Inclusive and exclusive production of η mesons by pions

B. V. Krippa

Institute for Nuclear Research of the Russian Academy of Sciences, Moscow Region, 117812, Russia

J. T. Londergan

Department of Physics and Nuclear Theory Center, Indiana University, 2/01 Milo Sampson Lane,

Bloomington, Indiana 47408

(Received 6 April 1992)

Inclusive and exclusive (π, η) reactions are studied in the framework of a momentum space DWIA model. It is assumed that both reactions are dominated by formation of an $N(1535)$ resonance. Medium corrections for the self-energy of the intermediate N^* are taken into account. Cross sections are calculated for (π^+, η) inclusive reactions on ¹²C and compared with experiment and with existing theoretical calculations. We show that quantitative agreement is obtained with experimental data, assuming this reaction is dominated by quasifree single-nucleon knockout, and using an N^* free width of 150 MeV. Differential and total cross sections are calculated for the exclusive reaction ¹³C (π^+ , η)¹³N.

PACS number(s): 25.80.Hp

I. INTRODUCTION

The past few years have seen considerable interest in nuclear production of η mesons. First, the Saclay threshold measurements of $p + d \rightarrow \eta + ^3$ He revealed cross sections much larger than expected [1]. Theoretical calculations by Liu and collaborators [2] suggested that bound nuclear states of the η may exist. The large threshold cross sections suggest the possibility of creating a "tagged eta meson" facility in the future [3—5]. In addition to this possible facility, the new generation of machines being constructed at Jülich and Uppsala, and ϕ factories [6], promise excellent energy resolution and the possibility of efficient detection of η mesons. These developments may herald the beginning of a series of experiments involving production and detection of η mesons in a variety of nuclear interactions.

Nuclear interactions of the η meson have been the subject of several recent investigations [2,7—11]. At present, data are sufficiently scarce that our understanding of η reactions with nuclei is incomplete. However, as a good starting point we can assume that near threshold the η meson is produced predominantly through coupling to the $N(1535)$, an S_{11} π -N resonance [12]. Consequently, observation of η production in nuclei can provide information on production and propagation of the $N(1535)$ in the nucleus. Since the $N(1535)$ is an isospin 1/2 baryon, the (π, η) process in nuclei allows us to study an intermediate baryon resonance which is not completely dominated by intermediate $\Delta(1232)$ production.

Much of the theoretical interest in η meson production is due to the fact that the $s\bar{s}$ strength in the pseudoscalar mesons is concentrated in the η and η' . To date, no nuclear η meson production experiments make use of the $s\bar{s}$ character of the η , or allow us to infer the $s\bar{s}$ content of the nucleon [11]. In this paper we present a "conventional" picture of η meson production on nuclei; i.e., we use a DWIA formalism which has been widely used to describe nuclear scattering and meson production reactions. We shall show that the limited data on (π, η) reactions can be described rather well in such a calculation.

We focus on two possible reactions arising in pioninduced production of η mesons. First, we briefly review inclusive production via the (π, η) reaction on nuclei, following the methods outlined in a previous paper [10]. Next, we consider the exclusive process $A(\pi, \eta)B$, where ^A and B are bound nuclear states. Our paper is organized as follows. In Sec. II, we outline our formalism and review the results for inclusive (π, η) reactions. In Sec. III we apply this same formalism to exclusive reactions in certain light nuclei, and we present predictions for cross sections there. In Sec. IV we summarize the main results from our calculations and review the advantages and drawbacks of our present model. We conclude with suggestions for future improved calculations of these processes.

II. INCLUSIVE (π, η) REACTIONS

There are now data on the inclusive (π^+, η) process on nuclear targets [9]. There have been two previous theoretical models applied to this process. The first was an intranuclear cascade calculation [13,14]. Such a emiclassical calculation could be expected to give the gross features of the reaction, but probably not a quantitative explanation. The second is a DWIA calculation of this reaction. Two existing calculations are by Kohno and Tanabe [8], and by the present authors [10]. Here we briefly review the results of the two DWIA models.

