Qualitative behavior of halo nuclei elastic scattering angular distributions

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(Received 17 December 1992)

We propose a novel decomposition of the scattering amplitude appropriate for a qualitative understanding of angular distributions in elastic collisions of neutron-rich nuclei. This decomposition allows one to isolate the contributions of the nuclear attraction to the elastic scattering amplitude.

PACS number(s): 25.70.Bc, 24.10.Ht

The recently measured angular distribution of (quasi)elastic scattering of ^{11}Li + ^{12}C at 637 MeV [1] shows a considerably enhanced ratio σ/σ_R as compared with that of neighboring stable nuclei. This noticeable feature has been attributed [1,2] to an additional attraction in the surface region due to neutron excess (*halo* effect).

The aim of the present Brief Report is to introduce a special decomposition of the scattering amplitude appropriate for a better understanding of the elastic angular distribution when both refraction and absorption effects contribute. Neglecting spin effects, the elastic scattering amplitude reads

$$f(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)(e^{2i\Delta_l}-1)P_l(\cos\theta)\,, \qquad (1)$$

where the (complex) phase shift Δ_l is determined by the total nucleus-nucleus interaction. This is described by a complex optical potential plus a Coulomb term:

$$V = V_{\rm Coul} + V_{\rm nucl}\,, \quad V_{\rm nucl} = U(r) + iW(r)\,. \qquad (2)$$

Next we define the (real) amplitude η_l and phase δ_l according to

$$e^{2i\Delta_l} = \eta_l e^{2i(\sigma_l+\delta_l)}\,, \quad \delta_l = {\rm Re}\,\Delta_l - \sigma_l\,, \qquad (3)$$

where $\sigma_l = \arg\Gamma(l+1+i\eta)$ is the Coulomb phase shift and $\eta = Z_1Z_2e^2/\hbar v$ is the Sommerfeld parameter.

If both the incident energy and the significant l values in Eq. (1) are large enough, then Δ_l is well approximated by the sum of the phase shifts produced by the different components of the optical potential (2) [3]. This suggests to search for a decomposition of $f(\theta)$ which isolates the contributions due to the real part of the nuclear optical potential, responsible for the refractive effects. The latter are expected to predominate in the scattering of exotic nuclei. We therefore propose to split the amplitude (1) into a *shadow* part and a *surface* part:

$$f(\theta) = f_{\rm shad}(\theta) + f_{\rm surf}(\theta), \qquad (4)$$

where

$$f_{\rm shad}(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)(\eta_l e^{2i\sigma_l}-1)P_l(\cos\theta) \qquad (5)$$

and

$$f_{\rm surf}(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\eta_l e^{2i\sigma_l}(e^{2i\delta_l}-1)P_l(\cos\theta)\,. \qquad (6)$$

Notice that while the sum in Eq. (6) converges quite rapidly, the one in Eq. (5) inherits the convergence difficulties of the total scattering amplitude. Thus for a safe numerical evaluation of the *shadow* amplitude it is necessary to proceed in the usual way by first extracting from Eq. (5) the exactly known pure Coulomb scattering amplitude.

In the high-energy limit the contributions arising from the real part $U(r)$ of the optical potential are entirely contained in $f_{\rm surf}(\theta)$ via the phase shifts δ_l [3]. This is no longer true at the energies considered here, viz., $E/A \sim 60-100$ MeV. Nevertheless we shall see that even in this case changes in $U(r)$ may seriously affect $f_{\rm surf}(\theta)$ while leaving $f_{\rm shad}(\theta)$ almost unaltered, at least in the angular range of interest.

Before applying the *shadow-surface* decomposition to the analysis of experimental data let us get more insight into the scattering mechanism through a qualitative discussion of the two subamplitudes (5) and (6). The *shadow* amplitude $f_{\rm shad}(\theta)$ has a transparent physical meaning: it simply describes the scattering of incident particles interacting through the Coulomb force with an absorbing nonreflecting target whose opacity is given in l space by η_l [the contributions from reflexion by the imaginary part $W(r)$ are included in δ_l]. In the presence of strong absorption, η_l is assumed to rise sharply from very small values to virtually unity for some critical (or grazing) angular momentum $l = l_0$. If in addition $\eta \ll 1$, then except in the nearly forward directions where Rutherford scattering dominates, $|f_{\rm shad}(\theta)|^2$ should behave like a Fraunhofer (*shadow*) diffraction pattern.

