defined by Eq. (2). Therefore, for any IBA-predicted band, the symmetry triangle for energy ratios $R(I/K+2)$ as defined by (1) is a segment of straight line, and we expect universal systematics in the form of smooth curves close to the straight lines (2).

As a first application, we consider Mallmann's systematics for the qgb [2]. Apparently, the "Ejiri formula" [usually quoted only for the representation $R(I/2)$ vs $R(4/2)$] does not describe well the experimental data (see, for example, Ref. [10]). We have examined all the possible correlations $R(I/2)$ vs $R(J/2)$ in the qgb, for states up to 12^+ (below the backbending) for all the nuclei listed by Sakai [1], and observed the following. The systematics with $I-J=2$ [e.g., $R(6/2)$ vs $R(4/2)$, $R(8/2)$ vs $R(6/2)$, etc.] deviate very little from the straight lines (2). This is shown in Fig. 1 for the four plots up to spin 12. The slight deviations from linearity are very well described, in each case, by a parabolic dependence. The correlations with $I-J>2$ show indeed increasing deviations from the straight lines (2), but these are very well accounted for if we use the empirical parabolas from Fig. 1. Thus, by using the basic parabolas fitted to $I-J=2$ (Fig. 1), which represent only slight departures from the linear relationship (2), we are able to describe very well any kind of correlation of the type $R(I/J)$ vs $R(I'/J')$. In particular, the linear correlation observed by Cizewski [11] between $R(6/4)$ and $R(4/2)$ is a result of the parabolic dependence of $R(6/2)$ on $R(4/2)$. One should mention that not all nuclei from Ref. [1] for which the qgb is known at least up to 6^+ appear in the plots of Fig. 1. A number of 37 (out of the total of 307) were eliminated due to their considerable deviation from the generally smooth patterns observed for all the other nuclei. These deviations, which do not make the object of the present work, are discussed, for some of these nuclei, in Refs. [4,11,12].

We have verified further the prediction (2), for the qγb and qβb (referred to as the lowest $K=2$ and $K=0$ bands, respectively), which are known in some detail in relatively many nuclei [1]. Figure 2 shows that the experimental data are very well described by the straight lines (2), both for the γ band (separately for the even- and odd-spin members, respectively), and for the β band.

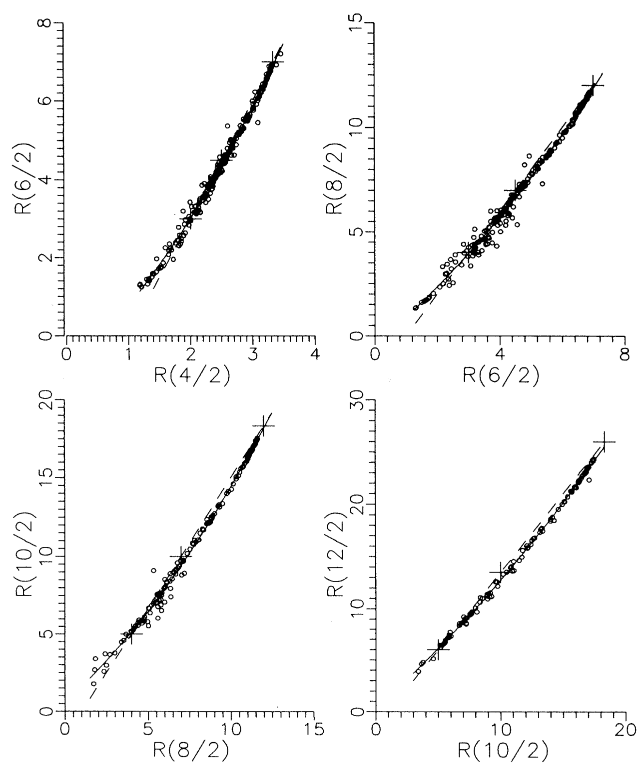

FIG. 1. Plots of energy ratios $R(I/2)$ vs $R(J/2)$ in the qgb, for states with $I-J=2$. The three big crosses in each plot are the IBA symmetry limits: U(5)—the lowest, O(6)—the middle, and SU(3)—the upper one, respectively. The dashed lines are the straight lines of Eq. (2). The continuous lines are phenomenological fits with a parabola.

