

Relation between the phenomenological interactions of the algebraic cluster model and the effective two-nucleon forces

K. Varga and J. Cseh

Institute of Nuclear Research of the Hungarian Academy of Sciences (MTA ATOMKI) Debrecen, Hungary

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We determine the phenomenological cluster-cluster interactions of the algebraic model corresponding to the most often used effective two-nucleon forces for the $^{16}\text{O} + \alpha$ system.

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I. INTRODUCTION

More than ten years ago the vibron model was proposed as a phenomenological algebraic description of the nuclear cluster or molecular states [1]. This is a model of the dipole collective motion, which is applied also in molecular [2] and in hadron spectroscopy [3]. Its formalism is very similar to that of the interacting boson model of the quadrupole collectivity of nuclei [4].

The interaction of the clusters in the vibron model is treated in a phenomenologic way, i.e., the Hamiltonian is expressed in terms of boson operators, and the expansion coefficients are fitted to experimental data. It is an interesting question of how these phenomenological algebraic cluster-cluster interactions are related to the effective two-nucleon forces, which are applied, e.g., in microscopic cluster studies [5]. Except for a brief discussion in [6] this question has not been investigated so far, although it is of great importance from the viewpoint of the microscopic foundation of the algebraic cluster model. The present paper is meant to be a contribution to this task.

The application to some well-known cluster bands in light nuclei revealed that (i) the $U(4) \supset U(3) \supset O(3)$ basis of the vibron model is preferred to the other possible basis, and (ii) this basis has to be truncated in a well-defined way in order to get rid of the Pauli forbidden states [7–9]. In particular, the n_π quantum number, which is the representation label of the $U(3)$ group and gives the number of oscillator quanta in the relative motion of the two clusters, has to be larger than a limit obtained from the Wildermuth condition [10]. By taking into account the Pauli blocking in this way, the model space of the algebraic description becomes a subset of the model space of the microscopic $SU(3)$ cluster model [5, 10]. [Due to the finite value of the $U(4)$ representation index n_π has an upper limit, too.]

When the internal degrees of freedom of the clusters play an important role, i.e., in case of non-closed-shell clusters, the model space is larger and the group structure of the algebraic description is more complicated. Nevertheless, for systems of non-closed-shell clusters the model space can also be constructed in such a way that it is free from the Pauli forbidden states and the spurious center-of-mass motion [6, 11]. It is done by using the $SU(3)$ shell

model for the description of the internal cluster degrees of freedom, instead of the previously applied phenomenological interacting boson [12], or interacting fermion [13] models. Again, the model space is a subspace of that of the microscopic $SU(3)$ cluster model.

The algebraic approach, in which the model space is free from the forbidden states and the interactions are treated phenomenologically, is called the semimicroscopic algebraic description. Since the basis states of this description have a one-to-one correspondence with the $SU(3)$ basis states of the microscopic cluster models, one can relate the phenomenological cluster-cluster interactions to the effective two-nucleon forces simply by equating the corresponding matrix elements. Here we present such a relation for the example of the $^{16}\text{O} + \alpha$ system.

In what follows, in Sec. II we give the matrix elements of the vibron model Hamiltonian, and show how it fits the energy spectrum of some selected bands in the ^{20}Ne nucleus. In Sec. III the matrix elements of some frequently applied effective two-nucleon forces are calculated. Finally, the relation between the phenomenological and microscopic interactions is discussed in Sec. IV.

II. THE PHENOMENOLOGICAL HAMILTONIAN AND ITS MATRIX ELEMENTS

In the algebraic description of clusterization the spectrum is generated by the interactions of a finite number (N) of bosons, which can occupy single-particle states with angular momentum and parity: 0^+ (σ bosons) and 1^- (π bosons). The total number of particles is conserved, therefore the creation and annihilation operators appear only in number conserving bilinear forms. They generate the $U(4)$ group, and the group structure of the model manifests itself in a twofold way: not only the basis states are characterized by the representation labels of the group chain

$$\begin{aligned}
 &U(4) \supset U(3) \supset O(3), \\
 &|N, n_\pi, L\rangle, \\
 &n_\pi = N, N-1, \dots, 0, \quad L = n_\pi, n_\pi-2, \dots, 1 \text{ or } 0,
 \end{aligned} \tag{1}$$

but also the physical operators are obtained in terms of the generators of the U(4) group [2]. (We consider here, as mentioned in the Introduction, the simple case of closed-shell clusters.) An especially important limiting situation, called dynamical symmetry, is reached when the Hamiltonian can be expressed in terms of the Casimir invariants of the group chain (1). Then the eigenvalue problem has an analytical solution. If we consider only one- and two-body interactions, the Hamiltonian of the U(3) dynamical symmetry can be written as

