Nucleus-nucleus reaction cross section at low energies: Modified Glauber model

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The Glauber model approach for the description of the heavy-ion reaction cross section is extended to lower energies. A simple analytic expression for the transparency function is obtained by evaluating the overlap integral of the nuclear densities over the hyperbolic trajectory. Using this transparency function the reaction cross section of a large number of heavy-ion systems is predicted reasonably well at low energies.

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The reaction cross section of heavy ions has been studied extensively both theoretically and experimentally. At high energies a good description of the heavy-ion reaction cross section has been given in the framework of the Glauber model [1]. At low energies strong absorption models have been used [2] to calculate the reaction cross section. In such models the interaction radius is obtained by parametrizing it as a function of the masses of the interacting nuclei and by fitting the experimental data. There is no first-principles prediction of the interaction radius, and such models cannot be extrapolated much beyond the data set with which they are fitted. We extend the scope of the Glauber model to low energies by calculating the overlap integral of the nuclear densities over a hyperbolic trajectory. The numerically computed values of the reaction cross section, using this transparency function, agree reasonably well with the data at low energies.

The nucleus-nucleus reaction cross section in the Glauber theory framework is given by [3,4]

$$\sigma_R = 2\pi \int_0^\infty b\, db \{1-\exp[-\bar{\sigma}_{NN}\chi(b)]\} , \tag{1}$$

where the transverse motion of the nuclei is neglected when they pass each other. Here $\bar{\sigma}_{NN}$ is the nucleon-nucleon cross section suitably averaged over the interacting n-n, p-p, and n-p pairs and b is the impact parameter.

In terms of the transparency function $T(b)$, that is, the probability that at the impact parameter b of the projectile will pass through the target without interacting, σ_R can be written as

$$\sigma_R = 2\pi \int_0^\infty b\, db [1-T(b)] . \tag{2}$$

For two nuclei in the form of two spheres $\chi(b)$ is given as [3,5]

$$\chi(b) = \int d^2 b_1\, \varphi_z^1(\mathbf{b}_1)\varphi_z^2(|\mathbf{b}_1-\mathbf{b}|) . \tag{3}$$

Equation (3) reduces to a simple closed-form analytic expression by assuming a Gaussian density distribution of the form [1]

$$\varphi_i(r) = \varphi_i(0)\exp(-r^2/R_i^2) . \tag{4}$$

The essence of Karol's method [1] is that $T(b)$ can be written as

$$T(b) = \exp[-\bar{\sigma}_{NN} K \exp(-b^2/2\sigma^2)] , \tag{5}$$

where

$$K = \pi^2 \varphi_1(0)\varphi_2(0) R_1^3 R_2^3 / 2\sigma^2$$

and

$$2\sigma^2 = R_1^2 + R_2^2 .$$

The above expression does not take into account the modification in the heavy-ion trajectory due to the Coulomb field. We incorporate this deviation in the eikonal trajectory due to the Coulomb field by replacing the impact parameter b by the coordinate y so that instead of $y^2=\text{const}=b^2$ we have $y^2=b'^2+Cz^2$, where b' is the distance of closest approach on the Coulomb trajectory, as suggested by Cole [6]. This replacement of b

