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Effective fermion SO(6) dynamical symmetry in the platinum nuclei

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It is shown that a very accurate fermion SO(6) dynamical symmetry exists for ^{196}Pt, even though the coupled system has no such formal dynamical symmetry in the fermion dynamical symmetry model. Implications of this effective dynamical symmetry are discussed.

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Dynamical symmetry is an important concept in many branches of physics [1]. Operationally, it can be thought of as spectroscopic pattern recognition. The SO(6) dynamical symmetry in nuclear physics has become an archetypal pattern. Nuclear SO(6) dynamical symmetry was first realized for the interacting boson model (IBM-1) [2], and very quickly was shown to be manifest in ^{196}Pt [3]. Some years later, this dynamical symmetry was found in the xenon and barium isotopes as well [4]. Briefly, for ^{196}Pt it was found that by using the dynamical group chain SO(6) $\supset$ SO(5) $\supset$ SO(3) in IBM-1 (which does not distinguish between protons and neutrons), both energy levels and selection rules could be reproduced extremely well. These results suggest that in the "real" nuclear wave functions there is a symmetry akin to the irreducible representations of the SO(6) dynamical symmetry.

But the building blocks of nuclei are fermions and the appearance of accurate SO(6) dynamical symmetries should be present in the fermion degrees of freedom. Ginocchio demonstrated that his schematic fermion model [5] exhibits such a symmetry, and the fermion dynamical symmetry model (FDSM) [6] established a connection between the fermion dynamical symmetries of the Ginocchio scheme and the underlying shell structure. The FDSM suggests that the SO(6) dynamical symmetry can occur easily in xenon and barium regions, since for the known nuclei it is only the 50–82 shell where the valence shells for both the protons and the neutrons can exhibit a clear SO(8) $\supset$ SO(6) dynamical symmetry chain. This is consistent with what has been observed. However, it is not obvious that ^{196}Pt can be SO(6)-like in the FDSM, because the shell symmetry for rare earth nuclei is $\mathrm{Sp}^{\nu}(6) \times \mathrm{SO}^{\pi}(8)$, which has no *n-p* coupled SO(6) chain. Thus, it is a challenge for the FDSM to explain why there is an SO(6) dynamical symmetry for the platinum isotopes. As a corollary, it is also important to see whether the model can sustain a transition between rotors that are deformed [SU(3)-like] and γ soft [SO(6)-like], as demanded by the data in the rare earth nuclei.

Wu and Vallieres [7] have shown numerically that for the symmetry $\mathrm{Sp}^{\nu}(6) \times \mathrm{SO}^{\pi}(8)$ it is possible to obtain an approximate SU(3) dynamical symmetry in the rare earth nuclei, even though there is no exact SU(3) dynamical symmetry from the coupled symmetry $\mathrm{Sp}^{\nu}(6) \times \mathrm{SO}^{\pi}(8)$. They showed that under the highest symmetry $\mathrm{Sp}^{\nu}(6) \times \mathrm{SO}^{\pi}(8)$, the numerical results for a system of three proton pairs and five neutron pairs outside the closed shells is very close to the SU(3) results when proper effective interactions are employed. Likewise, although there is no exact vibrational [SU(2)] dynamical symmetry limit for an *n-p* system in any valence shells, it was shown that with a proper *n-p* interaction one can obtain an approximate vibrational limit near the beginning or the end of the shells [7,8]. The results of Wu and Vallieres have led us to investigate the platinum isotopes using their FDU0 code [9] to determine if an approximate SO(6) dynamical symmetry can be realized numerically in the heavy rare earth nuclei. If this were true, then the quantum numbers which are exactly in the SO(6) group chain are approximately lurking in the system. We mention that in a slightly different context, the concept of approximate quantum numbers for a fermion system was previously examined in various shell-model studies [10–12]. In this Rapid Communication we report the discovery of a very well defined SO(6) fermion dynamical symmetry for this region.

