Δ-excited nuclear matter in the derivative scalar coupling model

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The equation of state of the baryonic matter containing nucleons and delta particles using the derivative scalar coupling model is studied. Comparison of the results with those obtained in a nonlinear σ-ω model yields the same equilibrium value of the effective mass and the compressibility constant.

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I. INTRODUCTION

The relativistic mean-field study of the hot and dense baryonic matter using the Walecka model has received wide attention in recent years due to its remarkable success in predicting many properties of nuclear matter and finite nuclei [1]. The model is simple, renormalizable, and uses explicit mesonic degrees of freedom. Moreover, two undesirable features of the model, viz., large decrease of the nuclear effective mass $m^*(Q_0)$ value at the equilibrium density Q_0 and the high value of the compressibility constant $K(Q_0)$ can be eliminated by adding a nonlinear cubic and quartic scalar meson interaction in the Lagrangian [2], which requires two additional parameters in the model. Recently Zimanyi and Moszkowski (ZM) have introduced a model of nuclear matter containing the scalar meson, vector meson, and nucleon as in the Walecka model except with a linear coupling between the scalar meson and the nucleon [3]. The model of Zimanyi and Moszkowski [3] yields not only the correct binding energy of -16.0 MeV at the saturation density Q_0 ($\cong 0.16$ fm^{-3}), but also a high value of the effective nucleon mass $m^*(Q_0) \cong 0.85 m_N$ and an acceptable value for the compressibility constant $K(Q_0)=225$ MeV at the above saturation density Q_0. The derivative scalar coupling model (DSC) of Zimanyi and Moszkowski has been successfully applied to the study of multilambda matter [4] and to the construction of the equation of state of the neutron star [5]. The relativistic Thomas Fermi calculations of the compression properties of the finite nuclear system show that near the saturation, the derivative scalar coupling model of ZM exhibits a behavior close to that in the standard Skyrme interaction SKM* but at high densities the trend departs away from that in SKM* [6]. Again, a comparative study of the nuclear properties using the DSC model of ZM and nonlinear σ-ω model of Walecka shows that while the former model reproduces the correct nuclear effective mass $m^*(Q_0)$ and the compressibility constant $K(Q_0)$, the latter yields correct spin-orbit splitting [7].

In this work, we report our studies of the properties of delta-excited nuclear matter using the derivative scalar coupling model of Zimanyi and Moszkowski [3]. Previous studies of delta-excited nuclear matter using the nonlinear σ-ω model of Walecka have shown that if universal couplings for the scalar meson with two baryons (N,Δ) are chosen, the nuclear effective mass $m^*(Q)$ becomes negative at high densities. However, when scalar meson-baryon couplings are chosen in the ratio of SU(6) symmetry breaking of the baryon masses, the nuclear effective mass $m^*(Q)$ becomes positive at all densities. Also, the matter becomes highly delta dominated at a small value of total baryon density ($\mu_B \cong 2000$ MeV) in the Walecka model under this choice of the scalar meson-baryon coupling [8]. In the present work, we have shown that the properties of Δ-excited nuclear matter in the DSC model are extremely insensitive to the above two different choices of the scalar meson-baryon couplings. Furthermore, in order to establish a meaningful comparison we have also shown the results of our calculations of the baryonic matter using the nonlinear σ-ω model of Walecka, which yields nearly the same value of the nucleon effective mass $m^*(Q_0)$ ($\cong 0.85 m_N$) and the compressibility constant $K(Q_0)$ ($\cong 210$ MeV) at saturation density, as obtained in the DSC model. This comparative study brings out the fact that the two nonlinear meson field theoretical models with two different types of nonlinearity, reproduce almost identical equations of state at zero temperature at all densities of pure nuclear matter (with no Δ present), due to nearly the same values of the effective mass $m^*(Q_0)$ and the compressibility constant $K(Q_0)$. We mention here that the mean-field approximation used in the present calculations does not provide an adequate description of the baryonic matter at high temperatures ($T \cong 100$ MeV). Hence, important corrections due to the contributions from vacuum polarization effect and nucleon correlations might have appreciable influence. One should also expect that the internal structures of the nucleons and mesons, i.e., quarks and gluons, exhibit the proper degree of freedom for discussion of the baryonic matter at these high temperatures. However, when Δ-excited nuclear matter at high temperature is studied, two models predict quite different behavior.

