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Realistic level densities in fragment emission at high excitation energies

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Heavy fragment emission from a ${}^{100}_{44}$Ru compound nucleus at 400 and 800 MeV of excitation is analyzed to study the influence of level density models on final yields. An approach is used in which only quasibound shell-model levels are included in calculating level densities. We also test the traditional Fermi gas model for which there is no upper energy limit to the single particle levels. We compare the influence of these two level density models in evaporation calculations of primary fragment excitations, kinetic energies and yields, and on final product yields.

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I. INTRODUCTION

Heavy ion reactions at energies of 10–100 MeV/A may be characterized by a fast process consisting of the emission of nucleons [1] (and perhaps some heavy fragments) followed by a slower stage in which quasiequilibrated nuclei decay, often with the emission of heavy clusters [2]. Statistical models are widely used for the interpretation of the latter quasiequilibrium stage [2,3]. These in turn generally use Fermi gas level densities for calculating the statistical weights of heavy and light partners in the decay.

The average excitation per nucleon relevant to many heavy ion experiments is in the range of nucleon binding energies. This means that many nucleons contributing to the level densities in the standard unrestricted Fermi gas model may be at excitations considerably in excess of the binding energy; their inclusion in a level density of an equilibrated system is questionable. In a recent work, we compared Fermi gas level densities with the shell-model results obtained for restricted schemes of bound and quasibound single particle levels [4]. At excitation energies of 8–10 MeV/A, the shell-model densities begin to decrease with increasing excitation, and are many tens of orders of magnitude less than the Fermi gas results. One example of such results is shown in Fig. 1. In this work we wish to address the question of the changes predicted in statistical model calculations for fragment decay using level densities calculated for quasibound and bound levels versus unrestricted Fermi gas level densities. This will be done by calculating the first step, binary decay of a ${}^{100}_{44}$Ru nucleus at 400 and 800 MeV of excitation, and comparing results based on a Fermi gas level density model with those using more realistic models for the nuclear level density in which only bound or quasibound levels are included. We also compare final yields after deexcitation of excited primary fragments to stable products.

We do not know how to define the level density at such high excitations in a completely rigorous and unambiguous fashion; but counting levels up to the total nuclear excitation, as in the Fermi gas model, is clearly unrealistic. We feel that the alternative used in our comparisons will serve as a source of guidance to the deviations expected from the use of unrestricted Fermi gas level densities.

II. EVAPORATION CALCULATION

The evaporation calculation has been described in some detail elsewhere [2]. It is basically a Weisskopf-Ewing formulation [5] in which the excitation energy in

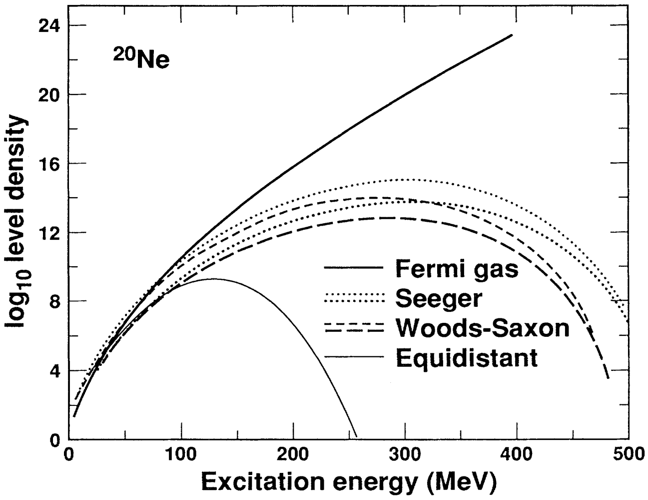

FIG. 1. Calculated level densities versus excitation energy for ^{20}Ne. The unrestricted Fermi gas result is given by the solid line. The thermodynamic calculation using quasibound levels due to Seeger is represented by the light dotted curve; the heavy dotted curves use the same levels but in a combinatorial calculation. The light and heavy dashed curves are as the dotted curves, but using Woods-Saxon model single particle levels. The thin solid line gives results of using quasibound equidistant doubly degenerate levels with spacing corresponding to $a = A/8$.

each channel is apportioned in all combinations between the channel energy and the statistical factors for ejectile and heavy residue. The statistical factor for the heavy residue, and for ejectiles (clusters) of $A > 5$, is given by a statistical level density. In this way the level density for clusters enters the calculation. The code used allows n, p, α emission, plus up to 20 additional clusters from each nuclide considered in the evaporation cascade.