Both DWIA calculations assume that the dominant re- \arctan process in (π^+, η) reactions is resonant excitation of a neutron by a π^+ to an $N(1535)$, which propagates

through the nucleus and undergoes quasifree decay to a free η and proton. Therefore only one particle-one hole final states of the nuclear system are considered. The final state proton-nucleus interactions are treated as in quasifree (p, p') inclusive reactions [15]. In paper I the inclusive cross sections were written as the square of a transition amplitude $T^{\pi \eta}$ with the general form

$$
T^{\pi\eta}(\mathbf{p}, \mathbf{k}_{\pi}, \mathbf{q}_{\eta}) = V^{\pi\eta}(\mathbf{p}, \mathbf{k}_{\pi}, \mathbf{q}_{\eta}) + \int \frac{d\mathbf{k}'}{(2\pi)^{3}} \frac{V^{\pi\eta}(\mathbf{p}, \mathbf{k}', \mathbf{q}_{\eta}) T^{\pi\pi'}(\mathbf{k}', \mathbf{k}_{\pi})}{\varepsilon(k') - \varepsilon(k_{\pi}) + i\epsilon}.
$$
\n(1)

In Eq. (1), k_{π} denotes the incident pion momentum, q_{η} the outgoing η momentum, and p the momentum of the (unseen) final proton. The first term $V^{\pi\eta}$ corresponds to pion-induced η production with no distortion of the incident pion, and the second term includes the effects of incident pion rescattering followed by η production. The initial-state pion-nucleus rescattering is accounted for in both models; the Kohno-Tanabe calculation uses a local pion optical potential assumed to be directly proportional to the one-body nuclear density, while the calculation of paper I uses a Glauber model calculation for pion rescattering together with an assumed separable offshell vertex function.

In both calculations the elementary $\pi + N \rightarrow \eta + N$ t matrix was taken from the form proposed by Bhalerao and Liu [7]. The free resonance position and width of the $N^{*}(1535)$ propagator are modified by including nuclear interactions of the isobar. In the Kohno-Tanabe calculation a constant imaginary part is assumed for the N^* :he
ear
cu-
N*
paself-energy (between $-50i \rightarrow -100i$ in strength). In paper I, the imaginary part of the self-energy was calculated by estimating the effects shown in Fig. 1. The imaginary self-energy contribution from these terms was evaluated using the Cutkosky rules [16,17], and the resulting Lindhard functions were evaluated in the Fermi gas approximation [18]. We assumed a real N^* -nucleus potential of -50 MeV, in agreement with Ref. [17]. The various contributions are discussed in paper I; the net result is to increase the isobar width in the nuclear medium.

In Fig. 2 we show (π^+, η) inclusive cross sections on a ¹²C target, in $\mu b/\text{MeV}$ sr, vs the η kinetic energy in MeV, for pion laboratory momentum 680 MeV/c. Experimental points are those of Peng et aL [9]; the solid

FIG. 1. Diagrammatic representation of self-energy contributions from various processes in the nucleus. (a) Pauli blocking contribution to N^* width; (b) one-meson $(\pi \text{ or } \eta)$ decay of N^* in nucleus; (c) two-pion decay of N^* . . Γ s fr $\mathop{\rm con}\limits_{N^*}$

FIG. 2. (π^+,η) inclusive cross sections on ¹²C target at $p_{\pi}^{\text{lab}} = 680 \text{ MeV}/c$. Inclusive cross sections, in $\mu\text{b}/\text{MeV}$ sr, vs η kinetic energy in MeV. The data are from Ref. [9]. Solid curve: full results of Ref. [10]; dashed curve: DWIA calculation of Ref. [8].

curve is the result of the full DWIA calculation of paper I, and the dashed curve is the DWIA calculation of Kohno and Tanabe [8]. The Kohno-Tanabe calculation gives good agreement with the shape of the experimental spectrum but is about a factor of 2 below the data, while the full theoretical curve of paper I gives good quantitative agreement with the experimental data.