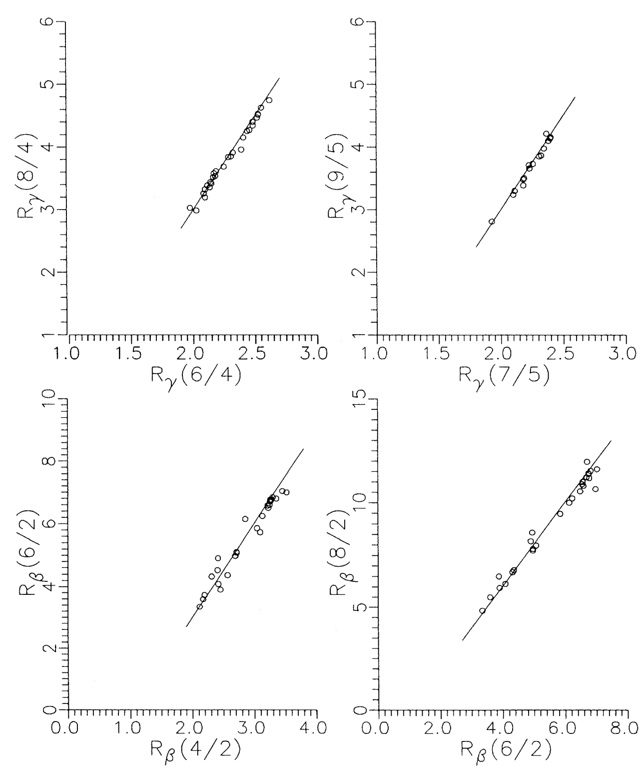

FIG. 2. Energy ratio systematics within the γ band (upper part) and β band (lower part). These plots are analogous to the Mallmann plots for the quasiground band (see also the text). The lines are not fits, but the straight lines calculated with Eq. (2).

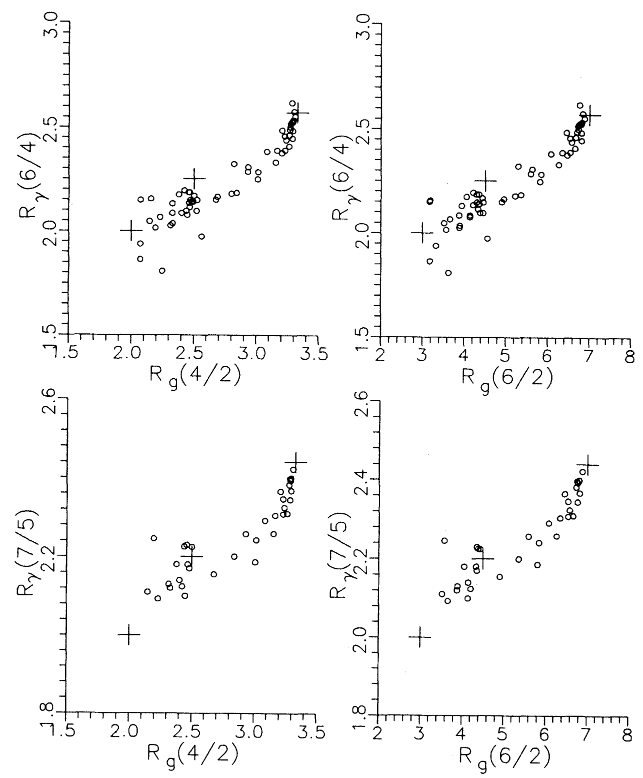

FIG. 3. Correlations between energy ratios from the quasigamma band and the quasiground band. The three big crosses are the symmetry limits [U(5)—the lowest, O(6)—the middle, and SU(3)—the upper one, respectively].

Usually, deviations from the universal curves indicate interesting phenomena. Thus, for example, the ^{184}Hg and ^{186}Hg nuclei (not shown in Fig. 2) deviate strongly from the general pattern since their β band is disturbed already at the 2^+ state by the shape coexistence phenomenon [13]. Other nuclei with large deviations and consequently not represented in Fig. 2 are ^{18}O and ^{24}Mg.