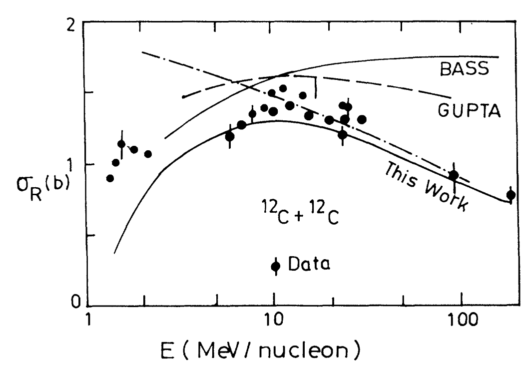
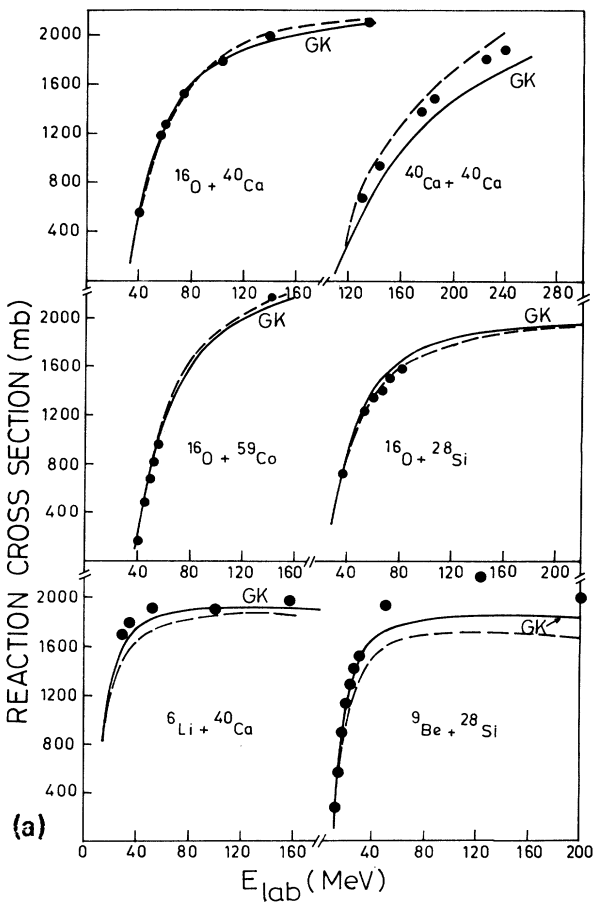
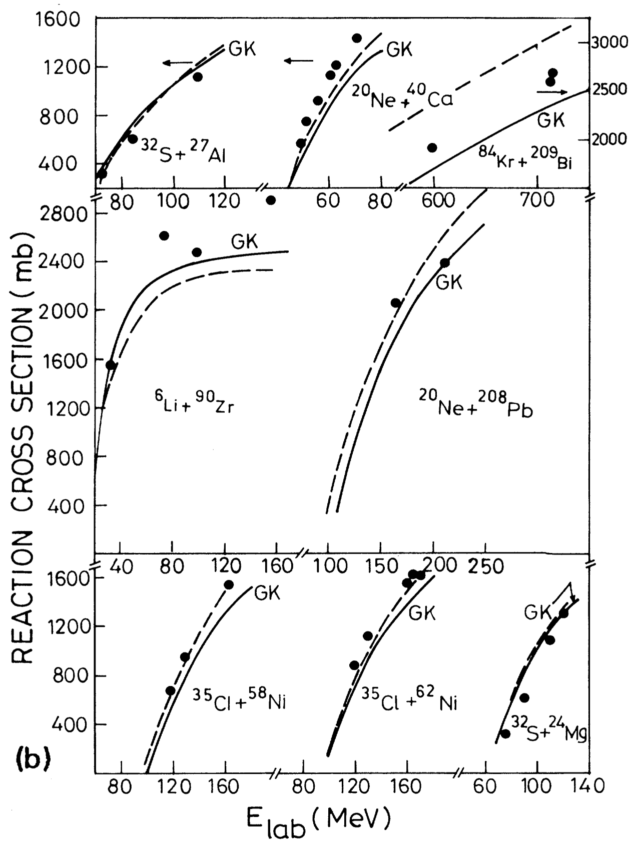


FIG. 1. Total reaction cross section for $^{12}C+^{12}C$ as a function of incident energy. The data points are from Ref. [8]. The solid and dashed curves are the results of parametrization of Bass (Ref. [2]) and Gupta and Kailas (Ref. [9]), respectively. The present calculations are labeled as this work. The unmodified Glauber model calculations are shown as a dot-dashed curve.