In paper I we showed that the discrepancy between our results and those of Kohno and Tanabe arises from two factors: a different treatment of the N^* -nucleus interac- $\begin{array}{l} \text{dat} \ \text{para} \ \text{be} \ \text{iv} \ \text{V*-1} \ \text{or} \ \text{v} \ \text{or} \ \text{v} \end{array}$ tions, and different descriptions of the π - 12 C initial-state rescattering. We first review the N^* -nucleus interactions.
Kohno and Tanabe assumed a constant imaginary N^* -Kohno and Tanabe assumed a constant imaginary N^* nucleus potential, as opposed to our Fermi gas calculation of the imaginary viev
sum
ppo:
V* : elf-energy. Our treatment of the isobar self-energy gave cross sections $20-25\%$ larger than Kohno and Tanabe, while both calculations predict nearly the same shape for the inclusive spectrum.

In our calculation, the dominant effect on the selfenergy comes from two-pion processes shown diagrammatically in Fig. 1(c). Although the branching ratio for this process is small, it has a relatively large effect on the isobar width in medium for two reasons. First, the phase space for this process is substantial because of the integration over the relative momentum of the pions. Second, energy sharing in two-pion decays can produce one pion with moderately low energy; such a low energy pion can have a substantial coupling to particle-hole nuclear states and hence produce a significant contribution to the isobar width in the nucleus. Previous calculations of N^* an
;es
;o-
V* medium effects have also observed this result [17,19].

The two calculations also differ in their description of the π -¹²C initial-state interactions. Pion-nucleus multiple scattering must be accurately accounted for, since the transition amplitude $\pi+N \to \eta+N$ is small compared to pion-nucleus scattering. Kohno and Tanabe used a pion optical potential of the form $t\rho$, where ρ is the one-body nuclear density and t is an isospin-averaged free πN amplitude. We made a Glauber calculation for pion-nucleus scattering using a symmetrized Fermi density [20] for the nuclear structure, and including Coulomb efFects, using the Glauber approximation model of Mach et al. [21].

In Fig. 3 we show results of our Glauber calculation of π^{+} -¹²C elastic scattering for 800 MeV/c pions. The experimental data are from Marlow et al. [22]. The solid curve is our full result and the dashed curve is the result without Coulomb effects. Our calculation agrees well with the data except in the deep minimum around 25° . Coulomb effects are negligible for angles greater than 10° . For large pion angles we expect our results to be larger than those calculated with harmonic oscillator nuclear wave functions, since such wave functions fall off rapidly at large momentum transfer. However, for pion angles from 10° to 20° , our predictions are larger than several calculations of pion scattering [23—25], all of which predict cross sections roughly 20% below the data in this angular region. Finally, we included both elastic and single charge-exchange pion scattering in our model; this makes only very small changes in our calculated cross sections.

The differences between the two DWIA calculations should not be taken too seriously, since neither calculation uses a true microscopic description of the N^* -nucleus interactions. We will discuss this further in the conclusions. Our result shown in Fig. 2 assumed a free N^* width of 150 MeV; this width is experimentally deter- ${\rm mined\,\, between\,\, 100\,\, and\,\, 250\,\, MeV}$ $[12].$ We repeated our sions. Our result shown
width of 150 MeV; this
mined between 100 and 2
calculation using an N^*
malizing the $\pi + N \rightarrow n$. calculation using an N^* free width of 200 MeV (renormalizing the $\pi + N \rightarrow \eta + N$ amplitude to fit the experimental cross section); the (π, η) inclusive cross sections decrease by about 40% when the width is changed from 150 to 200 MeV, and the shape of the spectrum changes only slightly.

