

## Rotational behavior in intermediate energy heavy ion collisions

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The rotational behavior together with in-plane collective flow for the intermediate energy  $^{40}\text{Ar} + ^{27}\text{Al}$  reaction is investigated by analyzing the rapidity dependent azimuthal distributions with the Boltzmann-Uehling-Uhlenbeck (BUU) model. The azimuthal distributions are fitted by a Legendre polynomial expansion up to the second order. By incorporating the uncertainties in the experimental reaction plane determination into our calculations, quantitative agreement between the calculations and data is obtained. It is found that the rotational behavior at midrapidity depends strongly on the impact parameter. Information about the in-plane collective flow is also extracted from the azimuthal distributions. Both the rotational behavior and the in-plane collective flow depend strongly on the in-medium nucleon-nucleon cross section and nuclear equation of state.

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The collective motion is found to be responsible for many observations in heavy ion collisions (HIC's) at bombarding energies from the Fermi energy to a few hundreds of MeV/nucleon. At the upper end of this energy region, the investigations reveal two collective effects in the reaction plane, i.e., the side-splash effect and the bounce-off effect, and one collective effect perpendicular to the reaction plane, i.e., the out-of-plane squeeze-out effect [1–5]. The squeeze-out effect demonstrates the existence of a strong azimuthal correlation between the emitted particles [6–8], which was first predicted by the hydrodynamical calculations [9]. At a later time, the molecular-dynamic model [10], Vlasov-Uehling-Uhlenbeck and quantum-molecular-dynamics approaches [11,12] were also used to reproduce the experimental azimuthal correlation between the emitted particles. At a few tens of MeV/nucleon, the collective phenomena have also been investigated experimentally [13–15] with  $4\pi$  spectrometers [16–20]. Of the collective phenomena, the azimuthal distribution or correlation between the emitted particles plays an important role in the study of reaction dynamics. The in-plane enhancement of particle emission, which stems from effects such as in-plane collective flow and/or rotational behavior, was found experimentally [21–23]. In this paper, we will discuss in detail the rotational behavior together with the in-plane collective flow by analyzing the rapidity dependent azimuthal distributions of particle emission within the Boltzmann-Uehling-Uhlenbeck (BUU) model.

Before discussing the azimuthal distributions of particle emission, we should know how to determine the reaction plane in the experiments. Generally the transverse momentum analysis method [24] can be used to deter-

mine the reaction plane. It gives  $\langle P_x/A \rangle$ , the average in-plane component of the transverse momenta which depends generally on the rapidity. The effect of the in-plane collective flow on the azimuthal distributions should result in peaking at  $\pm 180^\circ$  at low (target) rapidity and  $0^\circ$  at high (projectile) rapidity, and should vanish at midrapidity. At high energy HIC's the azimuthal distribution at midrapidity is sensitive to out-of-plane emission of particles, and clearly shows a preferential emission in the out of the reaction plane ( $\phi = \pm 90^\circ$ ) due to the squeeze-out effect. But this squeeze-out effect has not been found up to now for light systems. In intermediate energy HIC's, there exist clear peaks at  $\phi = 0^\circ$  and  $\pm 180^\circ$  for the azimuthal distribution at midrapidity in Refs. [21–23], which corresponds to the preferential particle emission in the reaction plane on the side of the projectile and target respectively due to the rotational behavior. This rotational effect is superimposed on the effect of the in-plane collective flow at low and high rapidities and appears sensitively at midrapidity. A similar in-plane enhanced emission of fragments was observed by Tsang *et al.* [25] in heavy ion reactions with U targets in which the reaction plane was determined using the azimuthal angles of fission fragments. It is noticed that the reaction-plane determination using the azimuthal correlation between light particles was well documented [21] when the mean-field interaction is primary attractive [26].

In the present paper we will analyze the rotational behavior together with in-plane collective flow using the BUU approach. The calculations for the reaction  $^{40}\text{Ar} + ^{27}\text{Al}$  at different bombarding energies and impact parameters are performed. The Boltzmann-Uehling-Uhlenbeck equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} f - \nabla_{\mathbf{r}} U \cdot \nabla_{\mathbf{p}} f = \frac{4}{(2\pi)^3} \int d^3 p_2 d^3 p_3 d\Omega \frac{d\sigma_{NN}}{d\Omega} V_{12} \times [f_3 f_4 (1-f)(1-f_2) - f f_2 (1-f_3)(1-f_4)] \delta^3(\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \quad (1)$$