II. THEORY

For the calculation, we consider the following Lagrangian obtained after rescaling the fermion wave functions as prescribed in the work of Zimanyi and

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Moszkowski [3]:

$$\mathcal{L}=\bar{\Psi}_N(i\gamma_\mu\delta^\mu-m_N^*-g_{\omega N}\gamma_\mu\omega^\mu)\psi_N-\tfrac{1}{4}F_{\mu\nu}+\tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$
$$+\tfrac{1}{2}(\delta_\mu\sigma\delta^\mu\sigma-m_\sigma^2\sigma^2)$$
$$+\bar{\psi}_\Delta(i\gamma_\mu\delta^\mu-m_\Delta^*-g_{\omega\Delta}\gamma_\mu\omega^\mu)\psi_\Delta\ . \tag{1}$$

In the case of the nonlinear Walecka model, the Lagrangian is

$$\mathcal{L}=\mathcal{L}=\tfrac{1}{3}b\sigma^3-\tfrac{1}{4}c\sigma^4\ . \tag{2}$$

The delta contribution in Eq. (1) has been added in a self-explanatory way and we have used the baryon effective masses

$$m_N^*=m_N/(1+g_{\sigma N}\sigma/m_N);\quad m_\Delta^*=m_\Delta/(1+g_{\sigma\Delta}\sigma/m_\Delta) \tag{3a}$$

for the Lagrangian (1) corresponding to the work of Zimanyi and Moszkowski and

$$m_N^*=m_N-g_{\sigma N}\sigma;\quad m_\Delta^*=m_\Delta-g_{\sigma\Delta}\sigma \tag{3b}$$

for the Lagrangian (2) corresponding to the nonlinear Walecka model. Thus, the baryon effective masses are related as

$$m_\Delta^*=m_\Delta\left[1+\frac{g_{\sigma\Delta}}{g_{\sigma N}}\frac{m_N}{m_\Delta}\left[\frac{m_N}{m_N^*}-1\right]\right]^{-1} \tag{4a}$$

for definition (3a) and

$$m_\Delta^*=m_\Delta\left[1+\frac{g_{\sigma\Delta}}{g_{\sigma N}}\frac{m_N}{m_\Delta}\left[\frac{m_N^*}{m_N}-1\right]\right] \tag{4b}$$

for definition (3b). $g_{\sigma B}$ and $g_{\omega B}$ in the expression for the Lagrangian are the meson-baryon couplings with B standing for the nucleon and the delta. It should be noted here that the spin-$\frac{3}{2}$ delta particles are treated as an effective particle and the reservation expressed regarding its inclusion in the original renormalizable Walecka σ-ω nuclear model due to nonrenormalizability of the spin-$\frac{3}{2}$ field [9] does not arise in this nonrenormalizable DSC model.

For the symmetric infinite nuclear matter, the above Lagrangian in the meson-field approximation [1] yields the following field equations:

$$(\Delta-m_v^2)W_0=-\sum_{B=N,\Delta}g_{vB}\rho_{vB}\ , \tag{5}$$

$$(\Delta-m_s^2)\sigma_0=-\sum_{B=N,\Delta}g_{sB}\xi\rho_{sB}+\delta\ . \tag{6}$$

The quantities δ and ξ are defined as

$$\delta=b\sigma_0^2+c\sigma_0^2\ , \tag{7a}$$

$$\xi=1\ , \tag{7b}$$

for the Lagrangian (2) of the nonlinear Walecka model and

$$\delta=0\ , \tag{8a}$$

$$\xi=(m_B^*/m_B)^2\ , \tag{8b}$$

for the Lagrangian (1) of the ZM model. ρ_{vB} and ρ_{sB} in Eqs. (5) and (6) are the vector and the scalar densities, respectively.