In Fig. 4 we show predicted cross sections for the (π, η) reaction on a ¹²C target, for pion laboratory momentum 620 MeV/ c . In our previous calculations off-shell effects in pion rescattering were found to be small. For this cal-

FIG. 3. Differential cross sections for the ¹²C(π^{+} , π^{+})¹²C reaction for 800 MeV/ c pions. The experimental data are from Marlow et al. [22]. Solid curve: full Glauber result including Coulomb effects; dashed curve: excluding Coulomb effects.

FIG. 4. (π^+, η) inclusive cross sections on ¹²C target at $p_{\pi}^{\text{lab}} = 620 \text{ MeV}/c$. Inclusive cross sections, in $\mu\text{b}/\text{MeV}$ sr, vs η kinetic energy in MeV. Solid curve: distorted waves for outgoing proton; dashed curve: plane waves for outgoing proton. These calculations include only on-shell pion rescattering effects.

culation we therefore included only on-shell pion rescattering; we expect small corrections to this estimate due to off-shell effects not included in Fig. 4. The solid curve in this figure includes distorted waves for the outgoing proton; the dashed curve uses plane waves.

III. EXCLUSIVE (π, η) REACTIONS

The same DWIA model which we used to calculate inclusive η production can also be utilized to predict cross sections for exclusive reactions, such as $\pi + A \rightarrow \eta + B$, where A and B are bound nuclear states. For this process the c.m. differential cross section for η production has the form

$$
\frac{d\sigma}{d\Omega_{\eta}} = \frac{q_{\eta}}{(2J+1)k_{\pi}} \overline{\sum} |F^{\pi\eta}(q_{\eta}, k_{\pi})|^2.
$$
 (2)

In Eq. (2), $\frac{1}{(2J+1)}\sum$ represents an average over the initial spins and sum over final spin states in this transition.

The amplitude $F^{\pi\eta}$ in Eq. (2) is related to the T matrix for the elementary reaction by the relativistic expression

$$
\langle \mathbf{Q}'|F(E)|\mathbf{Q}\rangle = -\frac{1}{2\pi} \left[\mathcal{M}(Q') \mathcal{M}(Q) \right]^{1/2} \langle \mathbf{Q}'|T(E)|\mathbf{Q}\rangle
$$

where

$$
\mathcal{M}_{ij}(q) = \frac{E_i(q) E_j(q)}{E_i(q) + E_j(q)}.
$$

We want to calculate the amplitude for a charged pion incident on a nuclear target to produce an η meson, leaving the final nucleus in a bound state. In DWIA this amplitude can be written as the sum of two terms,

$$
F^{\pi\eta}(\mathbf{k}_{\pi}, \mathbf{q}_{\eta}) = U^{\pi\eta}(\mathbf{k}_{\pi}, \mathbf{q}_{\eta}) - \frac{1}{(2\pi)^2} \sum_{\pi'} \int \frac{d\mathbf{k}'}{\mathcal{M}(k')} \frac{F^{\pi\pi'}(\mathbf{k}_{\pi}, \mathbf{k'}) U^{\pi'\eta}(\mathbf{k}', \mathbf{q}_{\eta})}{\varepsilon(k') - \varepsilon(k_{\pi}) + i\epsilon} \,. \tag{3}
$$

In Eq. (3), the first term represents production of an η by a single hard collision without initial-state nuclear scattering by the pion. The second term includes a sum over all initial-state pion interactions, followed by η production. In the second term we include both elastic and single charge-exchange scattering of the pion. The final-state nuclear interactions of the η are accounted for in the self-energy of the N^* in the nuclear medium. This calculation is identical to the self-energy calculation for the inclusive reaction and is given in paper I.