$$H = h_0 C_{1U4} + h_1 C_{2U4} + h_2 C_{1U3} + h_3 C_{2U3} + h_4 C_{2O3}, \quad (2)$$

where the C 's stand for the Casimir operators of the indicated order, e.g., C_{2U3} is the second-order Casimir of the U(3) group, and h_i 's are phenomenological parameters. The Hamiltonian matrix is diagonal in the basis (1), and the energy eigenvalues are

$$E = \epsilon + \beta L(L+1) + \gamma n_\pi + \delta n_\pi^2. \quad (3)$$

Here $\beta = h_3$, $\gamma = h_2 + 3h_3$, $\delta = h_3$, and the eigenvalues of the U(4) Casimir operators could be involved in the constant ϵ , because N is conserved.

Our aim here is to relate these phenomenological parameters to the effective two-nucleon forces. Nevertheless, in order to illustrate the ability of the vibron model Hamiltonian containing only one- and two-body terms, it is worthwhile to show to what extent can it fit an experimental spectrum. For this purpose we have chosen the ^{20}Ne nucleus, because its $K^\pi = 0_1^+, 0^-, 0_4^+$ bands are known to have a well-developed $^{16}\text{O} + \alpha$ cluster structure [14]. In addition, more recently the $K^\pi = 0_5^+$ band has been established based on alpha-scattering data [15], although the assignments of the $J \geq 4$ spins are less certain. The experimental spectrum that we have con-

sidered [15, 16] is plotted on the left-hand side of Fig. 1. Their description in terms of the Hamiltonian (2) and (3) with U(3) dynamical symmetry is shown on the right-hand side. For the $^{16}\text{O} + \alpha$ system the Wildermuth condition gives $n_\pi \geq 8$ [7, 8]. The parameters of the energy expression (3) were obtained from a least-squares fitting procedure, in which the weight of the uncertain states was 0.5, in comparison with the usual weight of 1.0 of the fully established states. (For the 6^+ and 8^+ members of the 0_5^+ band we have chosen the total weight of the 2 or 3 candidates to be 0.5.) The parameter values are (in MeV): $\beta = 0.161$, $\gamma = 13.601$, $\delta = -0.571$, $\epsilon = -71.040$.

III. MATRIX ELEMENTS OF THE EFFECTIVE TWO-NUCLEON INTERACTIONS

In the microscopic cluster model the wave function of the ^{20}Ne nucleus is given by

$$\Psi_{^{20}\text{Ne}} = n_0 \mathcal{A} \{ \phi_\alpha \phi_{^{16}\text{O}} \chi(\mathbf{r}_{\alpha-^{16}\text{O}}) \}, \quad (4)$$

where \mathcal{A} is an intercluster antisymmetrizer [17] and ϕ_α and $\phi_{^{16}\text{O}}$ are normalized antisymmetric internal wave functions of the alpha particle and the ^{16}O nucleus, respectively. Furthermore, $\chi(\mathbf{r}_{\alpha-^{16}\text{O}})$ is the wave function of the relative motion and n_0 is a normalization constant. If the internal states are harmonic oscillator shell model ground states of common size parameter $\nu (= \frac{m\omega}{2\hbar})$ [SU(3) scalars: $(\lambda, \mu) = (0, 0)$] and the wave function of the relative motion is a harmonic oscillator wave function φ_{nlm} of size parameter $\frac{16 \times 4}{16+4} \nu$ [belonging to the $(2n+l, 0)$ representation], then using the language of Elliott's SU(3) group [18]

$$\psi_{nl} = n_0 \mathcal{A} \{ \phi_\alpha \phi_{^{16}\text{O}} \varphi_{nlm}(\mathbf{r}_{\alpha-^{16}\text{O}}) \} \quad (5)$$

belongs to the SU(3) irreducible representation $(2n+l, 0)$ [19, 20]. (We note here the $n_\pi = 2n+l$ relation of the quantum numbers [7, 8].)

