Examination of inconsistencies between the deformed potential model and folding models for analysis of inelastic hadron scattering

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It is customary to extract deformation lengths from inelastic scattering data by using a deformed optical potential. It is then assumed that this deformation length also characterizes the deformation of the underlying density distribution of the excited nucleus. This equivalence is exact for a dipole deformation, but this corresponds simply to a spurious excitation mode, namely a shift of the center of mass of the system. We show that, even when the potential is obtained by folding an effective interaction over the density distribution, the deformed potential model is not exact for other multipoles, except in the limit that the interaction has a zero range. The errors made when the interaction has a realistic finite range can be large.

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The deformed potential (DP) model [1] has been the primary tool for many years for the analysis of measurements on nuclear inelastic scattering. Designed to describe collective (vibrational or rotational) excitations, it has frequently been applied to weaker, noncollective transitions also. In the vibrational case, the standard model [2] associates a radial transition density (TD) of the form

$$g_l(r) = -\delta_l^m \frac{d\rho(r)}{dr} \tag{1}$$

with the excitation of one isocalar phonon. (We use the notation of Ref. [3].) Here, $\rho(r)$ is the ground-state mass distribution and δ_l^m is the deformation length. We note that in this model the radial shape of the TD is independent of the multipolarity l.

It is argued that, because the nuclear force has a short range, the interaction potential $U(r)$ between an incident projectile and the target nucleus also undergoes oscillations in shape that follow the density motion. This leads to a radial transition potential (TP)

$$G_l^{\mathrm{DP}}(r) = -\delta_l^U \frac{dU(r)}{dr} \tag{2}$$

for the excitation of one phonon. Again, the radial shape in this DP model is independent of l. Frequently it is also argued that this displacement of the potential surface should equal that of the mass distribution, so that

$$\delta_l^U = \delta_l^m \ . \tag{3}$$

A folding procedure is implicit in the argument relating the TP (2) and the TD (1). Without it there is no simple connection between the density and the potential. Consequently, there is little justification for the assumption (3) when phenomenological models such as the Woods-Saxon one are used for the potential $U(r)$. In the folding approach [1,3], the optical potential results from folding an effective central interaction $v(|\mathbf{r}-\mathbf{r}'|)$, between the projectile at position $\mathbf{r}$ and a target nucleon at $\mathbf{r}'$ with the ground state mass distribution ρ of the target,

$$U(r) = \int_0^\infty \rho(r') v_0(r,r') r'^2 dr' \ . \tag{4}$$

(For simplicity we discuss "single folding." A "double folding" procedure, which includes folding over the projectile mass distribution, leads to the same conclusions.) Similarly, the TP is obtained by folding (which we denote by F) with the TD of Eq. (1),

$$G_l^F(r) = -\delta_l^m \int_0^\infty \frac{d\rho(r')}{dr'} v_l(r,r') r'^2 dr' \ . \tag{5}$$

In these equations, $v_l(r,r')$ is the l^{th} term in the multipole decomposition of $v(|\mathbf{r}-\mathbf{r}'|)$ [1,3]. The form (2), with the relation (4), then follows exactly if the interaction has zero range, $v(s) = J\delta(s)$ where $s = \mathbf{r}-\mathbf{r}'$, for then

$$U(r) = J\rho(r) \tag{6a}$$

and

$$G_l^F(r) = Jg_l(r) = -\delta_l^m dU(r)/dr \ . \tag{6b}$$

Realistic nuclear forces have finite ranges, so the relation between potential and density is nonlinear. It has been remarked repeatedly over the last three decades, both for light [4] and heavy ion [5] projectiles, that one consequence of this finite range is that the radial shapes of the transition potentials $G_l^F(r)$, defined by (5), acquire a strong dependence on multipolarity l, even if the corresponding transition density shape is independent of l, as in (1). This has been demonstrated for particular examples and specific choices of the interaction $v(s)$. Here we show for general $v(s)$ that the relations (2) and (3) are *exact* for $l = 1$. As a corollary, we see that they are *not* correct for other multipoles. The results of a numerical survey of the errors that can be committed by using the DP model will be presented elsewhere [6]. These errors can be large. For example, the extracted nuclear transition rate for octupole excitation of ^{208}Pb can be in error by a factor of 1.5 for ^{16}O projectiles at $E/A = 84$ MeV. The deviations are somewhat smaller for lighter ions, but can for example, have large effects on the deduced values of the M_n/M_p, the ratio of neutron and proton matrix

elements for the transition [7].

A special case in which the relations (2) and (3) hold is a dipole mode, $l=1$, which does not depend upon folding, but is a consequence of the invariance of the system under translation of its center of mass. Unfortunately, this case is unphysical and Eqs. (1) and (2) then correspond to a spurious mode of excitation. For $l=1$, the TD (1) corresponds to a small displacement, of magnetidue δ_1^m, of the center of mass of the system, without change of shape. Similarly, the TP (2) corresponds to a displacement δ_1^U of the potential U when $l=1$. Since a simple translation of the system will carry with it the potential, however generated, unchanged, we have $\delta_1^m=\delta_1^U$ and Eq. (3) is satisfied.