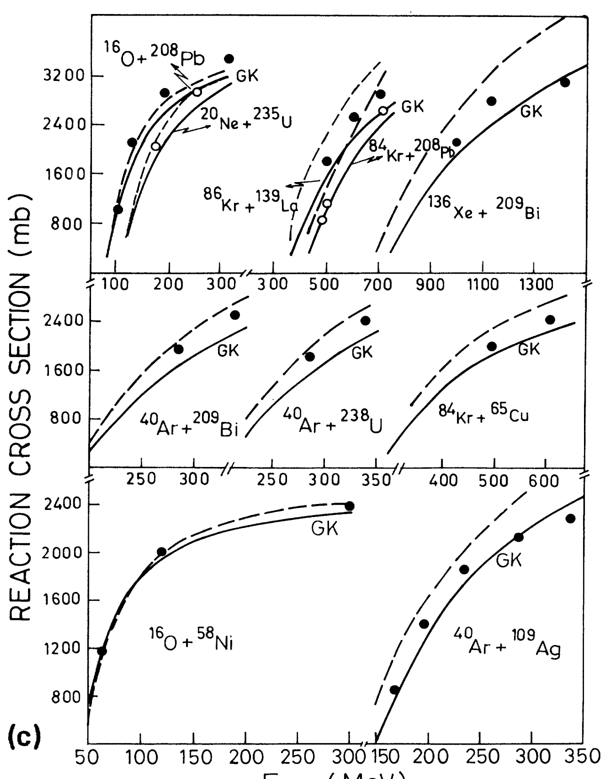


FIG. 2. (a)–(c) Comparison of the reaction cross section obtained from the analyses of the elastic scattering data given in Ref. [9] with the present calculations (dashed curves). Also shown as solid curves are the Gupta and Kailas results (Ref. [9]).

with y leads to $T(b)$:

$$T(b)=\exp\left[-\frac{\bar{\sigma}_{NN}K}{(C+1)^{0.5}}\exp\left[\frac{-b'^2}{2\sigma^2}\right]\right], \tag{6}$$

where

$$kb'=\eta+(\eta^2+k^2b^2)^{1/2}, \tag{7}$$

η being the Sommerfeld parameter and k the wave number. The quantity C is obtained by matching the hyperbola $y^2=b'^2+Cz^2$ to the actual Rutherford trajectory [7]. We obtain

$$C=\left[\frac{b'^2-b^2}{2bb'}\right]^2. \tag{8}$$

In this model there are no free parameters. The novel feature of the model is a simple analytic expression for the transparency function $T(b)$. We use Eq. (6) for the calculation of $T(b)$ and then numerically compute the integral [Eq. (2)] to obtain the reaction cross section.

The nucleon-nucleon cross section $\bar{\sigma}_{NN}$ averaged over neutron and proton numbers is calculated by the expression

$$\bar{\sigma}_{NN}=\frac{N_PN_T\sigma_{nn}+Z_PZ_T\sigma_{pp}+N_PZ_T\sigma_{np}+N_TZ_p\sigma_{np}}{A_PA_T}, \tag{9}$$

where A_P, A_T, Z_P, Z_T, and N_P, N_T are the projectile and target mass, charge, and neutron numbers, respectively. Fitted values of the experimentally determined nucleon-nucleon cross sections σ_{pp},σ_{np} were used [3]. The parameters $\varphi_i(0)$ and R_i are adjusted to reproduce the experimentally determined nuclear surface texture by matching the Gaussian density distribution to the two-parameter Fermi distribution [3].

The energy variation of the reaction cross section for the $^{12}C+^{12}C$ system, obtained in the present framework, is shown in Fig. 1 and is labeled as this work. The unmodified Glauber model results are shown as a dot-dashed line. The data points in this figure are from the compilation of Kox *et al.* [8]. The results of the calculation using the Bass [2] and Gupta models [9] are also given for comparison. The present model adequately describes this energy variation over an energy range right from few MeV/nucleon to 200 MeV/nucleon. The fit of the Gupta model is good but this model has a free parameter and this model fails in the high-energy domain [8].

In the low-energy domain we have studied a large number of the heavy-ion systems with ^6Li to ^{136}Xe as projectiles and ^{12}C to ^{238}U as targets. For these systems the energy variation of the reaction cross section predicted by the present study is plotted in Figs. 2(a)–2(c) as the dotted line. For comparison we have also plotted the calculated values of the reaction cross section based on the one-parameter model [9]. The data are given as solid circles in all of these systems. In most of these cases the data have been reproduced fairly well.

The success of this model in describing the heavy-ion reaction cross section in the low-energy range is due to the fact that we have been able to evaluate the overlap integral over the actual heavy-ion trajectories. The only ingredients required in this model are the nucleon-nucleon cross section and the nuclear densities. The scope of this model can be extended to the study of heavy-ion elastic scattering at low energies.

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