The FDSM Hamiltonian employed in the FDU0 code contains 11 parameters, corresponding to the most general set of symmetry-allowed one- and two-body interactions. However, the low-lying spectra are dominated by the pairing and quadrupole interactions, which allows us to employ a simplified five-parameter Hamiltonian for the platinums:

$$H = G'_{0\pi} S^{\dagger}_{\pi} S_{\pi} + G'_{0\nu} S^{\dagger}_{\nu} S_{\nu} + B'_{2\pi} P^2_{\pi} P^2_{\pi} + B'_{2\nu} P^2_{\nu} P^2_{\nu} + B_{2\pi\nu} P^2_{\pi} P^2_{\nu}, \quad (1)$$

where the operators are defined in [6]. We should note that in the FDSM the quadrupole pairing interactions can always be ignored if the spectrum is our only concern [6]; the quadrupole pairing is taken into account by redefining the parameters in Eq. (1): $G'_{0\sigma} = G_{0\sigma} - G_{2\sigma}$ and $B'_{r\sigma} = B_{r\sigma} - G_{2\sigma}$ ($\sigma = \pi, \nu$). We expect the quadrupole

interactions between neutrons and protons to be significantly larger than those among protons or among neutrons.

In the code, the model space is restricted to the S-D subspace in the normal-parity shells (heritage $u = 0$, corresponding to no broken pairs). Although the particles in abnormal-parity levels are not explicitly included, they do play a subtle and crucial role and are effectively included by the fact that there is a distibution of particles between the normal and the abnormal parity levels. For example, one physical consequence is that without particles in the abnormal-parity levels, stabilization of the prolate deformation and thus the abundance of such deformation will not be possible in the rare earths [13]. The number of pairs (N_1) in the normal-parity levels is treated as a good quantum number and is estimated from the semiempirical formula determined globally from the ground state spin of the odd-mass nuclei [13]:

$$N_1 = \begin{cases} N & \text{for } N < 1.5, \\ 0.75 + 0.5N & \text{for } 1.5 < N < 2\Omega_0 + 1.5, \\ N - \Omega_0 & \text{for } N > 2\Omega_0 + 1.5, \end{cases} \quad (2)$$

where N is the number of valence pairs and Ω_0 is the pair degeneracy of the abnormal-parity level. For ^{196}Pt, it has $N^\pi = 14$ and $N^\nu = 18$; according to this formula, $N_1^\pi = 8$ and $N_1^\nu = 11$ as input to the FDU0 code.

Using Eq. (1), the energy spectra of five even-even platinum isotopes $^{190-198}$Pt (there are about 13 levels for each isotope) were found to be satisfactorily described by one set of five parameters. The wave functions obtained in our calculations were used to compute various $E2$ branching ratios. The $T(E2)$ operator was taken to be

$$T(E2)^2_\mu = e_\pi P^2_\mu(i)_\pi + e_\nu P^2_\mu(k)_\nu. \quad (3)$$

For simplicity in the calculations described here, the "effective charges" e_π and e_ν were equated.