III. LEVEL DENSITIES

Fermi gas level densities are calculated based on the average level spacing at the Fermi energy [6]. The approach is an approximation for small excitations. It has long been recognized that nuclear structure causes considerable modification to a simple Fermi gas level density due to the irregular spacing and degeneracies of realistic single particle levels [7,8]. When the intrinsic degrees of freedom comprising the combinations of particle-hole degrees of freedom include unbound levels, these should probably be excluded from the count of the level density, since these nucleons are likely to be emitted in a "direct" single pass manner. However, nucleons of high angular momenta which are unbound but sufficiently below the centripetal barrier, and protons which are below the Coulomb barrier, may have longer lifetimes, and might legitimately be included in the level density count.

To calculate level densities we have used the thermodynamic shell-model approach [9] generalized to include negative temperatures [4]. These have been shown to agree well with results of direct counting calculations such as programmed, e.g., by Grimes [10]. Shell-model single particle levels due to Seeger and Howard [11], scaled with mass number, were used. Determination of what constituted quasibound levels was based on subjective criteria using optical model calculations for guidance. If the transmission coefficient for a nucleon of given energy and angular momentum was less than 0.1, the level was assumed to be quasibound; if greater than or equal to 0.1, the level was defined as unbound, and not included in the levels used to calculate level densities. For actual implementation, simple algorithms were used to approximate these cuts. Further details may be found in [4].

When using an unrestricted set of levels, nuclear structure effects average out at high excitations. Collective effects may change the ordering of levels, but the total number is conserved. The problems of calculating level densities using a constrained (e.g., quasibound) set of levels are greater. For example, within a spherical shell-model scheme the addition of a single nucleon may add one more quasibound level which may be of high spin and degeneracy, e.g., an $f\frac{7}{2}$ nucleon adds eight orbitals. This leads to a tremendous increase in calculated level density versus that of the neighbor nuclide, for there is no longer an averaging over more widely spread orbitals at higher excitations. Such an anomaly in calculated level densities is unphysical, because at high excitations and/or where nuclei are deformed, the nuclear potential and residual interactions mix different configurations and split the highly degenerate spherical shell-model orbitals, giving a more uniform sequence of lower degeneracy orbitals. Our use of spherical orbitals will give some of these "peaks and valleys" in the level densities calculated with the truncated level space. Nonetheless, these results will give guidance into the differences to be expected between restricted and unrestricted sets of levels.

In the following section we present statistical model results for the first chance binary decay of $^{100}_{44}$Ru nuclei, calculated using the level densities described above. These give primary distributions of excited fragment nuclei. We then calculate the stable yields resulting when the excited ejectiles have decayed to stable fragments. For the final deexcitation of light fragments we use Fermi gas level densities.

IV. RESULTS AND DISCUSSION

In Fig. 2 we show the spectra of primary excitations for $^{20}_{10}$Ne emitted from a $^{100}_{44}$Ru nucleus at 800 MeV of excitation using both an unrestricted Fermi gas level density (FGLD), and the shell-model scheme restricted to quasibound levels (SMQB). The internal energy distribution (primary fragment excitation) is broader with the less realistic Fermi gas level density. The narrower distribution in excitations for the case of shell-model quasibound levels results from the lower (than Fermi gas) level densities at higher excitations. We also show a result for which level densities were calculated using quasibound equidistant spaced levels, with a spacing corresponding to $a = A/8$. This gives an extremely narrow energy distribution with respect to the other two cases. It results from too few levels being available, coupled with a relatively shallow well, exaggerating the turnover point of the

