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Simple model for deriving *sdg* interacting boson model Hamiltonians: ^{150}Nd example

Y. D. Devi and V. K. B. Kota
Physical Research Laboratory, Ahmedabad 380 009, India
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A simple and yet useful model for deriving *sdg* interacting boson model (IBM) Hamiltonians is to assume that single-boson energies derive from identical particle (*pp* and *nn*) interactions and proton, neutron single-particle energies, and that the two-body matrix elements for bosons derive from *pn* interaction, with an IBM-2 to IBM-1 projection of the resulting *p-n sdg* IBM Hamiltonian. The applicability of this model in generating *sdg* IBM Hamiltonians is demonstrated, using a single-*j*-shell Otsuka-Arima-Iachello mapping of the quadrupole and hexadecupole operators in proton and neutron spaces separately and constructing a quadrupole-quadrupole plus hexadecupole-hexadecupole Hamiltonian in the analysis of the spectra, $B(E2)$'s, and $E4$ strength distribution in the example of ^{150}Nd.

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In order to make progress in applying the *sdg* interacting boson model (IBM), which is demonstrated to be useful [1–10] in analyzing hexadecupole ($E4$) properties of nuclei, it is essential that one derives *sdg* Hamiltonians with some microscopic input, so that the number of free parameters [3 single-particle energies (SPE's) and 32 two-body matrix elements (TBME's)] will reduce to a minimal number (say 4–6). Broadly speaking, two approaches to this rather complicated problem are available: (i) phenomenological, and (ii) microscopic (based on the shell model and its relatives). The symmetry defined Hamiltonian H_{sym} of Devi *et al.* [3,4,9,11], the boson surface delta interaction H_{BSDI} of Chen *et al.* [12], and the Hamiltonian H_{com} based on the commutator method given by Kuyucak *et al.* [5,8] belong to the first class, while the Otsuka-Arima-Iachello (OAI) [13] mapped and IBM-2 to IBM-1 projected Hamiltonian $H_{\text{OAI-proj}}$ proposed by Devi [3], the seniority transformed Dyson boson mapped and IBM-2 to IBM-1 projected Hamiltonian $H_{\text{DYS-proj}}$ of Navratil and Dobes [14], and the single-*j*-shell seniority mapped Hamiltonian $H_{\text{OAI-full}}$ of Yoshinaga [15] belong to the second class. Yoshinaga's $H_{\text{OAI-full}}$ Hamiltonian is not useful in analyzing real nuclei unless it is extended to multi-*j*-shell cases, and also to proton-neutron systems. These extensions render the mapping procedure rather complicated, as there is no unique correspondence between four-fermion and two-boson states. This problem can be circumvented by adopting the model where one assumes that *single-boson energies derive from identical particle (pp and nn) interactions and proton and neutron single-particle energies, and the two-body matrix elements for bosons derive from pn interaction and carrying out an IBM-2 to IBM-1 projection of the resulting p-n sdg IBM Hamiltonian.* This model [hereafter referred to as SPE($pp+nn$)-TBME(pn)-proj] was recently used by Navratil and Dobes [14], together with the similarity transformed Dyson boson mapping in the multi-*j*-shell case, to give a reasonably good description of the spectroscopic properties (spectra, $E2$, and $E4$) of vibrational ^{148}Sm, nearly rotational ^{150}Nd, and γ-unstable ^{196}Pt nuclei. However, it is not clear whether the agreements obtained by Navratil and Dobes are due to the elaborate multi-*j*-shell mapping scheme they used or the model SPE($pp+nn$)-TBME(pn)-proj employed. In order to conclusively establish the latter, in this report, using a simple single-*j*-shell OAI mapping in the above model, an IBM-1 Hamiltonian is derived, and the spectra, $B(E2)$ values, and $E4$ strength distribution are analyzed in the example of ^{150}Nd.