Now, after taking the ensemble average of the meson fields σ and ω_μ in the standard way [1], the following self-consistent equation for the determination of the dimensionless nucleon effective mass x_N,

$$1-x_N-C_S^2\left[x_N^4n_S^N(\vartheta,\pm\mu)+\left[\frac{g_{\sigma\Delta}}{g_{\sigma N}}\right]\left[\frac{m_\Delta}{m_N}\right]^3x_Nx_\Delta^3n_S^\Delta\right.$$
$$\left.\times(\vartheta,\pm\mu)-b(1-x_N)^2-C(1-x_N)^3\right]=0\ , \tag{9}$$

can be obtained. The scalar densities n_S^B and the vector densities n^B $(B=N,\Delta)$ in Eq. (9) are related to the Fermi-distribution function $n_F^B\ (\vartheta\pm\mu_B)$ at the temperature ϑ and with the chemical potential μ_B in the following way:

$$n_S^B=\frac{\gamma_B}{(2\pi)^3}\int\frac{d^3tx_B}{(t^2+x_B^2)^{1/2}}[n_F^B(\vartheta,\mu)+n_F^B(\vartheta,-\mu)]\ , \tag{10}$$

with $B=N,\Delta$,

$$n^B=\frac{\gamma_B}{(2\pi)^3}\int d^3t[n_F^B(\vartheta,\mu)-n_F^B(\vartheta,-\mu)]\ . \tag{11}$$

In Eq. (9) we have used the dimensionless coupling constants

$$C_s^2=\frac{g_{\sigma N}^2m_N^2}{m_s^2};\quad b=\frac{B}{m_Ng_s};\quad C=\frac{C}{g_{\sigma N}^4} \tag{12}$$

and we also define $C_v^2=g_{\omega N}^2m_N^2/m_\omega^2$ to obtain the total baryon density Q_B and the chemical potential μ_B as

$$Q_B=n_v^N+\frac{g_{\omega\Delta}}{g_{\omega N}}n_v^\Delta\ ,$$

$$\mu=\nu+C_vQ_B\ .$$

The total baryon energy density of the baryonic matter can be calculated from x_B using the relation

$$\varepsilon=\frac{1}{2C_s^2}(1-x_N)^2+\frac{C_v^2}{2}n_B^2$$
$$+\sum_{B=N,\Delta}\frac{\gamma_B}{(2\pi)^3}\int(t^2+x_B^2)^{1/2}d^3t[n_F^B(\vartheta,\mu)$$
$$+n_F^B(\vartheta,-\mu)]\ . \tag{13}$$

III. RESULTS AND DISCUSSION

Results of our calculations, shown in Figs. 1–4, are based on the phenomenological law of universal vector coupling, $g_{\omega\Delta}=g_{\omega N}$. Regarding the scalar meson-baryon couplings, different choices and their consequences have been discussed before in detail in Ref. [8]. We have car-

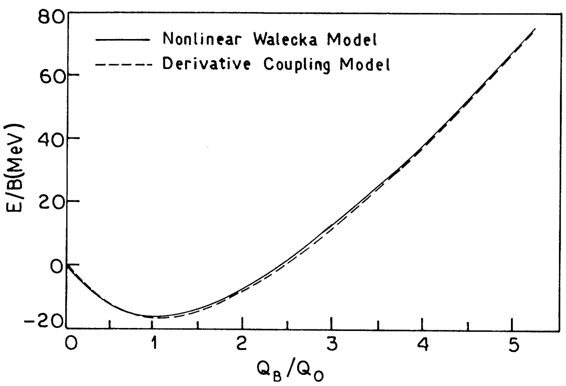

FIG. 1. Binding energy per nucleon (E/B, $E/B=\varepsilon/B-M$) against normalized density Q_B/Q_0 ($Q_0=0.16$ fm^{-3}) in the two models at zero temperature.