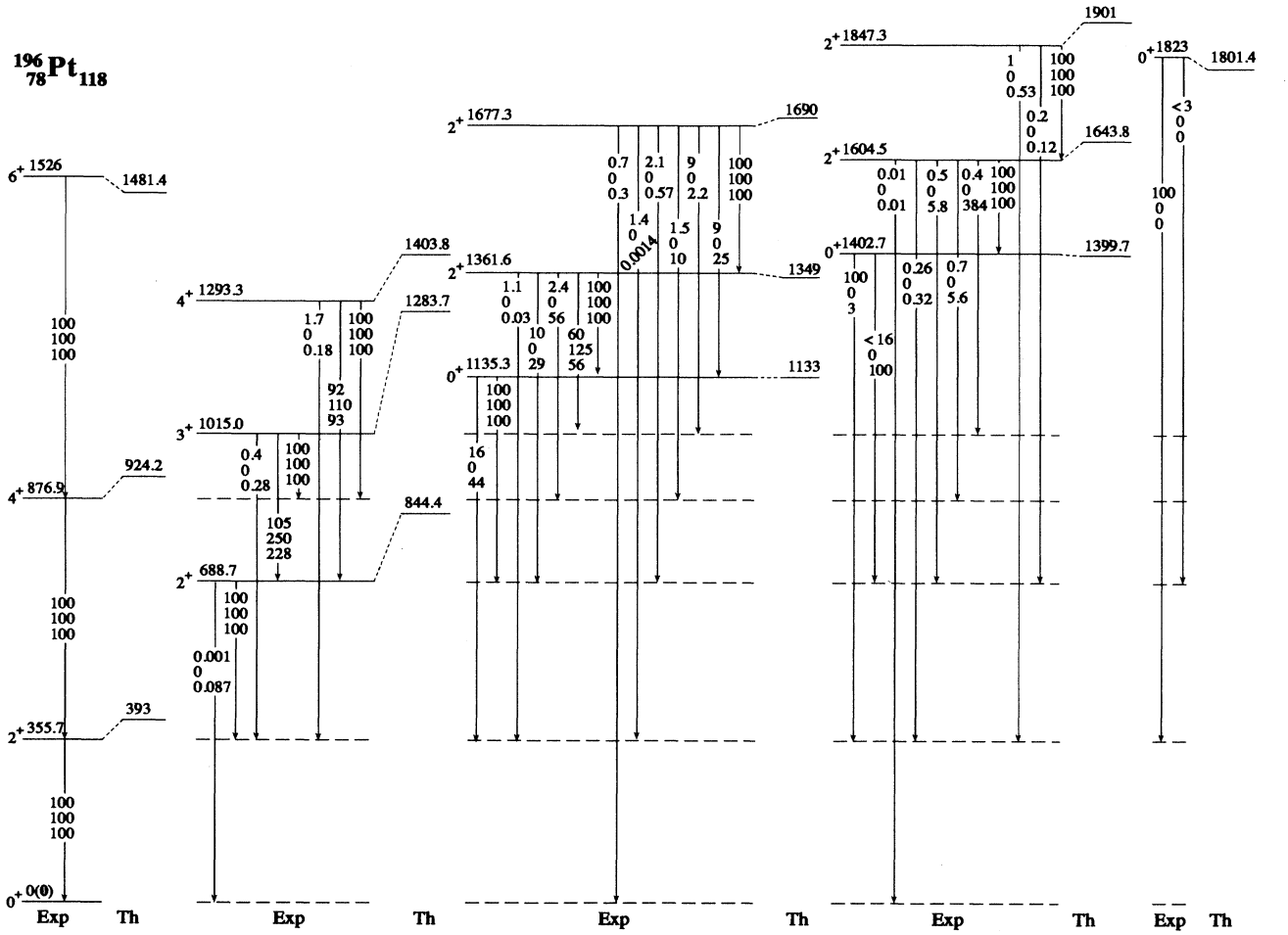

FIG. 1. Level scheme for positive-parity states in ^{196}Pt. Experimental levels are taken from [3]. The theoretical levels are from the FDSM calculation using the FDU0 code. The parameters are $G_{0\nu} = -48$, $G_{0\pi} = -65$, $B_{2np} = -300$, $B_{2\nu} = 66$, $B_{2\pi} = 32$, with all units in keV. The upper number on the transition arrows is the measured relative $B(E2)$ value; the middle number is the IBM-1 predicted value; the lowest number on each transition arrow is the FDSM prediction. There are only two transitions where the FDSM calculation does not agree with the IBM-1 prediction: the $2_4^+ \to 3_1^+$ and $0_3^+ \to 2_2^+$ transitions are forbidden in the IBM-1, but not in the FDSM. For weak $\Delta\tau = 0, \pm 2$ transitions the FDSM results agree with data quite well. An additional $d^\dagger \tilde{d}$ term is required in the $E2$ operator to reproduce these transitions with IBM-1. The IBM O(6) forbidden transitions in the present calculation are usually 1 to 2 orders of magnitude weaker than the IBM O(6) allowable transitions, which is in agreement with the findings of Ref. [20].

In Fig. 1 the energy level as well as the branching ratios are compared with the observations and with IBM calculations for ^{196}Pt. The overall FDSM results are in quantitative agreement with the IBM-1 O(6) level pattern and branching ratios. This agreement confirms that while the FDSM does not have an explicit mathematical SO(6) dynamical symmetry, it has a remarkably accurate practical one for this case.