As regards the *surface* amplitude $f_{\rm surf}(\theta)$ we notice that in the product $\eta_l(e^{2i\delta_l}-1)$ the first factor almost vanishes for $l \ll l_0$, while the second does the same for $l \gg l_0$. Thus the only significant contributions to the sum in Eq. (6) come from a narrow *window* centered at

$l \approx l_0$, which means that $f_{\rm surf}(\theta)$ behaves like an amplitude for quasielastic *surface* collisions [4]. This suggests to evaluate $f_{\rm surf}(\theta)$ in a manner similar to that used by Strutinsky [5] in the study of quasielastic surface transfer reactions. For the present qualitative purposes one can adopt the following very simple model. First we assume the target to be a totally absorbing sphere surrounded by an attractive diffuse *halo*, i.e.

$$\eta_l = \begin{cases} 1, & l \geq l_0 \\ 0, & l < l_0 \end{cases} \tag{7}$$

and

$$2\delta_l = \rho e^{-(l-l_0)/\Delta} \quad (\rho > 0) \qquad \text{for } l \geq l_0 \,. \tag{8}$$

Second we neglect Coulomb effects which at intermediate and high energies do not contribute significantly outside the near-forward direction. Further, we replace the Legendre polynomials by their asymptotic expression valid when $\theta > 1/l_0$ and change the sum over l into an integration. Using the Taylor expansion

$$e^{2i\delta_l} - 1 = \sum_{n=1}^{\infty} (i\rho)^n e^{-n(l-l_0)/\Delta}/n! \tag{9}$$

one then finds

$$f_{\rm surf}(\theta) \simeq \frac{\Delta}{ik}\left(\frac{2l_0+1}{\pi \sin\theta}\right)^{1/2} \times \sum_{n=1}^{\infty} \frac{(i\rho)^n}{n!(n^2+\Delta^2\theta^2)^{1/2}} \times \cos[(l_0+1/2)\theta - \pi/4 + \phi_n], \tag{10}$$

where

$$\phi_n = \sin^{-1}(\Delta\theta/\sqrt{n^2+\Delta^2\theta^2})\,. \tag{11}$$

In the case of strong attraction one has $\rho \gg 1$ and therefore the number of significantly contributing terms in Eq. (10) is large. Nevertheless several interesting conclusions can be drawn from this formula.

(i) Small angle scattering $(1/l_0 < \theta \ll 1/\Delta)$.—For $\theta \ll 1/\Delta$ one has $(n^2+\Delta^2\theta^2)^{1/2} \simeq n$ and $\phi_n \simeq 0$, so that all the terms in Eq. (10) oscillate approximately in phase. We therefore obtain

$$f_{\rm surf}(\theta) \simeq \frac{\Delta}{ik}\left(\frac{2l_0+1}{\pi \sin\theta}\right)^{1/2} \times \cos[(l_0+1/2)\theta - \pi/4] \sum_{n=1}^{\infty} (i\rho)^n/(n!n)\,, \tag{12}$$

which shows that $|f_{\rm surf}(\theta)|^2$ should exhibit pronounced surface diffraction oscillations for $1/l_0 < \theta \ll 1/\Delta$. These, however, cannot be observed in the actual experiments because at small angles *shadow* effects dominate the total scattering amplitude. We remark that as a consequence of the extended Babinet principle [6,7] the oscillations of $|f_{\rm surf}(\theta)|^2$ and those of $|f_{\rm shad}(\theta)|^2$ must be *out of phase* with respect to each other. It is easy to check that this rule is satisfied indeed. Ignoring again the Coulomb phases and taking Eq. (7) into account, evaluation of the sum in Eq. (5) leads to

$$f_{\rm shad}(\theta) \sim -\frac{2l_0+1}{2ik\theta} J_1[(l_0+1/2)\theta] \simeq -\frac{1}{ik\theta}\left(\frac{2l_0+1}{\pi\theta}\right)^{1/2} \times \sin[(l_0+1/2)\theta - \pi/4]\,, \tag{13}$$

for $\theta > 1/l_0$.

(ii) Large angle scattering $(\theta \gg 1/\Delta)$.—For large angles the phases ϕ_n are no longer negligible so that destructive interference between successive terms in (10) gradually sets in as θ increases. This results in a progressive damping of the oscillations at large angles. We also should like to emphasize that the nonoscillatory θ dependence in Eq. (13) decreases faster than the corresponding factor in Eq. (12). Thus, in the presence of strong refraction, the *shadow* diffraction oscillations are almost completely masked at large angles [8,9].