The conventional technique to calculate the matrix elements of the microscopic Hamiltonian

$$H = \sum_i T_i + \sum_{ij} V_{ij} \quad (6)$$

(sandwiched between the wave function ψ_{nl}) is based on the fact that the "shifted" Gaussian function

$$\varphi_s^\alpha(\mathbf{r}) = \left(\frac{2\alpha}{\pi} \right)^{3/4} e^{-\alpha(\mathbf{r}-\mathbf{s})^2} \quad (7)$$

is a generating function of the harmonic oscillator function [17]:

$$\varphi_{nlm}(\mathbf{r}) = A_{nl} \int d\hat{s} Y_{lm}(\hat{s}) \frac{d^{2n+l}}{ds^{2n+l}} \varphi_s^\alpha(\mathbf{r}) e^{\frac{\alpha}{2} \mathbf{s}^2} |_{s=0}, \quad (8)$$

where

$$A_{nl} = (-1)^n \sqrt{\frac{(2n+l)!}{4\pi(2n)!!(2n+2l+1)!!}}. \quad (9)$$

Using Eqs. (8) the matrix elements of the states ψ_{nl} can be derived as

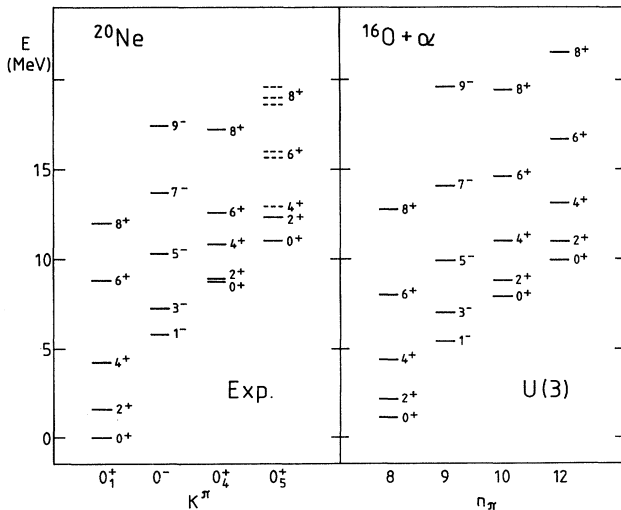


FIG. 1. Experimental states of ^{20}Ne in comparison with the phenomenological model calculation in terms of U(3) dynamical symmetry. The dashed lines indicate uncertain band assignments.

$$\langle \psi_{nl} | H | \psi_{nl} \rangle = A_{nl}^2 \int d\hat{s} Y_{lm}(\hat{s}) \int d\hat{s}' Y_{lm}(\hat{s}') \frac{\partial^{4n+2l}}{\partial s^{2n+l} \partial s'^{2n+l}} H(\mathbf{s}, \mathbf{s}') \Big|_{s=s'=0}. \quad (10)$$

The matrix elements on the right-hand side, the so-called generator coordinate method (GCM) matrix elements of ^{20}Ne (see, e.g., [17])

$$H(\mathbf{s}, \mathbf{s}') = \langle \mathcal{A}\{\phi_\alpha \phi_{1s_0} \varphi_{\mathbf{s}}^{\frac{16 \times 4}{16+4}\nu}(\mathbf{r}_{\alpha-1s_0})\} | H | \mathcal{A}\{\phi_\alpha \phi_{1s_0} \varphi_{\mathbf{s}'}^{\frac{16 \times 4}{16+4}\nu}(\mathbf{r}_{\alpha-1s_0})\} \rangle, \quad (11)$$

can be calculated easily. Owing to the well-known theorem of Elliott and Skyrme [21], one can express the wave function $\mathcal{A}\{\phi_\alpha \phi_{1s_0} \varphi_{\mathbf{s}'}^{\frac{16 \times 4}{16+4}\nu}(\mathbf{r}_{\alpha-1s_0})\}$ by a Slater determinant of harmonic oscillator single-particle orbits centered around \mathbf{s}_α and \mathbf{s}_{1s_0} :

$$n_0 \mathcal{A}\{\phi_\alpha \phi_{1s_0} \varphi_{\mathbf{s}'}^{\frac{16 \times 4}{16+4}\nu}(\mathbf{r}_{\alpha-1s_0})\} = \left(\frac{\pi}{2 \cdot 20\nu} \right)^{3/4} e^{+20\nu(\mathbf{S}-\mathbf{R})^2} \frac{1}{\sqrt{20!}} \det\{(000)_{\mathbf{s}_{1s_0}}^4 (010)_{\mathbf{s}_{1s_0}}^4 (011)_{\mathbf{s}_{1s_0}}^4 (01-1)_{\mathbf{s}_{1s_0}}^4 (000)_\alpha^4\}. \quad (12)$$