The idea that whenever the symmetry triangle is very elongated we may expect systematic correlations in the form of smooth (universal) curves, has also been tested in the case of the energy ratios (1) belonging to two different bands. One such case is that of the correlation between the lowest such ratios of even-spin and odd-spin qγb [$R_\gamma(6/4)$ and $R_\gamma(7/5)$], where the three symmetry limits lie almost on a straight line and the existing experimental data follow this line rather closely. A second example is given in Fig. 3, showing correlations between ratios of the qgb and qγb. In this case the symmetry triangle is again rather elongated and the experimental data are well confined within narrow strips close to a curve which joins the symmetry limits. An exception here are the light mass ($A \leq 70$) nuclei (data exist only for ^{24}Mg, ^{58}Fe, ^{60}Ni, ^{62,64}Zn, and ^{70}Se [1], not shown in Fig. 3), which appear to lie systematically well above the average pattern of the heavier ones.

We next examine some cases in which the symmetry triangle is well extended. We have found it interesting to consider ratios of a "mixed" sort, such as

$$R_{\gamma/g}(I) = \frac{E_\gamma(I) - E_\gamma(2)}{E_g(2)}. \tag{4}$$

In such cases, we have found that the symmetry triangle shows various shapes, and the available experimental data [1] fill in regions which reproduce this triangle, sometimes a bit distorted and/or displaced. As an illustration, Fig. 4 shows two such plots in which only a few chains of isotopes were selected, which are well known as representative of transitions between two symmetries [7]. The usefulness of such plots is immediately clear, as they reveal, qualitatively, these transitions.

In conclusion, we have observed correlations between different experimental energy ratios related to band structures in all the even-even nuclei. These correlations obey well the expectations based on simple IBA-1 predictions. The character of any such correlation between two energy ratios depends on its associated IBA symmetry triangle. When this triangle has a very elongated shape, or is a segment of straight line, the data usually follow a smooth (universal) curve. The Mallmann systematics [2] for the qgb are an example of such a situation, and we have shown that similar systematics exist for the quasigamma and quasibeta bands, and presumably for other bands as well. When the symmetry triangle is ex-

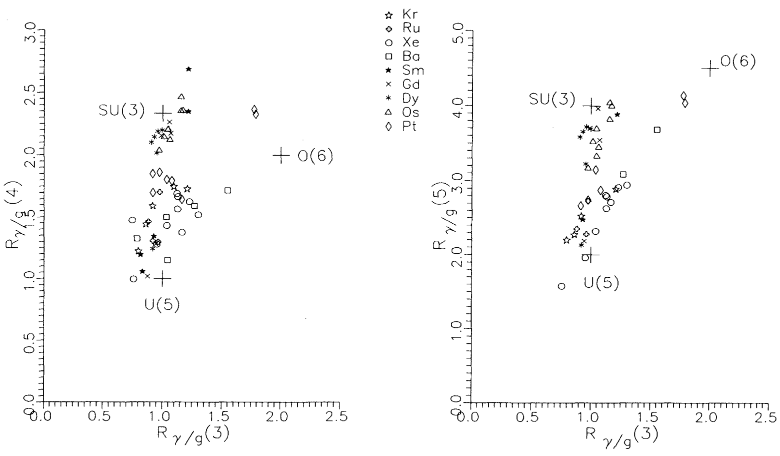

FIG. 4. Correlations between different mixed energy ratios as defined by Eq. (4), illustrating the IBA symmetry triangle. The three symmetry limits are explicitly indicated. Only several isotopic chains have been selected for display (see also the text).

tended, the energy ratio correlation plots can be used as a simple method to examine qualitatively the position of individual nuclei or of (transitional) classes of nuclei with respect to the dynamical symmetry limits.

Systematics such as those presented in this work can be used as means of predicting the whole low-energy collective level scheme of even-even nuclei (and, notably, of transitional ones) on the basis of a minimum of information (e.g., a few energy ratios). A more increased predictive power, having in view unknown nuclei as well, can be attained by adding other types of systematics, such as those of the N_pN_n family [14], that of the 2_γ^+ state energies [15], that of Ref. [16], and possibly others.

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