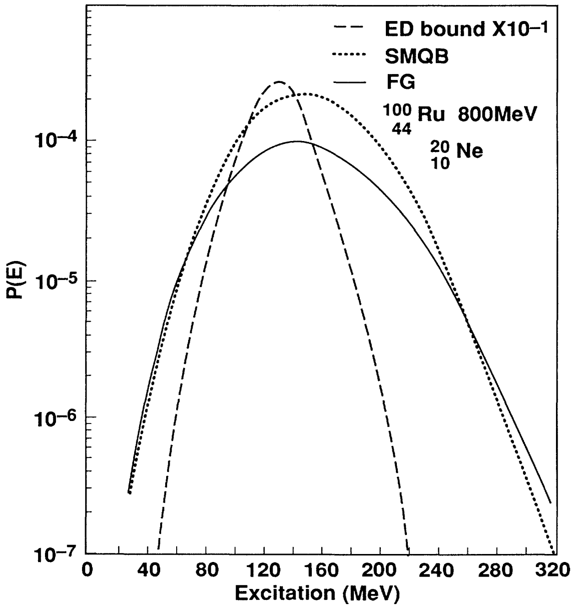

FIG. 2. Primary excitations for a ^{20}Ne fragment emitted from $^{100}_{44}$Ru at 800 MeV of excitation. The solid line results from a Fermi gas level density, while the dotted line results from a quasibound shell-model level scheme to calculate level densities. The dashed curve results from an equidistant spacing set of quasibound single particle levels with spacing corresponding to $a = A/8$.

level density versus excitation, e.g., as shown in Fig. 1.

The channel energy spectra for neutron and ^{20}Ne emission with the two choices of level density (FGLD, SMQB) are shown in Fig. 3. The internal energy (Fig. 2) is less with the SMQB level density, resulting in higher average kinetic energies and higher temperatures than for results calculated with the unconstrained Fermi gas level densities. Thus the choice of level density may be seen to have a considerable influence on the spectra and excitations of the fragments.

We consider next the primary yields of 20 clusters from ^{6}Li to ^{36}S (plus n, p, α) emitted from $^{100}_{44}$Ru at 400 and 800 MeV of excitation (first chance emission only). In Figs. 4 and 5, we compare calculated primary yields at 800 and 400 MeV of excitation with results using unrestricted Fermi gas level densities, and level densities restricted to quasibound shell-model levels as defined in Sec. III. The latter results show rather unphysical changes in yield with respect to results using a Fermi gas density, in particular the extremely large changes in yields for clusters differing by only one or two nucleons in the SMQB case. As discussed in Sec. III, this can result when one additional nucleon causes one additional level to be included in the count. The trend at 800 MeV for some enhancement of heavier clusters may represent a reasonable physical trend, whereas the detailed structure of the yields is an artifact of the spherical shell-model scheme.

The yields shown in Figs. 4 and 5 are primary yields for clusters, each with a spectrum of excitations. Results, e.g., for the ^{20}Ne excitation spectrum, are shown in Fig. 2. These fragments may also decay by nucleon or cluster emission until they reach particle stable products. The latter are what are observed in experiments; the primary channel energy spectra, e.g., of Fig. 3, will be considerably altered by secondary deexcitation of the primary fragments. The secondary deexcitation was calculated using only Fermi gas level densities, since most excitations were lower per nucleon than those of the original compound nuclei. The stable yield patterns predicted

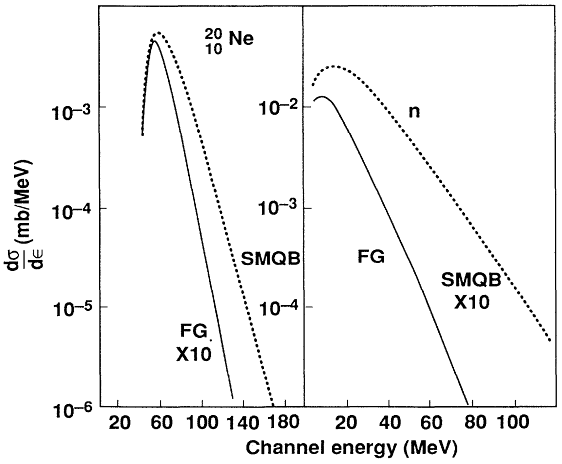

FIG. 3. Channel energy spectra for neutron and ^{20}Ne emission from $^{100}_{44}$Ru at 800 MeV, with level densities as described for Fig. 2.