is solved with the method of Bertsch and Das Gupta [27]. In Eq. (1),  $d\sigma_{nn}/d\Omega$  and  $V_{12}$  are the in-medium nucleon-nucleon cross section and relative velocity for the colliding nucleons, and  $U$  is the mean-field potential including the isospin-dependent term:

$$U = A\rho/\rho_0 + B(\rho/\rho_0)^\gamma + C\varepsilon_i(\rho_n - \rho_p)/\rho_0 \quad (2)$$

where  $\rho_0 = 0.17 \text{ fm}^{-3}$ , and  $\rho = \rho(\mathbf{r})$  is the local density of nuclear matter. In the calculations, one chosen parameter set of  $A = -356$ ,  $B = 303$ , and  $\gamma = 7/6$  corresponds to a soft nuclear equation of state (EOS) with the incompressibility coefficient  $K = 200 \text{ MeV}$ , another set of  $A = -124$ ,  $B = 70.5$ , and  $\gamma = 2$  corresponds to a stiff EOS with  $K = 375 \text{ MeV}$ . In the isospin-dependent term,  $\rho_n$  and  $\rho_p$  are the neutron and proton densities, the value of  $C$  is  $25 \text{ MeV}$ , and  $\varepsilon_i$  is the isospin operator with  $+1$  and  $-1$  for neutrons and protons, respectively. The differential nucleon-nucleon cross section  $d\sigma_{NN}/d\Omega$  is chosen to be isotropic. In the calculation we distinguish the neutrons and protons in the projectile and target.

Since the BUU model is a one-body theory and does not include complex fragment emission, all the protons which are referred to as pseudoprotons are included in the simulation whether they are free or bound in clusters. Here the nucleons are considered as emitted once the local density (freeze-out density)  $\rho(\mathbf{r}) < \alpha\rho_0$  ( $\alpha$ : 0.1–0.5) and the distance from the considered nucleon to the center of the corresponding target or projectile is greater than the radius of the corresponding target or projectile [28]. The choice of a different  $\alpha$  value ranging from 0.1 to 0.5 can influence the multiplicity, but it does not influence the main character of azimuthal distributions as a function of rapidity. To accumulate sufficient statistics for emitted nucleons, 500 test particles per nucleon were used.

Before comparison with the experimental data, we should know how to select the impact parameter in the experiment [29]. The basic assumption is that the larger the violence of the collision, the larger is the interaction volume of two nuclei, i.e., the smaller is the impact parameter. The violence of the collision is expressed through the value of global variable. Several global variables have been tested by means of simulated events, produced by a code which simulated the reaction mechanisms. Each event is filtered through a software replica of the detection setup to take into account the actual detector limitations. The quality of the impact parameter determination is expressed by the correlation between the real  $b$  value and the “experimentally” determined value. This correlation is broad when the global variable is the multiplicity, the total detected charge or midrapidity charge; it is slightly better with total transverse momentum. The resolution of such impact parameter selection of  $1 \text{ fm}$  [full width at half maximum (FWHM)] is achieved for the system studied here at energies above  $40 \text{ MeV/nucleon}$ .

For a nonzero impact parameter, the beam direction ( $z$ ) and the line joining the centers of the nuclei determine the reaction plane, i.e., the  $x$ - $z$  plane. The azimuthal angle of a fragment in this coordinate system is

$$\phi = \arctan(P_y/P_x). \quad (3)$$

In order to show clearly the dependence of azimuthal distribution on rapidity, we divide the rapidity into six windows from rapidity  $y = 0$  to  $y = 0.42$ . Figure 1 shows the calculated azimuthal distributions of pseudoprotons and of emitted protons in different rapidity intervals for  $^{40}\text{Ar} + ^{27}\text{Al}$  collision at  $b = 4.5 \text{ fm}$  and a bombarding energy  $E_{\text{in}} = 45 \text{ MeV/nucleon}$  with a stiff EOS and  $\sigma_{NN} = 33 \text{ mb}$ . In Fig. 1 the open and solid circles represent the original calculated results for pseudoprotons and emitted protons, respectively. The solid lines show the Legendre polynomial fits up to second order for the original calculated results which will be discussed in the following. Such azimuthal distributions are evaluated at  $t = 100 \text{ fm/c}$ . At about  $t = 80 \text{ fm/c}$  the azimuthal distributions start to be more or less stable. They change slowly at the later stage, maybe, due to evaporation process. The similar behavior is also observed at 36, 55, and 65 MeV/nucleon. The anisotropy of azimuthal distributions at the same impact parameter decreases with increasing the bombarding energy in this energy range. Clearly, the azimuthal distributions of the pseudoprotons are basically similar to those of the emitted protons. It implies that the azimuthal anisotropy of the fragments as a function of rapidity also shows a similar rotational collective motion. As an example, the azimuthal distribution of particle also shows the rotational behavior, and its anisotropy is stronger than that of protons [23]. At low rapidity the azimuthal distribution peaks at  $\phi = \pm 180^\circ$ , indicating that the protons are preferentially emitted to the target side. At midrapidity the contribution of the in-plane collective flow vanishes; hence, the azimuthal distribution will be very sensitive to the pattern of particle emission. Here the azimuthal distribution peaks at  $\phi = \pm 180^\circ$  and  $0^\circ$ ; i.e., there exists rotational behavior. The rotation of the interaction region around an axis perpendicular to the reaction plane favors the emission of particles in the reaction plane. At high rapidity the azimuthal distribution peaks at  $0^\circ$ , corresponding to a positive average in-plane component of the transverse momenta. To better study the azimuthal distributions and to allow for a comparison with experimental data, we have performed a Legendre polynomial expansion up to the second order to fit our azimuthal distributions:

$$dN/d\phi = a_0 + a_1 \cos\phi + a_2 \cos(2\phi), \quad (4)$$

where the parameters  $a_0$ ,  $a_1$ , and  $a_2$  depend upon the rapidity  $y$  as in Refs. [1,30]. The ratio  $a_1/a_0$  mainly reflects the collective flow effect on the azimuthal distribution in the reaction plane [31]. The in-plane collective

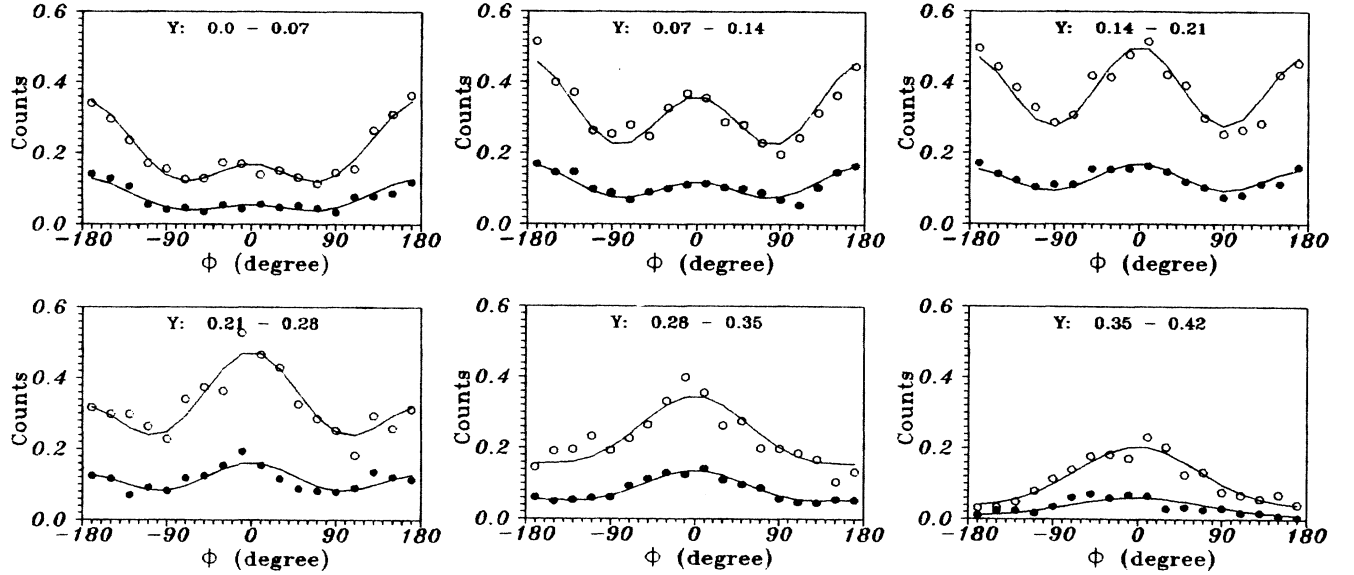


FIG. 1. The azimuthal distributions of pseudoprotons and emitted protons in the different rapidity interval for 45 MeV/nucleon  $^{40}\text{Ar} + ^{27}\text{Al}$  at  $b = 4.5$  fm. The open and solid circles represent the original calculated results for pseudoprotons and emitted protons, respectively. The solid lines show the Legendre polynomial fits up to second order for the original calculated results. Notice that the reaction plane is chosen as  $x - z$  plane which is known *a priori* in the BUU model.

