Null tests of time-reversal invariance

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Because null tests of parity conservation exist in nuclear and particle reactions, it has been possible to measure very precisely the (weak-interaction) parity nonconserving contribution to the process. There is, however, a proof of the nonexistence of a comparable null test of time-reversal invariance. As a result, reaction tests of T symmetry have, at best, achieved precisions several orders of magnitude below that of the tests of P symmetry. Since transmission experiments are not included in the nonexistence proof, the existing formalism used to describe spin observables in neutron transmission experiments has been expanded to include explicitly the target spin. Through this formalism, the time-reversal-violating (and parity nonconserving) forward scattering amplitudes are identified, along with the corresponding spin observables. It is noted that new and more precise tests of T symmetry are provided in transmission experiments, and that such investigations are applicable more generally in nuclear and particle physics.

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I. INTRODUCTION

It has been proved that there exists no null test of time-reversal invariance (TRI) in nuclear and particle physics in any reaction with two particles in and two particles out [1]. That is, there exists no single experimental observable which is required to be zero by TRI. This follows from the fact that TRI equates a reaction observable to an observable in the inverse reaction, and so the difference (or sum) of the two is zero. Even in elastic scattering, which is its own inverse reaction, two different observables are related by TRI, e.g., the polarization and the analyzing power, so that $P_y - A_y = 0$. Because of this requirement to compare two experimental observables, one of which is often difficult to measure with precision, it is easy to understand why such tests of T symmetry have rarely attained the 1% level of experimental error. In strong contrast, since null tests of parity conservation are available, e.g., the longitudinal analyzing power $A_z = 0$ from P symmetry, the weak-interaction parity-nonconserving (PNC) contribution to A_z in pp scattering has been determined with the remarkable precision of $\pm 2\times 10^{-8}$ [2,3]. Thus it is clear that a comparable *null test of TRI* would permit an improvement in experimental precision of several *orders of magnitude.*

Since transmission experiments, which correspond to forward scattering, are neither explicitly included nor excluded in the proof of the nonexistence of a null test of TRI, it is of considerable interest to examine the possibilities for such a test there. Transmission experiments with slow neutrons have shown remarkable enhancements in two PNC observables, the neutron spin rotation [4] and the neutron analyzing power A_z [5]. These enhancements are explained in terms of close-lying parity-mixed nuclear levels and the p-wave barrier hindrance of the parity-conserving transitions, and Stodolsky [6] and Kabir [7] have suggested that nuclear effects might also provide enhancements in time-reversal-violating (TRV) neutron transmission observables which become accessible with polarized targets. They have developed a formalism to describe the spin aspects of coherent neutron transmission and show that, in principle, a straightforward null test of TRI (and PNC) is a transmission asymmetry associated with the reversal of the neutron or target spin. They also note, however, that the spin-spin (pseudomagnetic) interaction causes a coherent rotation of the neutron spin around the target polarization direction, and this rotation, followed by an enhanced PNC analyzing power A_z, could provide a nonzero value of this asymmetry and, thus, a false signal of TRV. They have suggested some other TRV observables to be measured, but these are not null-test observables. They involve two observables and can be viewed as the polarized-target transmission analogues of the difference between the polarization and analyzing power, which is required to be zero by TRI.

Since the neutron-scattering amplitude builds up coherently in the forward direction only for wavelengths $\lambda \gg R$, where R is the radius of the scattering nucleus [8], this condition is fulfilled only in slow (s-wave) neutron transmission. Thus it is important to look for a TRV observable in ordinary transmission experiments (without the limitation to slow neutrons), where the coherent spin rotation is absent and the only experimental observable is the (spin-dependent) total cross section. For example, recent measurements near 10 MeV have been made of the total-cross-section spin-correlation coefficient

$$A_{x,y} = \frac{\sigma_{x,y}(++) - \sigma_{x,y}(+-)}{\sigma_{x,y}(++) + \sigma_{x,y}(+-)} , \tag{1}$$

in the transmission of polarized neutrons through a vector-polarized holmium target [9]. In Eq. (1), taking z along the beam direction, $\sigma_{x,y}(++)$ [$\sigma_{x,y}(+-)$] is the total cross section with the projectile and target transverse polarizations $p_x, p_y = 1$ ($p_x, -p_y = 1$) [10]. Since $A_{x,y}$ is a PNC and TRV experimental observable, as will