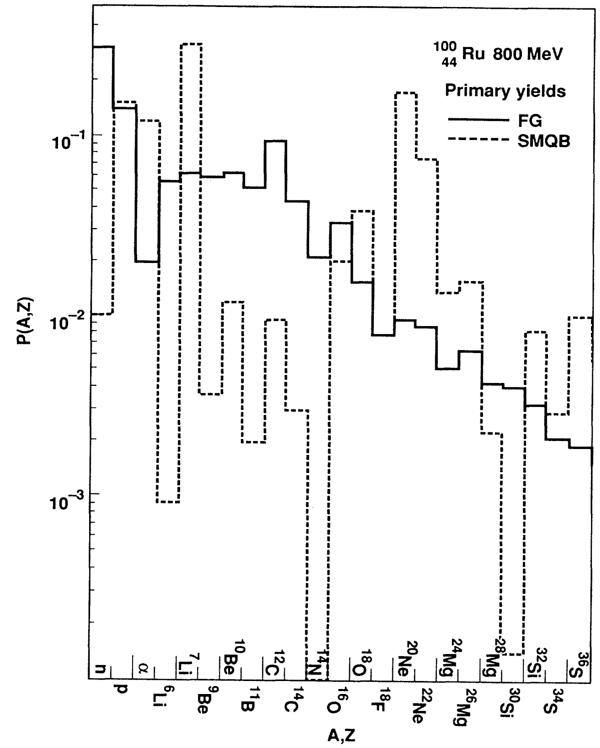

FIG. 4. Primary yield patterns for first chance decay of a $^{100}_{44}$Ru compound nucleus at 800 MeV of excitation using an unrestricted Fermi gas (FGLD, solid histogram) or a level density generated from quasibound shell-model single particle levels (dotted curves) as described in the text.

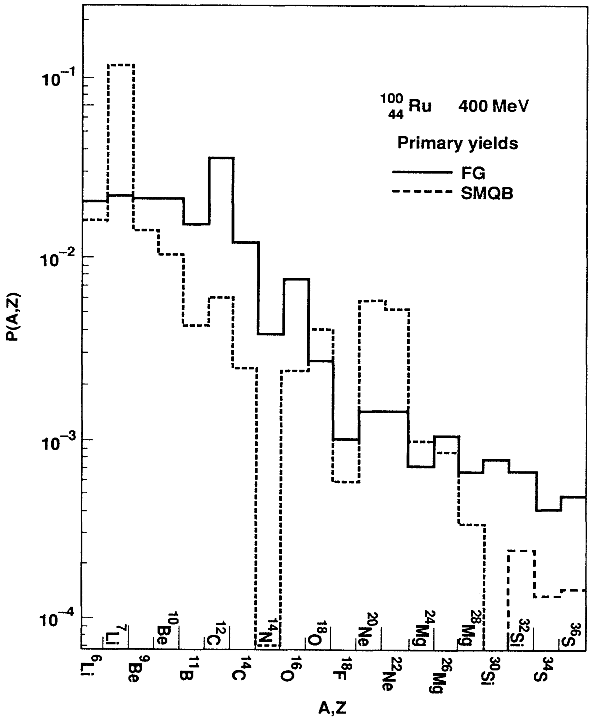

FIG. 5. As in Fig. 4, for the compound nucleus at 400 MeV of initial excitation.

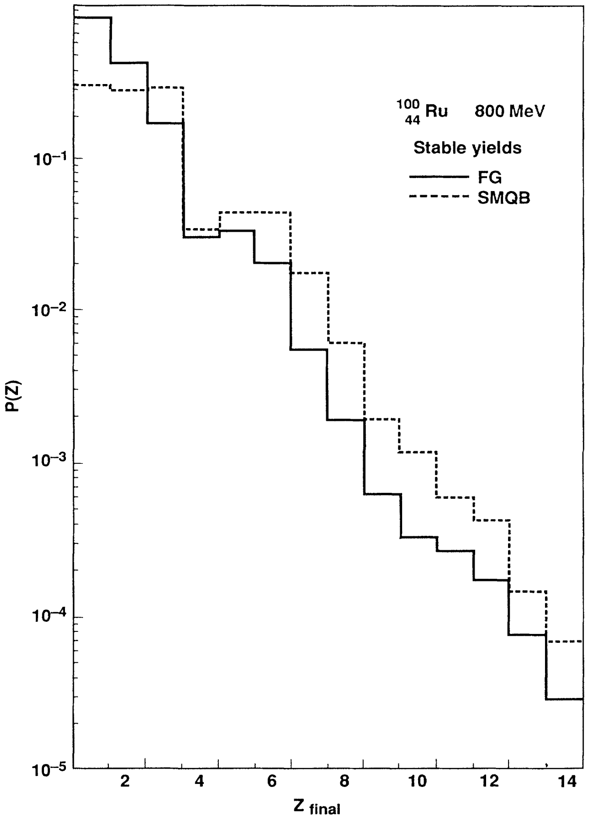

FIG. 6. Calculated final, stable yields up to $Z=14$ resulting from the decay of $^{100}_{44}$Ru compound nuclei initially at 800 MeV of excitation. This result is for the first chance emission products, calculated with the two level densities indicated, plus deexcitation of excited ejectiles to stable products using Fermi gas level densities.