In order to construct *sdg* Hamiltonians with a microscopic (shell model) basis, one has to start with proton-neutron (p-n) degrees of freedom. Then, using the simple model SPE($pp+nn$)-TBME(pn)-proj [3,14] and employing a quadrupole-quadrupole plus hexadecupole-hexadecupole form for the p-n force, the *p-n sdg* IBM Hamiltonian takes the form

$$H_{pn\,\text{sdg IBM}}=\sum_{\rho=\pi,\nu}(\varepsilon_{d\rho}\hat{n}_{d\rho}+\varepsilon_{g\rho}\hat{n}_{g\rho})+\kappa_{\pi\nu}^{(2)}Q_\pi^2\cdot Q_\nu^2+\kappa_{\pi\nu}^{(4)}Q_\pi^4\cdot Q_\nu^4 \ . \tag{1}$$

In (1), $\hat{n}_{d\rho}$ and $\hat{n}_{g\rho}$ are d and g boson number operators for $\rho=\pi$ for proton bosons and ν for neutron bosons. Similarly, $\varepsilon_{d\rho}$, $\varepsilon_{g\rho}$, $\kappa_{\pi\nu}^{(2)}$, and $\kappa_{\pi\nu}^{(4)}$ are free parameters. Using the OAI correspondence [13],

$$|(j_\rho)^{2N_\rho},\ v_\rho=0,\ J_\rho=0\rangle\leftrightarrow|n_{s;\rho}=N_\rho,\ L_\rho=0\rangle \ ,$$

$$|(j_\rho)^{2N_\rho},\ v_\rho=2,\ J_\rho=2\rangle\leftrightarrow|n_{s;\rho}=N_\rho-1,\ n_{d;\rho}=1,\ L_\rho=2\rangle \ ,$$

$$|(j_\rho)^{2N_\rho},\ v_\rho=2,\ J_\rho=4\rangle\leftrightarrow|n_{s;\rho}=N_\rho-1,\ n_{g;\rho}=1,\ L_\rho=4\rangle \ ,$$

where $2\Omega_\pi$ ($2\Omega_\nu$) and N_π (N_ν) are the shell degeneracy and boson numbers for protons (neutrons) respectively

$[j_\rho=(2\Omega_\rho-1)/2]$, and equating the matrix elements of multipole operators in fermion $[r_\rho^\lambda Y_\mu^\lambda(\theta_\rho,\phi_\rho)]$ and boson $(Q_{\mu;\rho}^\lambda)$ spaces, one obtains the effective charges $e_{ll';\rho}^{(\lambda)}$ that define $Q_{\mu;\rho}^\lambda$. They are [10]

$$e_{l0;\rho}^{(\lambda)}=e_{0l;\rho}^{(\lambda)}=\left[\frac{2(\Omega_\rho-N_\rho)}{\Omega_\rho(\Omega_\rho-1)(2\lambda+1)}\right]^{1/2},$$
$$e_{ll';\rho}^{(\lambda)}=e_{l'l;\rho}^{(\lambda)}=\mp\left[\frac{\Omega_\rho-2N_\rho}{\Omega_\rho-2}\right]\left[\frac{4(2l+1)(2l'+1)}{(2\lambda+1)}\right]^{1/2}\begin{Bmatrix} l & l' & \lambda \\ j_\rho & j_\rho & j_\rho\end{Bmatrix},\quad l\neq l' \text{ and } \lambda=2,4\ . \tag{2}$$

The minus sign for $e_{ll';\rho}^{(\lambda)}$ in (2) is for particle bosons [fermion number $N_f\le\Omega_\rho$, $N_\rho=N_f/2$] and the plus sign is for hole bosons [fermion number $N_f\ge\Omega_\rho$, $N_\rho=(2\Omega_\rho-N_f)/2$]. The factors $\langle j_\rho\|r_\rho^\lambda Y_\mu^\lambda(\theta_\rho,\phi_\rho)\|j_\rho\rangle$ that appear in the mapping are not shown in (2), and they are absorbed in the free parameters $\kappa_{\pi\nu}^{(\lambda)}$ in (1). Note that (j_π,j_ν) take the values (31/2,43/2) and (43/2,57/2) for rare earths and actinides respectively. Now, carrying out an IBM-2 to IBM-1 projection [16] by assuming that the low-lying levels belong to the F spin [17] $F=F_{\max}=(N_\pi+N_\nu)/2$, and using the simple result that

$$\langle FF_Z|e_\pi(b_\pi^\dagger\tilde{b}_\pi)+e_\nu(b_\nu^\dagger\tilde{b}_\nu)|FF_Z\rangle=[(e_\pi N_\pi+e_\nu N_\nu)/N]\langle FF|b^\dagger\tilde{b}|FF\rangle\ ,$$