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be shown in Sec. II, this is a null test of both P and T symmetry. Having determined that their experimental result was consistent with zero at the $5\times10^{-3}(2\sigma)$ level, the authors suggested that it would be more appropriate to investigate the low-energy (neutron) regime where PNC resonance enhancements have been seen. In this regard, Gould *et al.* [11] have reviewed the formalism that describes PNC and TRV terms in low-energy resonance total cross sections with polarized neutrons and/or polarized targets. It should be noted that this formalism does not apply for coherent neutron transmission, since the appropriate neutron spin rotations are not included.

Clearly, it is important to examine the question of a TRV observable in the more ordinary and widespread possibilities for charged-particle transmission experiments in nuclear and particle physics at all energies. Here one would be giving up the potential for enhancements from nuclear effects, but the improvement by several orders of magnitude in the experimental precision that can be attained in a *null* test of T symmetry is an equally definite, and more certain, "enhancement."

II. FORMALISM

In his treatment of coherent transmission through a polarized target, Stodolsky [6] describes the forward-scattering matrix simply in the 2×2 neutron spin space with no explicit inclusion of the corresponding target spin-space matrix. Since both projectile and target polarizations are required in order to provide a TRV term in the forward-scattering matrix, I choose to follow the procedure that is standard for (nonforward) scattering. That is, the corresponding observable, the spin-correlation coefficient, is given by

$$A_{j,k}(\theta)=\mathrm{Tr}MS_jS_kM^\dagger/\mathrm{Tr}MM^\dagger \ , \tag{2}$$

where S_j (S_k) is the projectile (target) spin operator corresponding to polarization in the j (k) direction and $M(\theta)$ is the transition matrix which connects the initial and final spin states $X_f=MX_i$ [12]. Since S_k operates in the target spin space, it is clear that M must encompass the combined projectile-target spin space. In order to identify, then, a TRV observable in (noncoherent) transmission experiments, I consider in detail, as prototypes, the cases with spin-$\frac{1}{2}$ projectile and spin-$\frac{1}{2}$ or spin-1 targets.

A. Spin structure $\frac{1}{2}+\frac{1}{2}\rightarrow\frac{1}{2}+\frac{1}{2}$

In the simplest case with spin-$\frac{1}{2}$ projectile and target, the 4×4 M matrix can be expanded in terms of direct products of the 2×2 projectile and target Pauli spin matrices σ_j and σ_k, respectively [13],

$$M(\theta)=\sum_{j,k}a_{j,k}\sigma_j\otimes\sigma_k \ , \quad j,k=o,x,y,z \ , \quad \sigma_o=1 \ . \tag{3}$$

Choosing the projectile helicity frame, unit vectors along the coordinate axes are taken to be

$$\mathbf{z}=\mathbf{k}_i \ , \quad \mathbf{y}=\mathbf{k}_i\times\mathbf{k}_f \ , \quad \mathbf{x}=\mathbf{y}\times\mathbf{z} \ , \tag{4}$$

where $\mathbf{k}_i$ and $\mathbf{k}_f$ are along the initial and final momenta of the projectile. Then, with $\sigma_x\equiv\boldsymbol{\sigma}\cdot\mathbf{x}$, etc., we have the following transformations under the P, T symmetry operations:

$$\begin{array}{c|ccc} & \sigma_x & \sigma_y & \sigma_z \\ \hline P & -\sigma_x & \sigma_y & -\sigma_z \\ T & -\sigma_x & \sigma_y & \sigma_z \end{array} \ . \tag{5}$$