We now proceed with the analysis of experimental data by effectively summing up the partial-wave series in Eqs. (5) and (6), with η_l and δ_l obtained from the numerical integration of the Schrödinger equation for a given optical potential. In order to test the usefulness of the *shadow-surface* decomposition in understanding the behavior of the angular distributions when surface scattering is important, we consider two cases of interest namely ^{11}Li + ^{12}C (637 MeV) and the neighboring system ^{16}O + ^{12}C (1503 MeV). The latter has been subjected to an exhaustive optical model analysis [10] which revealed, in particular, that the real parts of the nuclear optical potentials yielding equivalent good fits of the data behave very similarly in the surface region. We have chosen the best fit ($\chi^2 = 1.07$) Woods-Saxon potential whose parameters are

$$U_0 = 80 \text{ MeV},\ r_r = 0.881 \text{ fm},\ a_r = 0.784 \text{ fm}, \tag{14}$$
$$W_0 = 28.8 \text{ MeV},\ r_i = 1.008 \text{ fm},\ a_i = 0.8 \text{ fm}.$$

In the upper part of Fig. 1 we have plotted the calculated ratio $|f(\theta)|^2/\sigma_R$ (full curve) for the system ^{16}O + ^{12}C (for experimental data see Ref. [10]). The lower part of the same figure shows the result of the *shadow-surface* decomposition (full curves). The curves corresponding to each of the two components are easily distinguished because

$$|f_{\rm shad}(\theta)|^2/\sigma_R \to 1\,, \qquad |f_{\rm surf}(\theta)|^2/\sigma_R \to 0\,, \tag{15}$$

when $\theta \to 0$. It is manifest that the oscillations in the *shadow* and in the *surface* components are nearly out of phase as expected. One may also notice that beyond about 5° the scattering is dominated by the *surface* component. We now raise the following question: How will these two components react to a simulated enhancement

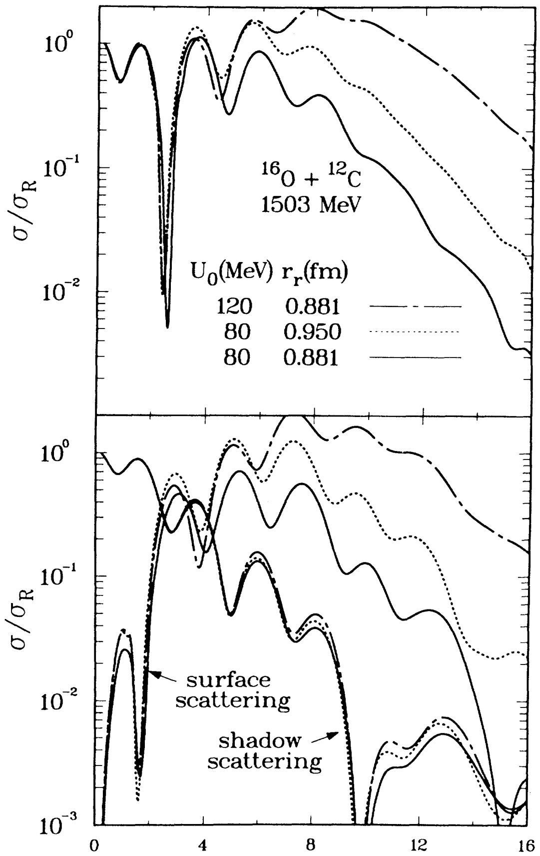

FIG. 1. Upper part: calculated elastic scattering angular distributions of $^{16}O + ^{12}C$ at 1503 MeV for three sets of optical potential parameters (see text). Lower part: *shadow-surface* decomposition corresponding to the same sets of parameters.

of the nuclear attraction? This can be done by increasing either the strength or the range of $U(r)$ (or both of them). We have varied separately U_0 and r_r while keeping the other parameters of the potential fixed at their values (14). The upper and the lower parts of Fig. 1 show the results obtained by taking successively $r_r = 0.95$ fm (dashed curve) and $U_0 = 120$ MeV (chain-dashed curve). It is apparent that additional attraction leads to a strong increase of the ratio $|f(\theta)|^2/\sigma_R$ at large angles. Furthermore, since the *shadow* amplitude varies very little, this is clearly due to the *surface* contribution. We have here a nice example of the separation of *shadow* and *surface* effects accomplished by the decomposition in Eqs. (4)–(6).