The form (1) with $l=1$ does correspond to a physical excitation if it is applied to the neutron and proton distributions separately, provided their displacements are in the ratio $\delta_1^n/\delta_1^p=-Z/N$ in order to preserve the center of the total mass. This is just the Goldhaber-Teller model for an isovector dipole excitation in which the neutrons and proton oscillate against one another [1,3]. However, the connection with the optical potential, which is primarily isocalar, then requires additional assumptions [1,3].

In order to demonstrate these features explicitly for the folding model, and to examine other multipoles, we use Fourier transform techniques [1,8]. For simplicity we consider interactions $v(s)$ that do not depend upon the density.

The optical potential (4) can be rewritten in the form

$$U(r)=\frac{1}{2\pi^2}\int_0^\infty j_0(kr)\tilde{\rho}(k)\tilde{v}(k)k^2dk \tag{7}$$

and the folded TP (5) becomes

$$G_l^F(r)=\frac{1}{2\pi^2}\int_0^\infty j_l(kr)\tilde{g}_l(k)\tilde{v}(k)k^2dk\ , \tag{8}$$

where we use a tilde to denote the Fourier transform. For example,

$$\tilde{g}_l(k)=4\pi\int_0^\infty g_l(r)j_l(kr)r^2dr\ . \tag{9}$$

By using the TD of Eq. (1) and integration by parts, Eq. (9) becomes

$$\tilde{g}_l(k)=4\pi\delta_l^m\int_0^\infty\rho(r)\frac{d}{dr}[r^2j_l(kr)]dr\ . \tag{10}$$

For the TP in the DP model we require

$$\frac{DU(r)}{dr}=\frac{1}{2\pi^2}\int_0^\infty\frac{dj_0(kr)}{dr}\tilde{\rho}(k)\tilde{v}(k)k^2dk\ ,$$

$$=-\frac{1}{2\pi^2}\int_0^\infty j_1(kr)\tilde{\rho}(k)\tilde{v}(k)k^3dk\ . \tag{11}$$

In general this bears no simple relation to the folded TP of Eq. (8). However, when $l=1$ we can use the property

$$\frac{d}{dr}\left[r^2j_1(kr)\right]=kr^2j_0(kr) \tag{12}$$

in Eq. (10) to give

$$\tilde{g}_1(k)=\delta_1^mk\tilde{\rho}(k)\ . \tag{13}$$

Then comparing (8) and (11) we see the identity

$$G_1^F(r)=-\delta_1^m\frac{dU(r)}{dr} \tag{14a}$$

$$=G_1^{DP}(r) \tag{14b}$$

if $\delta_1^U=\delta_1^m$. This result is independent of the form chosen for $v(s)$ but, as discussed earlier, the TD of Eq. (1) and the TP of Eq. (2) do not describe a physical excitation when $l=1$.

Since the radial shape of the folded $G_l^F(r)$ is known to depend on l, while the shape of the deformed potential $G_l^{DP}(r)$ is independent of l, it follows as a corollary that there can be no identity between them for $l\neq1$ (except in the limit of an interaction v with zero range). Thus the folding procedure provides no justification for using the deformed potential model (2) for excitations with $l>1$.

Our conclusions depend upon the use of the folding procedures (4) and (5) to provide a link between potentials and the underlying densities. Some such link is necessary, and folding has proven quite successful. The derivation depends upon the ansatz (1) for the transition density. This choice is supported for strong (collective) transitions both by experiment, such as electron scattering, and by microscopic structure calculations such as those using the RPA, but primarily for $l=2$ and 3 multipolarities. There is no similar justification for weaker excitations or higher multipolarities. Using the DP model to analyze these transitions is even more suspect, especially as the difference between folding and the DP model increases as l increases [4,5]. We conclude by quoting from Chap. 14 of Ref. [1]: There is no strict justification for this [the DP model] except for a point projectile and in the unrealistic limit of folding with an interaction v with zero range. Nonetheless, the model incorporates the appropriate physics in a qualitative way and has the advantage of simplicity.

The price paid for this simplicity is that the unambiguous connection between the density deformation and the potential deformation that is provided by folding is no longer available, and one must not be surprised if the excitation of a given transition by different probes results in different values for the deformation parameter when this model is used.

An additional price paid for the simplicity of the DP is that the explicit dependence on l of the shape of the transition potential is being neglected. Numerical studies [4,5] have shown that the differences between using the DP and folding approaches increase as $l-1$ increases. In practice, the most extensive applications of the DP model have been to $l=2$ transitions. It is also for these transitions that there is most likely to be independent information [e.g., $B(E2)$ value] against which extracted deformation lengths can be checked. The apparent success of the DP model for quadrupole excitations may then be attributed to $l=2$ having the smallest deviation from $l=1$, although the quantitative degree to which this is so requires further investigation [6]. This does not justify the DP model for $l=3$ and higher multipolarities, for which the deviations become greater.

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