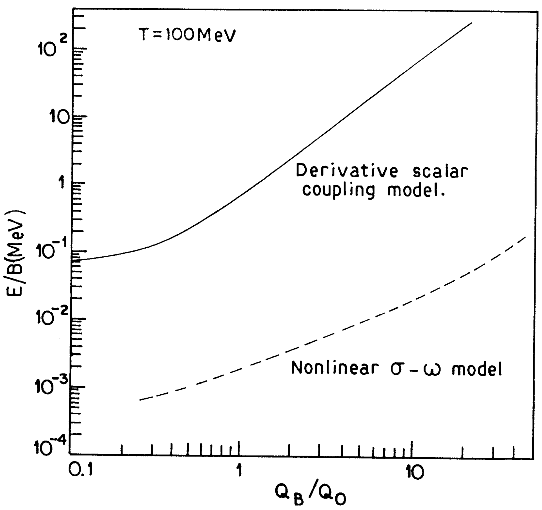

FIG. 3. Energy density of the baryonic matter at various normalized density values Q_B/Q_0.

ried out calculations using two different choices: (i) $g_{\sigma\Delta}=g_{\sigma N}$, and (ii) $g_{\sigma\Delta}=m_\Delta/m_N g_{\sigma N}$. The value of the other coupling constants utilized in this work to fit the normal nuclear matter are (i) the DSC model—$C_s^2=143.0$, $C_v^2=46.6$; (ii) the nonlinear model—$C_s^2=183.683$, $C_v^2=64.545$, $b=0.17788e-1$, $C=0.396674e-1$. In Fig. 1, the equation of state of ordinary nuclear matter at zero temperature ($T=0$) is drawn using the derivative scalar coupling model and the nonlinear σ-ω model, which predicts almost identical equations of state over a wide range of densities. It is found that both the nonlinear models predict the same values for the nucleon effective mass $m^*(Q_0)$ ($\cong 0.85 m_N$) and the bulk modulus $K(Q_0)$ ($\cong 210$ MeV) at the saturation density Q_0 ($\cong 0.16$ fm^{-3}). Similarly, the nucleon effective mass $m^*(Q)$ in the two nonlinear models is found to be almost equal over a large region of densities. This behavior is consistent with the earlier observation [2] that the equation of state of the nuclear matter in the relativistic mean-field theory depends chiefly upon the equilibrium value of the effective mass $m^*(Q_0)$ and the compressibility constant $K(Q_0)$ of the model.

However, $m^*(Q)$ of the delta-excited nuclear matter at a temperature of $T=100$ MeV in the above two models as drawn in Fig. 2 shows that the effective mass in the two nonlinear models diverges widely with the increase of the total baryon density. The two curves in Fig. 2 have been drawn using the relation $g_{\sigma\Delta}=(m_\Delta/m_N)g_{\sigma N}$ to yield positive values of the effective mass over a large region of density. However, choosing the universal coupling constant $g_{\sigma\Delta}=g_{\sigma N}$, we have not obtained a nega-

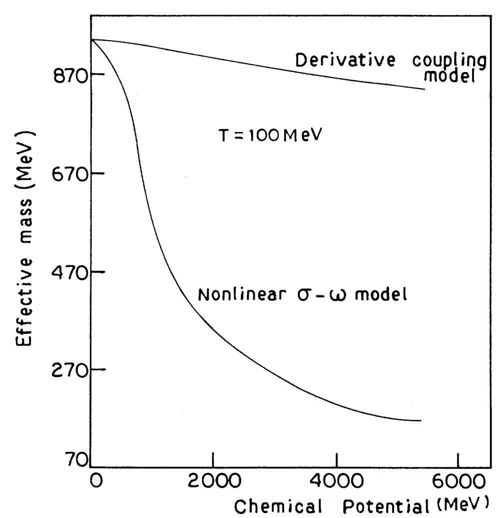

FIG. 2. Nucleon effective mass against total chemical potential value of the baryonic matter at a temperature of 100 MeV.