The 16 M-matrix amplitudes $a_{j,k}$ in Eq. (3),

$$\begin{gathered} a_{oo}, a_{ox}, a_o, a_o, a_{xo}, a_{xx}, a_{xy}, a_{xz} \ , \\ a_{yo}, a_{yx}, a_{yy}, a_{yz}, a_{zo}, a_{zx}, a_{zy}, a_{zz} \ , \end{gathered} \tag{6}$$

can then be classified, from (5), according to their P and/or T symmetry. That is, an amplitude is

$$\text{PNC (TRV) if } n_x+n_z \ (n_x) \text{ is odd} \ , \tag{7}$$

where n_x (n_z) is the number of x (z) subscripts [14].

Since J_z, the z component of total angular momentum, is conserved and the orbital component $l_z=0$ in forward scattering, the total (channel) helicity s_z is conserved there. The equivalent condition is that $M(0)$ be invariant with respect to rotation around the z axis, R_z. Imposing this condition on Eq. (3), with $\sigma_j\otimes\sigma_k\equiv\sigma_j\sigma_k$, results in the forward-scattering matrix with its PNC and TRV term $a_{x,y}$:

$$\begin{aligned} M(0)=a_{o,o}&+a_{o,z}\sigma_o\sigma_z+a_{z,o}\sigma_z\sigma_o \\ &+a_{x,x}(\sigma_x\sigma_x+\sigma_y\sigma_y)+a_{z,z}\sigma_z\sigma_z \\ &+a_{x,y}(\sigma_x\sigma_y-\sigma_y\sigma_x) \ , \end{aligned} \tag{8}$$

where the y axis is now identified with the transverse target polarization [10]. The fourth and sixth terms come, respectively, from the R_z-invariant forms $(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$ and $\mathbf{k}\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)$. Since the TRV amplitudes $a_{x,z}$ and $a_{z,x}$ vanish in forward scattering, there can be no test of TRI alone in a transmission experiment with a spin-$\frac{1}{2}$ projectile and target. However, the amplitude $a_{x,y}$ suggests that a corresponding PNC and TRV observable is available in the *incoherent* transmission experiments that are available in nuclear and particle physics. Here one uses a treatment that features transmitted *intensities* rather than amplitudes, and the spin-dependent observables, i.e., the total cross sections, are then related to the spin-dependent forward-scattering amplitudes by the optical theorem.

The transmission factor, defined as the ratio of transmitted to incident beam intensities through a target of areal density d (number of nuclei per cm^2), is

$$I(d)/I(0)\equiv T(d)=\exp[-\sigma_T d]\equiv\exp[-\sigma] \ , \tag{9}$$

where σ_T is the unpolarized total cross section and thus σ is a dimensionless "total cross section" which includes the areal density factor d. The corresponding spin-dependent cross sections are

$$\sigma_{j,k}=\sigma(1+p_jp_kA_{j,k}) \ , \quad j,k=x,y,z \ , \tag{10}$$

where p_j (p_k) is the projectile (target) polarization along the j (k) direction and $A_{j,k}$ is the corresponding (total-cross-section) spin-correlation coefficient, which is essentially defined by this equation. Then, with $\sigma_{j,k}(++)$ $[\sigma_{j,k}(+-)]$ defined as the cross section for the pure spin states $p_j=p_k=1$ ($p_j=-p_k=1$), we have

$$\sigma_{j,k}(++)=\sigma_{j,k}(--)=\sigma(1+A_{j,k}) ,$$
$$\sigma_{j,k}(+-)=\sigma_{j,k}(-+)=\sigma(1-A_{j,k}) . \tag{11}$$

Using these spin-dependent cross sections, the corresponding transmission factors are defined as

$$T_{j,k}=\tfrac{1}{2}\{\exp[-\sigma_{j,k}(++)]+\exp[-\sigma_{j,k}(+-)]\} \tag{12a}$$

and

$$\Delta T_{j,k}=\frac{\exp[-\sigma_{jk}(++)]-\exp[-\sigma_{jk}(+-)]}{\exp[-\sigma_{jk}(++)]+\exp[-\sigma_{jk}(+-)]} . \tag{12b}$$