$F_Z=(N_\pi-N_\nu)/2$ (IBM-1 states correspond to $F=F_Z=N/2$, $N=N_\pi+N_\nu$), which follows from the Wigner-Eckart theorem in F-spin space, the OAI mapped and IBM-2 to IBM-1 projected Hamiltonian $H_{\text{OAI-proj}}$ is

$$H_{\text{OAI-proj}}=\varepsilon_d\hat{n}_d+\varepsilon_g\hat{n}_g+\kappa_2(Q_\pi^2\cdot Q_\nu^2)_{\text{proj}}+\kappa_4(Q_\pi^4\cdot Q_\nu^4)_{\text{proj}}\ ,$$
$$\varepsilon_d=\sum_\rho \varepsilon_{d\rho}N_\rho/N,\quad \varepsilon_g=\sum_\rho\varepsilon_{g\rho}N_\rho/N,\quad \kappa_r=\frac{N_\pi N_\nu}{N(N-1)}\kappa_{\pi\nu}^{(r)},\quad r=2,4\ , \tag{3}$$
$$(Q_\pi^\lambda Q_\nu^\lambda)_{\text{proj}}=:\left[\left\{\sum_{l_1l_2}e_{l_1l_2;\pi}^{(\lambda)}(b_{l_1}^\dagger\tilde{b}_{l_2})^{(\lambda)}\right\}\cdot\left\{\sum_{l_3l_4}e_{l_3l_4;\nu}^{(\lambda)}(b_{l_3}^\dagger\tilde{b}_{l_4})^{(\lambda)}\right\}\right]:,\quad\lambda=2,4\ .$$

In (3), : : denotes normal ordering. Assuming ε_d and ε_g to be free parameters (instead of deriving them from pp and nn interactions) the Hamiltonian $H_{\text{OAI-proj}}$ is used to study the spectroscopy of ^{150}Nd; the boson number $N=9$ with $N_\pi=5$ and $N_\nu=4$. Furthermore, based on the success of earlier calculations for Sm isotopes [9] and nuclei in the Os-Pt region [3,4], the spherical basis defined by n_s, n_d, and n_g with the restrictions $n_s\ge n_s^{\min}$ and $n_g\le n_g^{\max}$, where $n_s^{\min}=2$ and $n_g^{\max}=2,3$, are adopted. Although calculations with both $n_g^{\max}=2$ and 3 are performed (for Sm and Pt-Os isotopes, the $n_g^{\max}=2$ restriction is used [3,4,9]) for comparison with the results given in [14], where $H_{\text{DYS-proj}}$ is used with $n_g^{\max}=3$, only the $n_g^{\max}=3$ results are discussed. It should be mentioned that the $n_g^{\max}=2$ results are essentially the same as the $n_g^{\max}=3$ results, the latter being slightly better. With $n_s^{\min}=2$ and $n_g^{\max}=3$ restrictions, the matrix dimensions for $L=0$, 1, 2, 3, 4, 5, and 6 are 65, 90, 203, 208, 286, 260, and 294, respectively.

In order to calculate $E2$ and $E4$ properties the consistent Q^2,Q^4 formalism is adopted, which leads to the following multipole operators ($T^{E\lambda}$):

$$T_\mu^{E\lambda}=\left[\sum_{\rho=\pi,\nu}e_\rho^{(\lambda)}Q_{\rho;\mu}^\lambda\right]_{\text{proj}}=\sum_{l,l'=0,2,4}\left[\frac{1}{N}\sum_{\rho=\pi,\nu}N_\rho e_\rho^{(\lambda)}e_{ll';\rho}^{(\lambda)}\right](b_l^\dagger\tilde{b}_{l'})_\mu^\lambda,\quad\lambda=2,4\ . \tag{4}$$

TABLE I. $B(E2)$ values for ^{150}Nd.