In this equation $(nlm)_\mathbf{s}^4$ stands for the harmonic oscillator shell model orbit nlm centered around \mathbf{s} and filled by four nucleons with different spin-isospin configuration. For example,

$$(000)_{\mathbf{s}_{1s_0}}^4 = \varphi_{000}(\mathbf{x}_1 - \mathbf{s}_{1s_0}) \eta_{p\uparrow}(1) \varphi_{000}(\mathbf{x}_2 - \mathbf{s}_{1s_0}) \eta_{p\downarrow}(2) \varphi_{000}(\mathbf{x}_3 - \mathbf{s}_{1s_0}) \eta_{n\uparrow}(3) \varphi_{000}(\mathbf{x}_4 - \mathbf{s}_{1s_0}) \eta_{n\downarrow}(4), \quad (13)$$

where \mathbf{x}_i is the single-particle coordinate, $\eta_{\sigma\tau}(i)$ is the spin-isospin function of the i th nucleon. The parameter coordinates can be related as $\mathbf{s} = \mathbf{s}_{1s_0} - \mathbf{s}_\alpha$ and $\mathbf{S} = \frac{1}{20}(16\mathbf{s}_{1s_0} + 4\mathbf{s}_\alpha)$, and \mathbf{R} is the center-of-mass coordinate of ^{20}Ne .

Using the reduction formulas given by [22], the matrix elements between Slater determinants can be expressed by the single-particle matrix elements of the Hamiltonian and by the overlap of the (nonorthogonal) single-particle orbits. To facilitate the analytical calculation the nucleon-nucleon interactions used in the microscopic cluster model are linear combination of Gaussian potentials

$$V_{ij} = \sum_k (W_k + M_k P_{ij}^x + B_k P_{ij}^\sigma + H_k P_{ij}^x P_{ij}^\sigma) V_k e^{-(\mathbf{x}_i - \mathbf{x}_j)^2 / d_k^2}, \quad (14)$$

where P_{ij}^x is the spatial, P_{ij}^σ the spin exchange operator between particles i and j , W_k , M_k , B_k , and H_k are the Wigner, Majorana, Bartlett, and Heisenberg parameters, V_k is the strength, and d_k is the diffusivity of the

potential. Using these Gaussian interactions the matrix element $H(\mathbf{s}, \mathbf{s}')$ has the following form:

$$H(\mathbf{s}, \mathbf{s}') = \sum_i c_i s^{2k_i} s'^{2k'_i} \mathbf{s}\mathbf{s}'^{m_i} e^{-a_i s^2 - a'_i s'^2 + b_i \mathbf{s}\mathbf{s}'}. \quad (15)$$

The c_i , a_i , a'_i , and b_i constants are expressed by the W_k , M_k , B_k , H_k , V_k , and a_k parameters of the potential and the ν parameter of the harmonic oscillator wave function. The explicit analytical expressions are too lengthy to be tabulated here and the interested reader can find them in Refs. [23–25].

The normalization constant n_0 can be determined in the same manner, putting the unity operator in place of H .

In determining the SU(3) matrix elements numerically, we have considered some of the most conventional effective interactions used in the cluster model calculations: a phenomenological one proposed by Volkov [Volkov force number 2 (V2)] [29], and a potential of reaction matrix type given by Hasegawa and Nagata (HN1 and HN2) [27], and a modified version of the latter force [28]. All

TABLE I. Parameters of the effective nuclear potentials.

	d_k (fm ²)	V_k (MeV)	W_k	M_k	H_k	B_k
HN1	2.5	-6.0	0.4583	0.4583	0.0417	0.0417
	0.94	-546.0	0.4148	0.4148	0.0852	0.0852
	0.54	1655.0	0.4229	0.4229	0.0771	0.0771
HN2	2.5	-6.0	-0.2361	1.1528	0.5972	-0.5139
	0.94	-546.0	0.4148	0.4148	0.1310	0.0394
	0.54	1655.0	0.4474	0.3985	0.1015	0.0526
MHN	2.5	-6.0	-0.2361	1.1528	0.5972	-0.5139
	0.94	-546.0	0.4240	0.4057	0.1401	0.0302
	0.54	1655.0	0.4474	0.3985	0.1015	0.0526
V2	1.80	-60.65	0.38	0.62	0	0
	1.01	61.14	0.38	0.62	0	0

TABLE II. SU(3) matrix elements of the Hamiltonian with Volkov number 2 force for ^{20}Ne (in MeV).