Thus $T_{j,k}$ is the transmission factor for a completely polarized beam transmitted through an unpolarized target (and vice versa), while $\Delta T_{j,k}$ is the *transmission asymmetry* of the polarized beam for opposite states of the target polarization. Using Eq. (11),

$$T_{j,k}=e^{-\sigma}\cosh\sigma A_{j,k} , \tag{13a}$$

$$\Delta T_{j,k}=-\tanh\sigma A_{j,k} . \tag{13b}$$

We now use the spin-dependent form of the optical theorem [15] to express σA_{jk} in terms of the imaginary part of the corresponding forward-scattering amplitude. That is,

$$\sigma_T(p_j,p_k)=\frac{4\pi}{k}\mathrm{Im}\,\mathrm{Tr}[\rho_{j,k}M(0)] , \tag{14}$$

where $\rho_{j,k}$ is the density matrix representing the initial polarizations and $\sigma_T(p_j,p_k)$ is the corresponding total cross section. The normalization $\mathrm{Tr}\,\rho=1$ has been chosen. Then, in the established notation,

$$\rho_{j,k}=\tfrac{1}{4}(\sigma_o+p_j\sigma_j)\otimes(\sigma_o+p_k\sigma_k) , \tag{15}$$

and so

$$\rho_{j,k}(+\pm)=\tfrac{1}{4}(1+\sigma_j\sigma_o\pm\sigma_o\sigma_k\pm\sigma_j\sigma_k) . \tag{16}$$

Taking

$$\frac{4\pi d}{k}\equiv K , \tag{17}$$

Eq. (14) becomes

$$\sigma_{j,k}=K\,\mathrm{Im}\,\mathrm{Tr}[\rho_{j,k}M(0)] , \tag{18}$$

and with

$$\sigma A_{j,k}=\tfrac{1}{2}[\sigma_{j,k}(++)-\sigma_{j,k}(+-)] , \tag{19}$$

we have

$$\sigma A_{j,k}=\tfrac{1}{4}K\,\mathrm{Im}\,\mathrm{Tr}[(\sigma_o\sigma_k+\sigma_j\sigma_k)M(0)] . \tag{20}$$

Then noting that

$$\mathrm{Tr}[(\sigma_j\sigma_k)(\sigma_{j'}\sigma_{k'})]=4\delta_{jj'}\delta_{kk'} , \tag{21}$$

the only terms from $M(0)$ [Eq. (8)] that survive in Eq. (20) are the terms $a_{o,k}$ and $a_{j,k}$, once the polarizations p_j,p_k have been selected. For our purpose here, to identify a PNC/TRV observable corresponding to the amplitude $a_{x,y}$, the appropriate choice is $p_j=p_x,p_k=p_y$ (or vice versa), for which

$$\sigma A_{x,y}=K\,\mathrm{Im}\,a_{x,y} . \tag{22}$$

Then, with

$$\sigma=K\,\mathrm{Im}\,a_{o,o} . \tag{23}$$

$$A_{x,y}=\mathrm{Im}\,a_{x,y}/\mathrm{Im}\,a_{o,o} . \tag{24}$$

Thus the spin-correlation coefficient $A_{x,y}$ is the observable to measure as a true *null test* for a combined PNC and TRV effect, since the amplitude $a_{x,y}$ vanishes when either symmetry holds. Finally, the corresponding transmission asymmetry, which is the directly measureable quantity, is

$$\Delta T_{x,y}=-\tanh(K\,\mathrm{Im}\,a_{x,y})\rightarrow -K\,\mathrm{Im}\,a_{x,y} \text{ for } a_{x,y}\ll 1 . \tag{25}$$

This is also the simplest experimentally, for which high precision can be achieved.