		$B(E2;\ L_i\to L_f)$ ($10^4\ e^2\,\text{fm}^4$)	
		sdg IBM	
$L_i\to L_f$	Expt.[a]	OAI-proj[b]	DYS-proj[c]
$2_1^+\to0_1^+$	0.563±0.002	0.537	0.560
$4_1^+\to2_1^+$	0.819±0.038	0.879	0.810
$6_1^+\to4_1^+$	0.980±0.09	0.934	0.883
$0_2^+\to2_1^+$	0.208±0.009	0.227	0.071
$2_\beta^+\to4_1^+$	0.095±0.028	0.037	0.033
$2_\beta^+\to2_1^+$	0.036±0.017	0.069	0.004
$2_\beta^+\to0_1^+$	0.0024±0.0005	0.001	0.012
$2_\gamma^+\to2_1^+$	0.034±0.007	0.001	0.073
$2_\gamma^+\to0_1^+$	0.015±0.0009	0.011	0.012

[a]Reference [18].
[b]Present calculation.
[c]Reference [14].

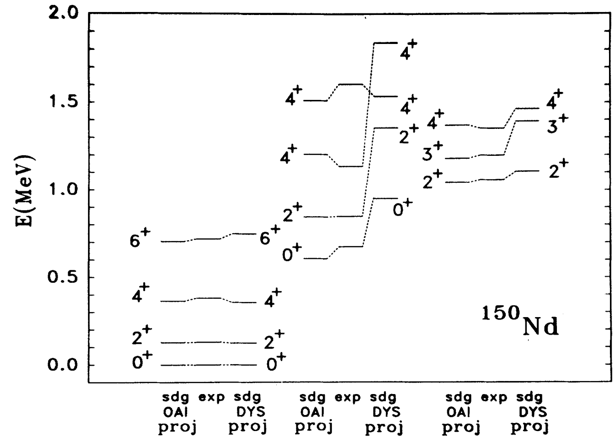

FIG. 1. Experimental and calculated energy levels for ^{150}Nd. Experimental data are from [18]. The results calculated using $H_{\text{OAI-proj}}$ (present calculations) and $H_{\text{DYS-proj}}$ [14] in *sdg* space with $n_g^{\max}=3$ are labeled as *sdg*-OAI-proj and *sdg*-DYS-proj, respectively. It is important to note that the simple $H_{\text{OAI-proj}}$ gives results that are in closer agreement to data, compared to those obtained with a more microscopic $H_{\text{DYS-proj}}$, although the number of free parameters are the same in both calculations.

The effective charges $e^{(\lambda)}_{ll';\rho}$ are the same as the ones used in the Hamiltonian (3), and they are defined in (2). The $e^{(\lambda)}_{\pi}$, $e^{(\lambda)}_{\nu}$ are the two free parameters in $T^{E\lambda}$.

The calculated spectrum for ^{150}Nd is shown in Fig. 1, and it is compared with data, as well as with the calculations of Navratil and Dobes [14]. The rms deviation from experimental energy levels is 37 keV. The description of the data obtained with $H_{\text{OAI-proj}}$ is as good as, if not somewhat better than, the $H_{\text{DYS-proj}}$. The parameters in the calculations are (in MeV) $\varepsilon_d=0.556$, $\varepsilon_g=1.378$, $\kappa_2=-0.498$, and $\kappa_4=-0.859$; the ε_d and ε_g values are from Ref. [14]. The $B(E2)$ values are calculated using the $E2$ operator (4) with $e^{(2)}_{\pi}=1.95\times10^2$ e fm^2 and $e^{(2)}_{\nu}=-5.04\times10^2$ e fm^2, and the results are given in Table I. Once again, the agreements between data and the present calculations are as good as those [14] in which a more elaborate mapping procedure is used. The nucleus ^{150}Nd is one of the few nuclei in the $100\le A\le200$ region where $E4$ strength distribution [$B(E4;\, 0^+_{\text{g.s.}}\rightarrow4^+_i)$ for 4^+ levels up to ~3 MeV)] is measured [1], the other two being ^{112}Cd [2] and ^{156}Gd [19]. Therefore, as a further test of the model SPE($pp+nn$)-TBME(pn)-Proj, which is used in deriving $H_{\text{OAI-proj}}$, the $E4$ strength distribution in ^{150}Nd is constructed using the $E4$ operator (4) with $e^{(4)}_{\pi}=7.486\times10^4$ e fm^4 and $e^{(4)}_{\nu}=-9.94\times10^4$ e fm^4, and the results are compared with the data in Fig. 2. Shown also are the results obtained with $H_{\text{DYS-proj}}$ [14] and Hartree-Bose plus Tamn-Dancoff approximation (HB+TDA) calculations of Wu *et al.* [1]. The details of the HB+TDA calculations where a phenomenological Hamiltonian is employed are given in [1]. From Fig. 2 it is seen that (i) the $H_{\text{OAI-proj}}$ calculation, although it reproduces the largest $0^+_{\text{g.s.}}\rightarrow4^+_1$ strength, underestimates the strength between 2 and 3 MeV, (ii) the $H_{\text{DYS-proj}}$ underestimates the overall strength by a factor of 2, and also the observed fragmentation between 2 and 3 MeV is not properly described, and (iii) the HB+TDA calculation describes the fragmentation of the $E4$ strength reasonably well, in spite of the fact that it overestimates the strength between 1 and 2 MeV and predicts none between 2.25 and 3 MeV, although experimentally there is sizeable strength in this domain. From this comparison it is clear that the observed $E4$ strength distribution in ^{150}Nd is reasonably well described by the *sdg* IBM, although the calculation HB+TDA overestimates and $H_{\text{OAI-proj}}$ underestimates the strength between 2 and 3 MeV. However, considering the microscopic nature of the model SPE($pp+nn$)-TBME(pn)-Proj employed in constructing $H_{\text{OAI-proj}}$ and the $E4$ transition operator, together with the agreements shown in Fig. 2, it can be concluded that it is a viable model for studying $E4$ properties.