The transition amplitude $U^{\pi\eta}$ in Eq. (3) is related to the T matrix for the process $\pi + N \to \eta + N$ through the relation

$$
U^{\pi\eta} = -\frac{\left[\mathcal{M}_{\eta B}(q_{\eta})\mathcal{M}_{\pi A}(k_{\pi})\right]^{1/2}}{2\pi} \left\langle B; q_{\eta} | t(\omega) | A; k_{\pi} \right\rangle \,. \tag{4}
$$

We confine ourselves to nuclear transitions involving only a single nucleon. In this case the nuclear matrix elements in Eq. (4) can be approximated by

$$
\langle B; q_{\eta} | t(\omega) | A; k_{\pi} \rangle \simeq F_{\text{c.m.}}^{\frac{1}{2}} t(\omega, q_{\eta}, k_{\pi}, \langle p \rangle) \int d\mathbf{r} \, \phi_{f}^{*}(\mathbf{r}) \phi_{i}(\mathbf{r}) \exp \left[i \frac{\mathcal{A} - 1}{\mathcal{A}} \mathbf{Q} \cdot \mathbf{r} \right] . \tag{5}
$$

In Eq. (5), ϕ_i and ϕ_f are respectively single-particle wave functions in initial and final states, the momentum transfer $\mathbf{Q} = \mathbf{k}_{\pi} - \mathbf{q}_{\eta}$, and the factor $F_{\text{c.m.}}^{\frac{1}{2}} = \exp \left[Q^2/4 \mathcal{A} \alpha \right]$ is
the standard correction which compensates for the lack of translation invariance in the single-particle shell model [26], where A is the nuclear mass number and α the harmonic oscillator parameter. To evaluate Eq. (5) we have made a factorization approximation in which the elementary amplitude is evaluated at some effective Fermi momentum $\langle \mathbf{p} \rangle = -[2\mathbf{k}_{\pi} + (\mathcal{A} - 1)\mathbf{Q}]/2\mathcal{A}.$

In Fig. 5 we show cross sections for the exclusive reaction ${}^{13}C(\pi^+, \eta) {}^{13}N$ leading to the ground state of ${}^{13}N$. The cross sections in μ b/sr are shown vs η c.m. angle in degrees. In Fig. 5, the dashed curve is for incident pion laboratory momentum 680 MeV/c, the solid curve for 650

FIG. 5. ¹³C(π^+ , η)¹³N cross sections, in μ b/sr, vs η c.m. angle in degrees. Dashed curve: at $p_{\pi}^{\text{lab}} = 680 \text{ MeV}/c$; solid curve: $p_{\pi}^{\text{lab}} = 650 \text{ MeV}/c$; long-dashed curve: $p_{\pi}^{\text{lab}} = 620$ MeV/c.

 MeV/c , and the long-dashed curve for 620 MeV/c. The cross sections show a typical diffractive behavior, with a first minimum which moves to lower angles as the incident pion momentum increases. The overall cross section falls off rapidly with increasing momentum transfer (scattering angle), reflecting the falloff in the single-particle momentum density. We predict a maximum cross section of roughly 15–20 μ b/sr in the forward direction.

In Fig. 6 we show the dependence of the exclusive cross sections on various parameters in our calculation. The dashed curve represents our full results including both pion single charge exchange and isobar self-energy corrections. The solid curve represents the same calculation

FIG. 6. ¹³C(π^+ , η ¹³N cross sections, in μ b/sr, vs η c.m. angle in degrees, for $p_{\pi}^{\text{lab}} = 650 \text{ MeV}/c$. Dashed curve: full calculation including pion single charge exchange; solid curve: calculation without pion charge exchange; long-dashed curve: includes both elastic and charge-exchange pion scattering but without $N(1535)$ self-energy corrections.

TABLE I. Total cross sections for ${}^{13}C(\pi^+,\eta)^{13}N$ reaction.

k_{π}	$\sigma_{\rm tot}$
(MeV/c)	(μb)
620	2.03
650	2.80
680	3.41

with pion charge-exchange excluded. The long-dashe curve includes elastic and charge-exchange pion scatter- $\mathop{\hbox{ind}}$ but without N^* self-energy corrections, e.g. assuming e-ex
last
N*
dth only the free width for the isobar. From Fig. 6 we see that inclusion of pion single charge exchange tends to decrease the cross sections by about 25%. The isobar self-energy tends to be a more important ingredient in this calculation; assuming the free width of the isobar increases the cross section by a factor of roughly 2.5. As we mentioned in Sec. II, the dominant process which increases the width of the isobar in the nuclear medium is two-pion decay of the isobar. The total exclusive cross sections for this reaction are listed in Table I, for the different incident pion momenta. The total cross sections increase with increasing incident pion momentum. Over this range the elementary cross section $\pi + N \rightarrow \eta + N$ varies slowly with energy, so the rising total cross section is due to an increase in phase space for the reaction.