There is one, important, case for which a measurement of $A_{x,y}$ does *not* provide a null test of the P,T symmetries, and that is for the case of identical projectile and target particles, e.g., proton-proton scattering [16]. The identical-particle interchange in the scattering matrix [Eq. (8)] corresponds to $\sigma_x\sigma_y\leftrightarrow\sigma_y\sigma_x$, and the last term changes its sign. Thus the amplitude $a_{x,y}$ and the corresponding observable $A_{x,y}$ are required to be zero by this identical-particle symmetry.

B. Spin structure $\frac{1}{2}+1\rightarrow\frac{1}{2}+1$

Since it is clear from the foregoing development that there is no uniquely TRV forward-scattering amplitude in the $\frac{1}{2}+\frac{1}{2}\rightarrow\frac{1}{2}+\frac{1}{2}$ spin system, it is important to examine the suggestion [17] for an additional T-odd, P-even term in the forward-scattering matrix of the form $\mathbf{k}\cdot(\boldsymbol{\sigma}\times\mathbf{I})(\mathbf{k}\cdot\mathbf{I})$, with target spin $I\geq 1$ since I^2 represents an alignment. Since the simplest $M(0)$ matrix that can furnish such a term is that for a system with the spin structure $\frac{1}{2}+1\rightarrow\frac{1}{2}+1$, one can proceed then in the same manner as in Sec. II A. Equation (3) now becomes the 6×6 matrix

$$M(\theta)=\sum_{j,k}a_{j,k}\sigma_j\otimes P_k+\sum_{j,lm}a_{j,lm}\sigma_l\otimes P_{lm} , \tag{26}$$

$$j,k=o,x,y,z , \quad lm=xx,yy,xy,xz,yz ,$$

with the sum over lm limited to the five independent terms. The P_k (P_{lm}) are the vector, rank-1 (tensor, rank-2) components of the spin-1 matrix operator [12]. Thus the 16-term first sum combined with the 20-term second sum provides the required 36 terms of the M matrix. Imposing R_z invariance, the forward-scattering matrix reduces to 10 terms,

$$M(0)=a_{o,o}+a_{o,z}\sigma_o P_z+a_{z,o}\sigma_z P_o+a_{x,x}(\sigma_x P_x+\sigma_y P_y)+a_{z,z}\sigma_z P_z+a_{x,y}(\sigma_x P_y-\sigma_y P_x)$$
$$+a_{o,zz}\sigma_o P_{zz}+a_{z,zz}\sigma_z P_{zz}+a_{x,xz}(\sigma_x P_{xz}+\sigma_y P_{yz})+a_{x,yz}(\sigma_x P_{yz}-\sigma_y P_{xz})\ . \tag{27}$$

The first six terms are the equivalent of Eq. (8), and the additional four terms arise from the R_z-invariant forms $(\mathbf{k}\cdot\mathbf{I})^2$, $(\mathbf{k}\cdot\boldsymbol{\sigma})(\mathbf{k}\cdot\mathbf{I})^2$, $(\mathbf{k}\cdot\mathbf{I})(\boldsymbol{\sigma}\cdot\mathbf{I})$, and $\mathbf{k}\cdot(\boldsymbol{\sigma}\times\mathbf{I})(\mathbf{k}\cdot\mathbf{I})$, respectively, and each of these latter terms has the $\mathbf{I}^2$ factor that provides the spin-1 tensor operator P_{lm} in Eq. (27). Then, from (7), $a_{x,yz}$ is the T-odd, P-even amplitude that has been the object of this development. So, in following the same procedure as for the $\frac{1}{2}+\frac{1}{2}\rightarrow\frac{1}{2}+\frac{1}{2}$ spin system in Sec. II A, one would like to choose the density matrix representing the spin-1 polarization as

$$\rho_{yz}=\tfrac{1}{3}(P_o+\tfrac{2}{3}p_{yz}P_{yz})\ , \tag{28}$$

where p_{yz} is the target tensor polarization, i.e., alignment along the $y=z$ direction. However, unlike the situation for vector polarization, one cannot isolate the tensor component p_{yz}. As prepared in a polarized-ion source, where the quantization (3) axis is an axis of cylindrical symmetry, the sole vector and tensor components are p_3 and p_{33}, respectively. When the axis is aligned along the direction $y=z$ in the chosen projectile helicity frame, the polarization components are [12]