(λ, μ)	$n = 0$	1	2	3	4	5	6	7
(8,0)	2.97	0.33	-1.90	-3.46	-4.16	–	–	–
(9,0)	16.46	12.93	10.23	8.33	7.27	–	–	–
(10,0)	26.75	23.29	20.49	18.41	17.08	16.51	–	–
(11,0)	37.97	34.48	31.44	29.19	27.63	26.76	–	–
(12,0)	48.36	44.80	41.84	39.47	37.72	36.61	36.13	–
(13,0)	58.70	55.17	52.19	49.74	47.85	46.54	45.80	–
(14,0)	68.68	65.23	62.26	59.77	57.77	56.29	55.34	54.94

parameters of the forces NH1, NH2, and MHN are fixed. The Volkov force has a parameter (Majorana exchange parameter) for the strength of the odd-parity state. This parameter can be adjusted to the separation energy of the clusters. These forces had been successfully applied in GCM calculations for ^{20}Ne [26] and for other light nuclei. The parameters of these effective nucleon-nucleon interactions can be found in Table I. As an illustrative numerical example the matrix elements $\langle \psi_{nl} | H | \psi_{nl} \rangle$ using V2 force are listed in Table II. In agreement with Matsuse *et al.* [26] the harmonic oscillator size parameter and the Majorana parameter of the Volkov force is chosen to be $\nu = 0.16 \text{ fm}^{-2}$ and $M_k = 0.62$ ($k = 1, 2$), respectively.

IV. PHENOMENOLOGICAL INTERACTIONS FROM MICROSCOPIC FORCES

The parameters of the phenomenological interactions (2) and (3) can be obtained from the effective two-nucleon forces by equating the corresponding matrix elements of the two descriptions. Taking, e.g., the H_{32} , H_{40} , H_{50} , H_{60} matrix elements, where we have used the simplified notation of $H_{nl} = \langle \psi_{nl} | H | \psi_{nl} \rangle$, with the straightforward relations

$$\begin{aligned}
 \beta &= \frac{1}{6}(H_{32} - H_{40}) , \\
 \gamma &= \frac{1}{4}(-11H_{40} + 20H_{50} - 9H_{60}) , \\
 \delta &= \frac{1}{8}(H_{40} - 2H_{50} + H_{60}) , \\
 \epsilon &= (15H_{40} - 24H_{50} + 10H_{60}) ,
 \end{aligned}
 \tag{16}$$

and using their analytical expressions in [23–25], we can obtain the Hamiltonian of the vibron model from the effective two-nucleon forces.

Since, however, these relations are rather complicated, the deduction of the phenomenological parameters from the numerical values corresponding to specific two-nucleon interactions can help to illuminate the situation. We have done that for the microscopic interactions mentioned in Sec. III.

In order to obtain the phenomenological interaction from the microscopic one, we can use a set of four appropriate matrix elements, e.g., as shown above; or we can extract the parameters from a large number of ma-

trix elements by a fitting procedure. We have followed the second way, which obviously gives more reliable parameters. When doing so, we have considered the 101 matrix elements with the $8 \leq n_\pi \leq 20$ quantum numbers. From such a calculation one obtains in a natural way also a quantitative measure of the average deviation between the microscopic and phenomenologic matrix elements. In Table III we have listed the parameters (in MeV) belonging to the different two-nucleon forces, and in the last column the average root mean square deviation of the microscopic and phenomenologic matrix elements are given, too. Considering the fact that the energy region of these matrix elements spans more than 100 MeV, the deviation is not very large. In other words, the phenomenological Hamiltonian containing only first- and second-order terms can approximate these effective two-nucleon forces reasonably well.

By comparing these sets of parameters with the one that gave the best description of the experimental spectrum of Fig. 1, we can realize some similarities both in the signs and in the overall magnitudes. But there are some differences as well; the most remarkable ones are the larger experimental β and δ values, giving rise to more definite splitting with respect to the L and n_π quantum numbers.

V. SUMMARY

In this paper we have obtained the phenomenological cluster-cluster interaction of the vibron model from effective two-nucleon forces. The example we have considered was the $^{16}\text{O} + \alpha$ system, but the same procedure can be applied to other cases, too. This relation is based on the similarities between the model spaces of the two descrip-

TABLE III. Parameters of the phenomenological cluster-cluster interactions obtained from effective two-nucleon forces. The last column gives the average rms deviation between the microscopic and phenomenologic matrix elements. (The values are given in MeV.)

Force	β	γ	δ	ϵ	d
V2	0.0493	12.289	-0.119	-91.329	1.15
HN1	0.0720	13.776	-0.149	-123.28	1.79
HN2	0.0687	13.042	-0.136	-106.01	1.77
MHN	0.0562	12.415	-0.121	-93.145	1.46

tions, which was established by the modification of the basic assumptions of the vibron model, via selecting out the forbidden states. The relation can be given in analytical expressions, but they are too complicated even for the simplest example, i.e., for the case of the two closed-shell clusters. On the other hand, the numerical values of the phenomenological parameters can be obtained easily

from the effective two-nucleon forces. For the most often used microscopic interactions we have given the corresponding Hamiltonians of the vibron model.

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