IV. CONCLUSIONS

We have previously shown that the inclusive (π, η) reaction on nuclei can be described quite well in a DWIA calculation. In this work we provide a detailed comparison of our results with earlier work by Kohno and Tanabe. We have extended this calculation to predict the differential and total cross sections for exclusive nuclear (π, η) reactions, for the reaction ${}^{13}C(\pi^+,\eta) {}^{13}N_{g.s.}$. We assume that the incident π^+ excites a neutron to an intermediate $N(1535)$ resonance, which then undergoes decay to $\eta+p$. A Glauber calculation was used to approximate the initial-state interactions of the incident pion, and we made estimates of the imaginary part of the N^* self-energy. We obtained quantitative agreement with experimental data for the inclusive reaction ${}^{12}C(\pi^+, \eta) {}^{12}N$, and we make qualitative predictions for the exclusive re- $\rm{action \ }^{13}C(\pi^+,\eta)^{13}N_{g.s.}, \rm{ for \ which \ no \ measurements \ ex-}$ ist at present.

These calculations need to be improved in a number of areas before they can be used with any confidence. The most pressing need is for a full microscopic treatment of the production and nuclear propagation of the $N(1535)$. The "isobar-hole" model used successfully for pion-nucleus reactions [27] should be extremely useful for η production reactions. Such models assume that meson formation occurs solely through production and propagation of isobar doorway states. For η mesons this should be an excellent approximation. Only two nonstrange baryons, the $N(1535)$ S_{11} and $P(1710)$ P_{11} π - N resonances, have any substantial coupling to $\eta + N$ [12]. Consequently the isobar-hole model should be quite reliable to describe the (π, η) reaction; this model can provide a realistic treatment of the full isobar self-energy, and can incorporate realistic nuclear densities.

Near threshold the reaction $\pi + N \rightarrow \eta + N$ is dominated to a large extent by the S_{11} resonance at 1535 MeV; higher resonances [particularly the $P(1710)$] should contribute substantially to this process at higher pion incident energies. For example, phenomenological fits to η production cross sections induced by pions assumed contributions from three different nucleon resonances [28]. Additional precise data on the basic production reaction $\pi + N \rightarrow \eta + N$ over a wide energy band would still be quite useful.

When these improvements have been carried out one can calculate η production cross sections for a wide variety of energies and nuclear targets. Finally, for simplicity we have made a factorization approximation for the exclusive reaction calculation. This involved evaluating the transition operator at some averaged value of the momentum rather than integrating over it. The validity of this approximation for such calculations needs to be checked.

In the future there are several possible apphcations of these DWIA calculations. First, for certain exclusive reactions one may be able to isolate diferent parts of the $\pi + N \rightarrow \eta + N$ transition amplitude; for example, one may be able to isolate the spin-dependent part of this transition amplitude for certain nuclear transitions. One can also use this same formalism to examine inelastic (π, η) transitions leading to excited states of the residual nucleus. This model could also be extended to treat quasifree reactions involving three bodies $(\eta, \text{ nucleon},$ and residual nucleus) in the final state. We might expect these states to be the dominant final states resulting from quasifree reactions in
and residual nucleus) i
these states to be the c
production of an N^*
similar model could b production of an N^* in the nuclear medium. Finally, a similar model could be constructed to examine protoninduced η production, e.g., (p, η) inclusive reactions, or (p, N^*) reactions [29].