Considering the system $^{11}Li + ^{12}C$, it must be kept in mind that for the time being the experimental energy resolution does not allow us to separate true elastic from quasielastic scattering [1]. Since surface effects are expected to play an important role here too, rather than trying to fit a new optical potential from the mixed data available we decided to use for a qualitative analysis a standard Woods-Saxon well with the same parameters (14) as for $^{16}O + ^{12}C$ (of course, for ^{11}Li this yields a

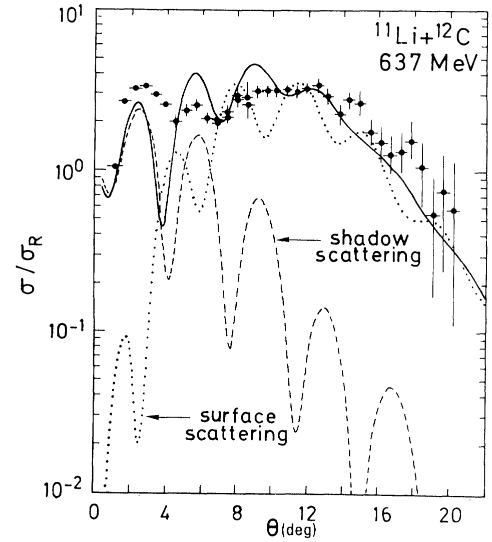

FIG. 2. Elastic scattering angular distributions for $^{11}Li + ^{12}C$ at 637 MeV. Full curve: calculated angular distribution for the same optical potential parameters as for $^{16}O + ^{12}C$. The dotted and dashed curves are, respectively, the *surface* and the *shadow* components (see text). The experimental points are taken from Ref. [1].

different range). Figure 2 displays $|f(\theta)|^2/\sigma_R$ (full curve) and the two components $|f_{\rm shad}(\theta)|^2/\sigma_R$ (dashed curve) and $|f_{\rm surf}(\theta)|^2/\sigma_R$ (dotted curve) for scattering of ^{11}Li on ^{12}C ($E/A = 60$ MeV) calculated in this way. We have also reproduced in this figure the experimental data of Ref. [1]. It is instructive to see what happens when the set of parameters (14) is used to do a similar calculation for the neighboring $^{11}C + ^{12}C$ system ($E/A \simeq 60$ MeV). The results are plotted in Fig. 3, together with the experimental points of Ref. [1]. In the $^{11}Li + ^{12}C$ case the

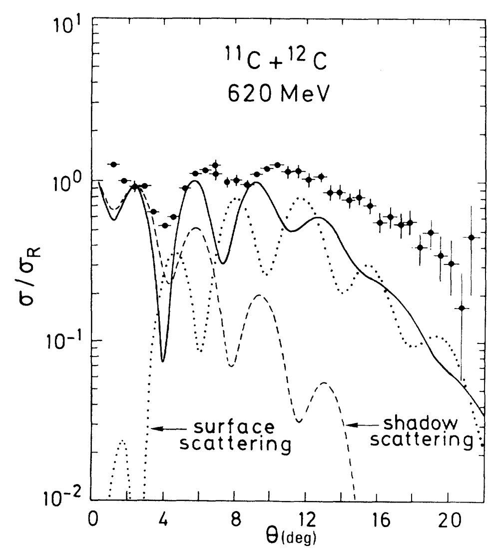

FIG. 3. Same as for Fig. 2, for elastic scattering of ^{11}C on ^{12}C at 620 MeV. Experimental points from Ref. [1].

overall agreement between the curve $|f(\theta)|^2/\sigma_R$ and the data seems to indicate that the refractive character of the optical potential (14) ($U_0/W_0 \simeq 3$) suffices to account for the excess-neutron effect. For the $^{11}C + ^{12}C$ system the calculated curve $|f(\theta)|^2/\sigma_R$ falls off too rapidly at large angles. In order to account for the poor angular resolution indicated by the horizontal error bars of the experimental data, the calculated curves have been averaged over small intervals of 0.4° amplitude around each point. Figures 2 and 3 show, in both the data and the calculated curves, a considerably enhanced ratio of σ/σ_R for ^{11}Li relative to ^{11}C scattering. This noticeable feature is probably due to a large extent to Coulomb effects. As a matter of fact the following may be noted:

(i) The repulsive Coulomb potential tends to undo the effects of the nuclear attraction. As a consequence, the contribution of the *surface* component diminishes when passing from ^{11}Li to ^{11}C scattering. This reduction can also be measured by the ratio $|f_{\rm surf}|^2/|f_{\rm shad}|^2$ which, as seen in the figures, is smaller for ^{11}C than for ^{11}Li beyond about 8°.

(ii) When the ratio σ/σ_R is plotted, such effects are amplified simply because for $^{11}C + ^{12}C$ the Rutherford cross section is four times larger than for $^{11}Li + ^{12}C$.

To summarize, we have proposed a special decomposition of the elastic scattering amplitude which proves to be useful for a better understanding of nuclear attraction effects in the presence of strong absorption at intermediate and high energies. Another remarkable virtue of the present *shadow-surface* decomposition is that it can be obtained directly from standard optical model codes without further ado. We hope that these features will be found useful in forthcoming analyses of experimental data.

The Division de Physique Théorique is Unité de Recherche des Universités Paris 6 et Paris 11 Associée au CNRS.

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