$$p_x=0\ ,\quad p_{xx}=-\frac{1}{2}p_{33}\ ,\quad p_{xy}=0\ ,$$
$$p_y=\frac{1}{\sqrt{2}}p_3\ ,\quad p_{yy}=\frac{1}{4}p_{33}\ ,\quad p_{yz}=\frac{3}{4}p_{33}\ , \tag{29}$$
$$p_z=\frac{1}{\sqrt{2}}p_3\ ,\quad p_{zz}=\frac{1}{4}p_{33}\ ,\quad p_{xz}=0\ .$$

Thus the spin-1 target density matrix

$$\rho_T=\tfrac{1}{3}[P_o+\tfrac{3}{2}p_yP_y+\tfrac{3}{2}p_zP_z+\tfrac{1}{6}(p_{yy}-p_{zz})(P_{yy}-P_{zz})$$
$$+\tfrac{1}{2}p_{xx}P_{xx}+\tfrac{2}{3}p_{yz}P_{yz}]\ , \tag{30}$$

for $p_3=p_{33}=1$, becomes

$$\rho_T=\tfrac{1}{3}\left[P_o+\frac{3}{2\sqrt{2}}P_y+\frac{3}{2\sqrt{2}}P_z-\frac{1}{4}P_{xx}+\frac{1}{2}P_{yz}\right]\ . \tag{31}$$

Then, taking $p_3=\frac{1}{3}$, $p_{33}=\pm 1$, which is a combination available from a polarized-ion source, the result corresponding to Eq. (20) is

$$\sigma A_{x,yz}=\tfrac{1}{6}K\ \mathrm{Im\,Tr}[(-\tfrac{1}{4}\sigma_oP_{xx}+\tfrac{1}{2}\sigma_oP_{yz}-\tfrac{1}{4}\sigma_xP_{xx}$$
$$+\tfrac{1}{2}\sigma_xP_{yz})M(0)]\ . \tag{32}$$

And with

$$\tfrac{2}{3}\mathrm{Tr}[(\sigma_jP_{yz})(\sigma_{j'}P_{lm})=6\delta_{jj'}\delta_{(yz)(lm)}\ , \tag{33}$$

the result corresponding to Eq. (24) is

$$A_{x,yz}=\tfrac{3}{4}\mathrm{Im}\,a_{x,yz}/\mathrm{Im}\,a_{o,o}\ . \tag{34}$$

As the notation indicates, this is the spin-correlation coefficient for the beam polarization p_x in combination with the target tensor polarization p_{yz}, as is shown in Fig. 1, and the corresponding transmission asymmetry is

$$\Delta T_{x,yz}=-\tfrac{3}{4}K\ \mathrm{Im}\,a_{x,yz}\quad \text{for } a_{x,yz}\ll 1\ . \tag{35}$$

Although these results have been derived for the specific case of a spin-1 target, they are valid generally for the case of rank-2 tensor polarization of a target of spin $I\geq 1$. A very recent determination of this spin-correlation coefficient has been made in the transmission of 2-MeV polarized neutrons through an aligned ^{165}Ho target with $I=\frac{7}{2}$ [18], and the result, $(1\pm 6)\times 10^{-4}$, clearly demonstrates the improvement that has been achieved in tests of T symmetry by using this null-test observable. It is also clear that a further improvement in precision of more than two orders of magnitude can be attained in such transmission experiments with proton beams at higher energies, since this has already been demonstrated in the analogous null tests of P symmetry.

Even though I have described this observable $A_{x,yz}$ in terms of a polarized spin-$\frac{1}{2}$ projectile and a tensor-polarized target, the present experimental facilities may be better suited to using "reverse kinematics," for example, a tensor-polarized deuteron beam and a polarized proton target. This follows from the fact that proton targets have achieved significantly higher polarizations than have tensor-polarized deuteron targets, and deuteron beams of high tensor polarization are available.