ACKNOWLEDGMENTS

The authors wish to thank Dr. L-C. Liu for enlightening discussions, and Dr. R. Mach for the pion rescattering Glauber code. This research was supported in part by the U.S. National Science Foundation under Contract NSF-PHY91-08036.

- [1] J. Berger et al., Phys. Rev. Lett. 61 , 919 (1988).
- [2] L.C. Liu and Q. Haider, Phys. Rev. C 34, 1845 (1986); Q. Haider and L.C. Liu, Phys. Lett. B 172, 257 (1986).
- [3] B.M.K. Nefkens, in New Vistas in Physics with High-

Energy Pion Beams, edited by B. Gibson and J. McClelland (World Scientific, Paris, 1988).

- [4] R.E. Chrien, Summary of Eta Meson Sessions, in [5].
- [5] Workshop on Future Directions in Particle and Nuclear
- $[6]$ G. Barbiellini et al., Particle World 1, 125 (1990).
- [71 R.S. Bhalerao and L.C. Liu, Phys. Rev. Lett. 54, 865 (1985).
- [8] M. Kohno and H. Tanabe, Nucl. Phys. A519, 755 (1990).
- [9] J.C. Peng et al., Phys. Rev. Lett. 63 , 2353 (1989); J.C. Peng, in Proceedings of the Third International Symposium of $\pi N/NN$ Physics, Gatchina, 1989 (unpublished), p. 315; Proceedings of 17th INS International Symposium on Nuclear Physics at Intermediate Energies, Tokyo, 1988 (unpublished), p. 233.
- [10] B.V. Krippa and J.T. Londergan, Phys. Lett. B 286, 216 (1992). Referred to hereafter as paper I.
- [11] J.T. Londergan, Nuclear Eta Production, in [5].
- [12] Particle Data Group, Phys. Lett. B 239, 1 (1990).
- [13] W. Gibbs and W. Kaufman, unpublished, cited by Peng, in Proceedings of the Third International Symposium of $\pi N/NN$ Physics [9].
- [i4] A.S. Iljinov, E.S. Golubeva, and I.A. Pshenichov (unpublished) .
- [15] S.V. Förtsh et al., Nucl. Phys. A485, 258 (1988).
- [16] C. Itzykson and J.B. Zuber, Quantum Field Theory (McGraw-Hill, New York, 1980); R.E. Cutkosky, J.

Math. Phys. 1, 429 (1960).

- 17] H.C. Chiang, E. Oset, and L.C. Liu, Phys. Rev. C 44, 2 (1991).
- 18] C. Garcia-Recio, E. Oset, and L.L. Salcedo, Phys. Rev. C 37, 194 (1988).
- [19] E. Oset, Nucl. Phys. **B304**, 820 (1988).
- [2o] V.V. Burov and V.K. Lukyanov, Dubna Report 3INR P4-11098, 1977 (unpublished).
- 21] R. Mach, M.G. Sapoznikov, and Yu.A. Shcherbakov Czech. J. Phys. B 26, 1248 (1976).
- 22] D. Marlow et al., Phys. Rev. C 30, 1662 (1984).
- [23] M. Arima, K. Masutani, and R. Seki, Phys. Rev. C 44, 415 (1991).
- 24] E. Oset and D. Strottman, Phys. Rev. C 44, 468 (1991).
- [25] V. Franco and H.G. Schlaile, Phys. Rev. C 41, 1075 $(1990).$
- 26] T. De Forest, Jr. and J.D. Walecka, Adv. Phys. 15, 1 (1966).
- 27] M. Hirata, J.H. Koch, F. Lenz, and E.J. Moniz, Phys. Lett. 70B, 281 (1977); E. Oset, H. Toki, and W. Weise, Phys. Rep. 83, 282 (1982).
- 28] R.M. Brown et al., Nucl. Phys. **B153**, 89 (1979).
- [29) L.C. Liu, 3.T. Londergan, and G.E. Walker, Phys. Rev. C 40, 832 (1989).