flow results in peaking at  $\phi = \pm 180^\circ$  (i.e.,  $a_1 < 0$ ) for low rapidity values and  $\phi = 0^\circ$  (i.e.,  $a_1 > 0$ ) for high rapidity values. A negative value of  $a_2/a_0$  reflects the squeeze-out effect and a positive value a rotational behavior. In Fig. 1 the solid lines represent the Legendre polynomial fits. Figure 2 shows the values of  $a_2/a_0$  at midrapidity as a function of impact parameter at  $E_{\text{in}} = 45$  MeV/nucleon. The  $a_2/a_0$  is always positive at all bombarding energies studied here, which indicates that the squeeze-out effect does not exist at this energy domain for the  $^{40}\text{Ar} + ^{27}\text{Al}$  system (this can also be confirmed by the azimuthal distributions which never exhibit any enhancement at  $\phi =$

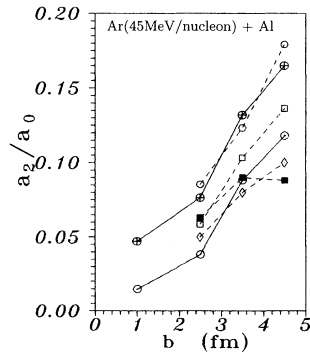


FIG. 2. The values of  $a_2/a_0$  versus  $b$  for 45 MeV/nucleon  $^{40}\text{Ar} + ^{27}\text{Al}$  collision, where the circles with dots and circles with crosses represent the experimental data for particles with charge  $z = 1$  and  $z = 2$ , respectively. The open circles, squares, and diamonds demonstrate the calculated results of a stiff EOS with  $\sigma_{NN} = 20, 33,$  and  $55$  mb, respectively. The solid squares correspond to calculations with a soft EOS and  $\sigma_{NN} = 33$  mb. All calculated values have been corrected for the uncertainties in the reaction-plane determination.

$\pm 90^\circ$ ). In this figure the experimental data for protons and for particles with  $z = 2$  are not corrected for the average difference between the real and determined reaction plane [23]. The dashed lines with symbols show our BUU calculations which are corrected by a finite rms dispersion  $\Delta\phi$  of the experimental reaction plane determination [21,24,32–34]. This fluctuation  $\Delta\phi$  of an experimentally estimated reaction plane about the true reaction plane depends on the bombarding energy, the impact parameter, and the rapidity (or the in-plane collective flow) [33]. By Monte Carlo simulation as in Ref. [33], we can get the fluctuation  $\Delta\phi$  of the reaction plane for 45 MeV/nucleon  $^{40}\text{Ar} + ^{27}\text{Al}$  collision at  $b = 2.5\text{--}4.5$  fm. It is in the range of  $50^\circ\text{--}60^\circ$ . By adding this fluctuation into BUU calculation, the corrected calculated result reduces the values of  $a_2/a_0$  strongly because the fluctuation to some extent smooths the azimuthal distributions. It is found that the

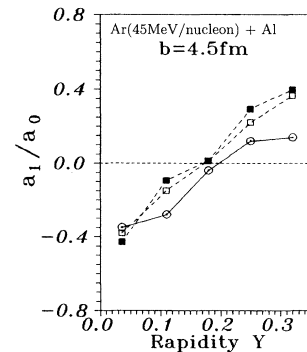


FIG. 3. The value of  $a_1/a_0$  versus rapidity for 45 MeV/nucleon  $^{40}\text{Ar} + ^{27}\text{Al}$  system at  $b = 4.5$  fm. The definition of symbols and the correction for the reaction-plane determination are the same as in Fig. 2.

bigger the fluctuations, the larger the reduction of coefficients. In the figure the increasing of  $a_2/a_0$  with the impact parameter implies that the rotational effect of the system gets stronger with the increasing impact parameter. Clearly the calculated  $a_2/a_0$  values are sensitive to the in-medium nucleon-nucleon cross section. The calculation with larger  $\Delta\phi$  yields a smaller value of  $a_2/a_0$ . For the calculation at  $b = 2.5$  fm, the  $a_2/a_0$  reveals to be insensitive to EOS, allowing  $\sigma_{NN}$  to be determined from the data. The sensitivity to both the EOS and  $\sigma_{NN}$  becomes comparable at larger impact parameters ( $b \geq 3.5$  fm). Such a character is very similar to the sensitivity of the in-plane collective flow to both the EOS and  $\sigma_{NN}$  [28,35,36]. Since our BUU calculation accounts for only nucleon emission, the calculated coefficients  $a_2/a_0$  and  $a_1/a_0$  should have the values between the experimentally extracted values for  $Z = 1$  and  $Z = 2$  since these two charge particles constitute the dominant part of the observed multiplicities in the experiment [37,38]. Obviously, a good agreement between the calculation and the experimental data is obtained with  $\sigma_{NN} = 33$  mb and a stiff EOS. This is also consistent with the conclusion from the analysis of collective flow in Refs. [28,35].