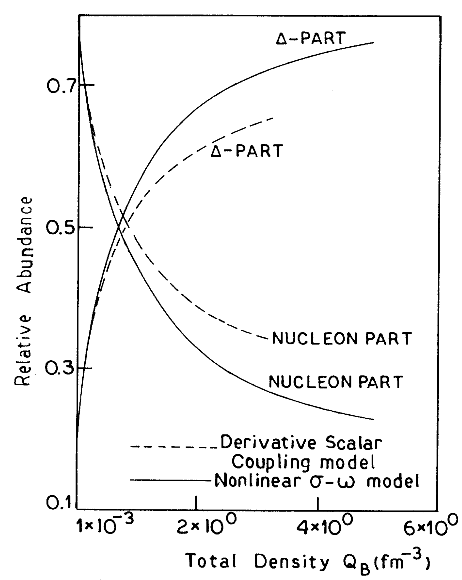

FIG. 4. Relative abundances (ρ_Δ/Q_B, ρ_N/Q_B) of the two baryons (Δ and N) at various densities.

tive value of the effective mass $m^*(Q)$ in the DSC model up to a density of $24Q_0$. In fact, it decreases very little ($\cong 0.8m_N$) from the vacuum value at the above density. The two choices of the scalar meson-baryon coupling constant yield practically the same curve for the effective mass in the DSC model, showing its insensitiveness to the different choices of coupling constants. On the other hand, the nonlinear σ-ω model yields an effective mass $m^*(Q)$ which falls off at a faster rate, reaching about $0.15m_N$ at a density of $24Q_0$. Whenever the universal coupling constant $g_{\sigma\Delta}=g_{\sigma N}$ is used, we obtain a negative value of $m^*(Q)$ equal to $-0.05m_N$ at a chemical potential of 30 000 MeV. The curves corresponding to the universal scalar meson-baryon coupling are not shown in the figure to avoid awkwardness.

It may be noted that the behavior of the Δ-excited nuclear matter in the linear Walecka model with two different choices of the coupling constants $g_{\sigma\Delta}$ is qualitatively similar to that in the nonlinear σ-ω model. Only the rate of decrease of $m^*(Q)$ with Q is much higher in the linear Walecka model and $m^*(Q)$ becomes negative at quite small values of the density ($\mu_B \cong 1800$ MeV) when $g_{\sigma\Delta}=g_{\sigma N}$ is used [8].

In Fig. 3 we have drawn the equation of state of the baryonic matter in the two models. The higher value of the effective baryon mass $m^*(Q)$ at any Q in the DSC model introduces an upward shift. In this case also, the curves corresponding to $g_{\sigma\Delta}=g_{\sigma N}$ are not shown because in the scale they are practically inseparable from those curves in the figure.

Finally, Fig. 4 shows the relative abundances of the nucleon and the delta particle in the two models. It is observed that in both models, the nucleon part dominates at smaller densities and as density increases the delta part dominates. While the density of the transition ($Q_t \cong 0.72$ fm^{-3}) from the nuclear matter is nearly equal, the rate of decrease of the nucleon and the increase in the delta part is much smaller in the model of Zimanyi and Moszkowski. Again, the transition from nuclear matter to delta matter in the present model [3] occurs at a much larger density in comparison to the Walecka model [8] characterized by a much smaller nucleon effective mass at the equilibrium. Heide and Ellis [10] have obtained similar result, which shows the influence of the bulk constants at equilibrium on the different types of phase transitions in the baryonic matter.

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