In comparing this calculation with the interacting boson model, there is one point that merits particular attention. In the boson case, the concept of F spin is crucial to classify the low-lying symmetric and mixed-symmetry states [14]. The "Majorana interaction" is then used to ensure that mixed-symmetry states do not come at unacceptably low energies in the spectrum. It is not clear how to introduce F spin in a fermion system and there is no analogous Majorana interaction in our fermionic Hamiltonian: For a fermion many-body system, the positions of the mixed-symmetry states are governed by the basic pairing and multipole interaction strengths [15]. This was also noticed by Arima *et al.* [16] in their application of the Ginocchio schematic model to the Samarian isotopes, and by Zamick in the shell-model studies in the fp shell-model region [17]. For the ^{196}Pt case that we discuss, the mixed-symmetry 1^+ state is at an excitation energy of approximately 2 MeV, and this is correctly predicted by our calculations without introducing the analog of a Majorana term. Thus, the effective fermion SO(6) symmetry produces a quantitative agreement with the spectrum using a simpler Hamiltonian than is required in the IBM-2.

Because of the different shell symmetries in the FDSM, the effective SO(6) dynamical symmetry that is realized in the platinum region and the SO(6) symmetry realized in the mass-130 region differ fundamentally at the microscopic level. The Xe-Ba region corresponds to a $SO^{\nu}(8) \times SO^{\pi}(8)$ coupling scheme in the FDSM; this scheme admits a coupled $SO(6)^{\pi+\nu}$ dynamical symmetry and a formal SO(6) symmetry is possible for these nuclei. For Pt isotopes, the realization of the SO(6)-like structure is critically dependent on a specific Hamiltonian and the number of valence pairs. Thus, in this view the SO(6) dynamical symmetry of the mass-130 region is required by the shell structure, while (ironically) the original example of O(6) symmetry in the Pt nuclei is more accidental. The shell structure permits it, but does not demand it. This would explain the more ubiquitous appearance of SO(6) symmetry in the Xe-Ba region. We note in this connection that the Xe-Ba region not only exhibits an SO(6) symmetry in the even-even isotopes but the even-odd ones as well, and that the level pattern for the latter is quite different from the Pt even-odd ones. A more extensive discussion of the shell structure dependence of the SO(6)-like symmetry in platinum and the mass-130 regions will be published elsewhere [18].

Finally, we emphasize that the parameters used in reproducing the SO(6)-like spectrum and the branching ratios of the platinum isotopes do not represent a peculiar set chosen solely for this purpose. Similar calculations in the remainder of the rare earth nuclei indicate that the Pt parameters are related to the effective interactions of the entire shell. For example, if the same five parameters are used in the FDU0 calculation but the appropriate valence particle numbers for heavy Gd isotopes are chosen, the spectrum and transition rates become SU(3)-like (axially symmetric rotors). With only small adjustments in these parameters (for example, less than 5% for the n-p quadrupole coupling, which is the dominant parameter) the spectrum and transition rates for the low-lying states in the heavier Gd isotopes are *quantitatively* reproduced. Likewise, a similar set of parameters leads to vibrational [SU(2)-like] spectra for valence particle numbers near the beginning or the end of the shell. These investigations are in progress.

These results imply that a fixed Hamiltonian with parameters varying slowly with particle number can produce spectra that evolve from SU(2) to SU(3), from SU(3) to SO(6), and from SO(6) to SU(2), as the proton or neutron number changes. This is similar to the SU(5)-O(6)-SU(3) triangle relation [19] in the IBM, except that we now have an SU(2)-SO(6)-SU(3) triangle. However, unlike the IBM, the triangle relationships in the FDSM are *shell dependent.* For the SO(8) $\times$ SO(8) shells there is no SU(3) limit and the SO(6) symmetry is the most collective limit (corresponding to γ-unstable rotations). In this case the triangle relationship is replaced by SU(2)–SO(7)–SO(6). We speculate that whenever there is a vibrational [SU(2)-like] symmetry limit on one corner of the triangle and an SU(3)-like rotational limit on a second corner of the triangle, an SO(6)-like effective transitional symmetry may occur, even though there is formally no such dynamical symmetry in the shells. Such speculation raises the possibility that effective SO(6)-like spectra might occur in the transitional regions of $\mathrm{Sp}(6)^{\pi} \times \mathrm{Sp}(6)^{\nu}$ shells, even though this symmetry has no formal SO(6) subgroup that satisfies basic conservation laws. This conjecture could be tested in the $Z = 82$–126 and $N = 126$–184 shells where the expected symmetry is $\mathrm{Sp}(6)^{\pi} \times \mathrm{Sp}(6)^{\nu}$, although candidate SO(6)-like nuclei in this region may not be stable nuclides.

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