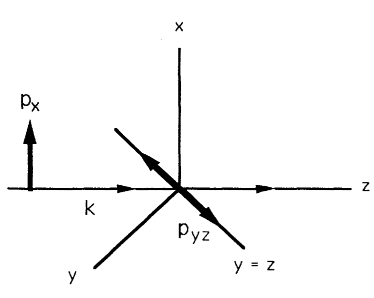

FIG. 1. Experimental arrangement for the measurement of the total-cross-section spin-correlation coefficient $A_{x,yz}$. The beam in the z direction, with polarization p_x, is transmitted through the target with tensor polarization p_{yz}, i.e., alignment along the direction $y=z$. This is symbolic only. Other tensor polarization components are present, as is described in the text, but they do not contribute in the determination of $A_{x,yz}$.

III. SUMMARY

The complete spin-space scattering matrix has been used in order to identify unambiguously the T-odd and/or P-odd forward-scattering amplitudes. These then provide, via the spin-dependent optical theorem, the total-cross-section observables that constitute null tests of the corresponding symmetries. Although these spin-dependent total cross sections are often measured via transmission experiments, they can also be determined from measurements of the scattered flux of particles integrated over the experimentally accessible solid angle (see, e.g., [2,3]). It is important to realize that precision in the determination of the absolute total cross section is not essential. The relevant precision is that attained in the ratio of the cross sections for the opposite spin orientations.

An especially important result is that the TRV observable is directly proportional to the imaginary part of the corresponding TRV forward-scattering amplitude, in contrast to the situation that exists for the standard (nonforward) scattering experiments. There, T symmetry tests can be accomplished only via comparison of two separate observables, e.g., $P_y - A_y$, where that difference is given in terms of a TRV amplitude in a bilinear combination with a TRI amplitude. As a result, there have been instances where the (unknown) TRI amplitude turned out to be so small that no significant test of TRI had, in fact, been made [20]. This kind of ambiguity does not exist in this null test, and one can directly state the precise level to which the (imaginary part of the) T-odd amplitude has been determined.

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[1] F. Arash, M. J. Moravcsik, and G. R. Goldstein, Phys. Rev. Lett. **54**, 2649 (1985).

[2] S. Kistryn, J. Lang, J. Liechti, Th. Maier, R. Müller, F. Nessi-Tedaldi, M. Simonius, J. Smyrski, S. Jaccard, W. Haeberli, and J. Sromicki, Phys. Rev. Lett. **58**, 1616 (1987).

[3] P. D. Eversheim, W. Schmitt, S. Kuhn, F. Hinterberger, P. von Rossen, J. Chlebek, R. Gebel, U. Lahr, B. von Przewoski, M. Wiemer, and V. Zell, Phys. Lett. B **256**, 11 (1991); P. D. Eversheim (private communication).

[4] B. Heckel, M. Forte, O. Schaerpf, K. Green, G. L. Greene, N. F. Ramsey, J. Byrne, and J. M. Pendlebury, Phys. Rev. C **29**, 2389 (1984), and references therein.

[5] V. P. Alfimenkov, S. B. Borazakov, Vo Van Thuan, Yu. D. Mareev, L. B. Pickelner, A. S. Khrykin, and E. I. Sharpov, Nucl. Phys. **A398**, 93 (1983); Y. Masuda, T. Adachi, A. Masaike, and K. Morimoto, *ibid.* **A504**, 269 (1989); C. D. Bowman, J. D. Bowman, and V. W. Yuan, Phys. Rev. C **39**, 1721 (1989); J. D. Bowman, C. D. Bowman, J. E. Bush, P. P. J. Delheij, C. M. Frankle, C. R. Gould, D. G. Haase, J. Knudson, G. E. Mitchell, S. Penttilä, H. Postma, N. R. Roberson, S. J. Seestrom, J. J. Szymanski, V. W. Yuan, and X. Zhu, Phys. Rev. Lett. **65**, 1192 (1990); C. M. Frankle, J. D. Bowman, J. E. Bush, P. P. J. Delheij, C. R. Gould, D. G. Haase, J. N. Knudson, G. E. Mitchell, S. Penttilä, H. Postma, N. R. Roberson, S. J. Seestrom, J. J. Syzmanski, S. H. Yoo, V. W. Yuan, and X. Zhu, *ibid.* **67**, 564 (1991).