after final deexcitation are shown in Figs. 6 and 7. The structure present for the primary yields has for the most part disappeared or has been very much reduced during the final deexcitation to stable yields. For this case we have summed final yields for each atomic number. The differences in yields between the FGLD and SMQB are not large within the overall uncertainties of the calculation. The increase in high Z yields at 800 MeV for SMQB results is a consequence of the decreased level density (versus Fermi gas) at high excitations.

V. CONCLUSIONS

At excitations per nucleon which are nearing the binding energy of the least tightly bound nucleon, Fermi gas level densities are poor approximations. The use of quasibound spherical shell-model states is questionable, because of the expected modifications of such a scheme due to residual interactions and shape oscillations. Nonetheless they are probably more realistic than unrestricted Fermi gas models. In this work we have compared several aspects of cluster emission from a $^{100}_{44}$Ru compound nucleus at 400 and 800 MeV of excitation predicted with a binary statistical decay model using two forms of level density: unrestricted Fermi gas, and shell-model quasibound. We consider the second choice to be more physically reasonable at very high excitations.

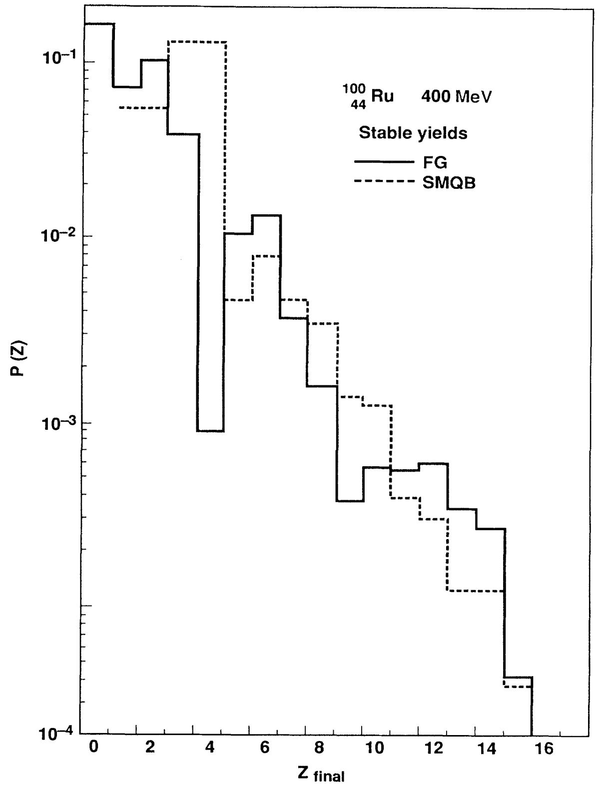

FIG. 7. As in Fig. 6, for a compound nucleus initially at 400 MeV of excitation.

We found that there were significant differences in the residual excitation spectra of the clusters when using the truncated (quasibound) shell-model level scheme to generate the level densities, with lower average excitations than for the FGLD. This means, e.g., that a calculation of multifragmentation using a Fermi gas level density would underestimate high multiplicity yields due to overestimating the cooling rate of the heavy residues (in a successive binary calculation) or due to overestimating the fragment excitations in a simultaneous decay calculation.

The kinetic energy spectra of primary fragments using the SMQB densities were "harder" than those using the FGLD. The observed spectra will have significant influences due to post emission deexcitation, making interpretations of emission spectra in terms of temperature very tenuous.

The distribution of elemental fragment yields for different level density choices does not show a great sensitivity to the choice of level densities, even though the primary yields do. This is due to a washing out of the primary yield dependency when summing over isotopic yields to get elemental yields, and a washing out of structure in the primary yields during the deexcitation to stable yields.

In this work we have shown consequences of first chance decay, so that the influence of excitation would be clear. In actual practice emission will take place from a wide range of excitations following fast nucleon emission, and from successive nucleon and cluster emission if a successive binary interpretation is reasonable. Consideration of constraints of level densities (statistical factors) should, however, be important at very high excitations in most multifragmentation models.

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