Figure 3 demonstrates the dependence of  $a_1/a_0$  on the rapidity at  $b = 4.5$  fm. A good linearity is found as well as the average in-plane transverse momentum as a function of the rapidity around the midrapidity region. The slope of  $a_1/a_0$  on rapidity closely relates to the in-plane collective flow. When the fluctuation  $\Delta\phi$  of the reaction plane is taken into consideration, the absolute values of  $a_1/a_0$  decrease like  $a_2/a_0$ , resulting in the decrease of the slope of  $a_1/a_0$  on rapidity; i.e., the large uncertainties between the estimated and true reaction plane result in reducing in-plane collective flow strongly. A good agreement with the experimental data is shown especially when a stiff EOS and  $\sigma_{NN} = 33$  mb are used. If the slopes of  $a_1/a_0$  and of the average transverse momentum ( $P_x/A$ ) at midrapidity on rapidity are denoted by  $S_{a1}$  and  $S_{P_x}$ , respectively (the latter is actually the in-plane collective flow per unit of rapidity), the relation of  $S_{a1}$  to  $S_{P_x}$  is written as

$$S_{P_x} = \lambda S_{a1}. \quad (5)$$

We found that the coefficient  $\lambda$  is not very sensitive to the impact parameter, bombarding energy  $E_{in}$ ,  $\sigma_{NN}$  and the EOS within a statistical error 10%; its value is about 95 MeV/c. This quantity has its physical background;  $\lambda$  should be approximately equal to half of the rms transverse momentum at midrapidity. Figure 4 shows the impact parameter dependence of  $S_{a1}$ . Clearly, the sensitivities of  $S_{a1}$  (the in-plane collective flow) and of  $a_2/a_0$  to the EOS and  $\sigma_{NN}$  are similar.

In summary, the azimuthal distributions of proton emission for the  $^{40}\text{Ar} + ^{27}\text{Al}$  system are discussed based on the BUU method. The rotational behavior together with the directed transverse flow is found. The in-plane enhanced particle emission for azimuthal distributions at midrapidity has an origin in the rotationlike effect. This effect generally increases with the increasing of the im-

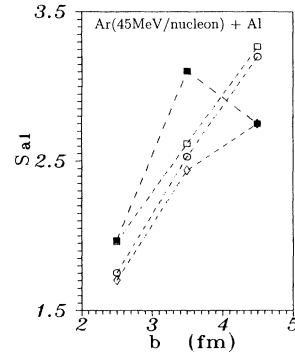


FIG. 4. The slope of  $a_1/a_0$  on rapidity versus  $b$  for 45 MeV/nucleon  $^{40}\text{Ar} + ^{27}\text{Al}$  reaction. The definition of symbols and the correction for the reaction-plane determination are the same as in Fig. 2.

pact parameter  $b$ , which is clearly seen from the impact parameter dependence of  $a_2/a_0$ . The slope of the  $a_1/a_0$  on rapidity at the midrapidity region closely relates to the in-plane collective flow. By the simulation of the fluctuation of the experimentally estimated reaction plane about the true reaction plane in calculations, quite good agreements between the calculation and the coefficients  $a_2/a_0$  and  $a_1/a_0$  extracted from the experiment are obtained. The coefficients are both sensitive to  $\sigma_{NN}$  at all impact parameters, and to the EOS at larger impact parameters, indicating that both collective motions—in-plane collective flow and rotational behavior—depend on both  $\sigma_{NN}$  and the EOS. By fitting the experimental data, it is suggested that the parameter set of  $\sigma_{NN} = 33$  mb and the stiff EOS ( $K = 375$  MeV) may be suitable for  $^{40}\text{Ar} + ^{27}\text{Al}$  reaction below 100 MeV/nucleon, which is consistent with the conclusions from the collective flow by using the transverse momentum analysis for the same system [28]. The measurement of azimuthal distributions could provide a quite convenient and fruitful analysis method of collective flow and rotational behavior. We are awaiting comparison with more experimental data before presenting more abundant conclusions about the rotational behavior in intermediate energy HIC's, and the azimuthal distributions should be further studied experimentally and theoretically above the balance energy where the disappearance of the in-plane flow occurs. It is very interesting to investigate the evolution of azimuthal distributions with the projectile energy and the target mass.

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