[6] L. Stodolsky, Phys. Lett. B **172**, 5 (1986).

[7] P. K. Kabir, Phys. Rev. Lett. **60**, 686 (1988); Phys. Rev. D **37**, 1856 (1988).

[8] A. G. Klein and S. A. Werner, Rep. Prog. Phys. **46**, 259 (1983), for example.

[9] J. P. Soderstrum, C. R. Gould, D. G. Haase, L. W. Seagondollar, M. B. Schneider, and N. R. Roberson, Phys. Rev. C **38**, 2424 (1988).

[10] References [6] and [7] choose the x axis to be along the direction of the transverse target polarization. I take that to be the y axis, so that its transformations under the P- and/or T-symmetry operations are the same as those for the usual choice of $\mathbf{y}=\mathbf{k}_i \times \mathbf{k}_f$ for (nonforward) scattering, as is detailed in Sec. II.

[11] G. R. Gould, D. G. Haase, N. R. Roberson, H. Postma, and J. D. Bowman, Int. J. Mod. Phys. A **5**, 2181 (1990).

[12] For example, G. G. Ohlsen, Rep. Prog. Phys. **35**, 717 (1972).

[13] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Annu. Rev. Nucl. Sci. **10**, 291 (1960).

[14] In elastic scattering, which is the application here, TRI requires a T-odd amplitude to vanish. More generally, TRI requires a T-odd transition amplitude to be equivalent, but opposite in sign, to the corresponding amplitude in the inverse process.

[15] R. J. N. Phillips, Nucl. Phys. **43**, 413 (1963).

[16] I am indebted to C. A. Miller for raising the question of an identical-particle constraint.

[17] P. K. Kabir, *The Investigation of Fundamental Interactions with Cold Neutrons,* Natl. Bur. Stand. (U.S.) Spec. Publ. No. 711 (U.S. GPO, Washington, D.C., 1986), p. 81. In V. G. Baryshevskii, Yad. Fiz. **38**, 1162 (1983) [Sov. J. Nucl. Phys. **38**, 699 (1983)], this T-odd P-even term was implicitly contained in the B_5 term following Eq. (2). However, it was not so identified explicitly, and the other B_i terms are all P odd.

[18] J. E. Koster, E. D. Davis, C. R. Gould, D. G. Haase, N. R. Roberson, L. W. Seagondollar, S. Wilburn, and X. Zhu, Phys. Lett. B **267**, 23 (1991). Here $A_{x,yz}$ is called a T-odd analyzing power. However, in the established terminology [19], an observable measured with both the beam and target polarized is a spin-correlation coefficient. I have extended this convention to the total-cross-section observables.

[19] The Madison Convention, in *Proceedings of the Third International Symposium on Polarization Phenomena in Nuclear Reactions,* edited by H. H. Barschall and W. Haeberli (University of Wisconsin Press, Madison, 1971), p. xxv. Its extensions have been compiled by N. Hoshizaki, J. Phys. Soc. Jpn. Suppl. **55**, 549 (1986).

[20] H. E. Conzett, in *Polarization Phenomena in Nuclear Physics—1980 (Fifth International Symposium, Santa Fe),* Proceedings of the Fifth International Symposium on Polarization Phenomena in Nuclear Physics, edited by G. G. Ohlsen, R. E. Brown, N. Jarmie, W. W. McNaughton, and G. M. Hale, AIP Conf. Proc. No. 69 (AIP, New York, 1981), p. 1452; J. Bystricky, F. Lehar, and P. Winternitz, J. Phys. (Paris) **45**, 207 (1984).