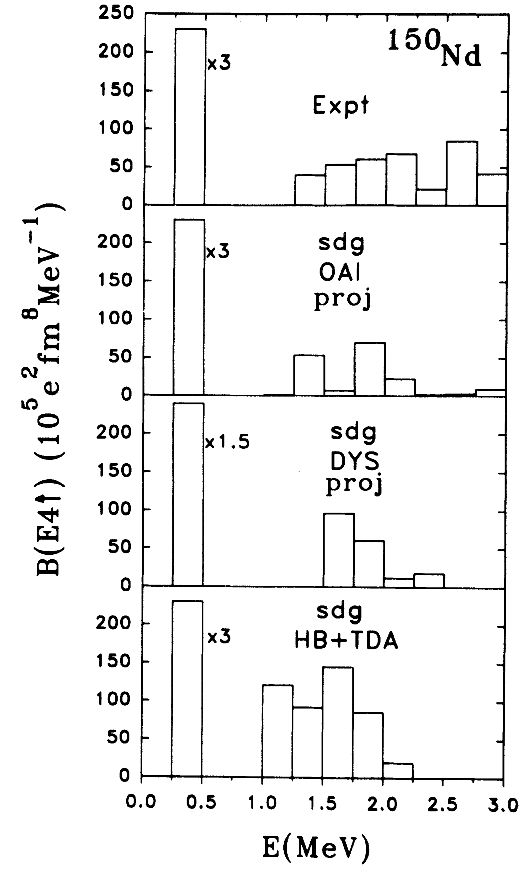

FIG. 2. $E4$ strength distributions in ^{150}Nd as measured in experiment [1] and the results of *sdg* IBM calculations. (i) Matrix diagonalization calculations with $H_{\text{OAI-proj}}$ denoted as *sdg*-OAI-proj (present calculation); (ii) matrix diagonalization calculations with $H_{\text{DYS-proj}}$ denoted as *sdg*-DYS-proj [14]; (iii) HB+TDA calculations denoted as *sdg*-HB+TDA [1]. Shown in the figure is $B(E4\uparrow)$ strength/MeV with 0.25 MeV bin size; $B(E4\uparrow)=B(E4;0^+_{\text{g.s.}}\rightarrow4^+_i)$. Note that the strength in the bin (0.25–0.5) MeV must be multiplied by the factors 3, 3, 1.5, and 3 in experiment, *sdg*-OAI-proj, *sdg*-DYS-proj, and *sdg*-HB+TDA calculations respectively.

The results given for spectra, $B(E2)$ values, and $E4$ strength distributions for ^{150}Nd clearly indicate that the simple model SPE($pp+nn$)-TBME(pn)-proj should be an essential ingredient of any microscopic procedure for deriving *sdg* IBM Hamiltonians. In order to conclusively establish this result, it is desirable to have a more systematic set of calculations employing the above model for a variety of nuclei.

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