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Hypertriton calculation with meson-theoretical nucleon-nucleon and hyperon-nucleon interactions

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Faddeev equations for the coupled ΛNN and ΣNN systems are solved precisely for meson-theoretical nucleon-nucleon and hyperon-nucleon interactions. In this force model the hypertriton is not bound.

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I. INTRODUCTION

The hypertriton ($^3_\Lambda$H) is the bound nuclear system of lowest mass including one hyperon Y (Λ or Σ). It is, obviously, the first system in which to test YN forces including the interesting Λ-Σ conversion. In a formalism keeping only a Λ and nucleons, that conversion induces a ΛNN three-body force; in a formalism where both Λ and Σ degrees of freedom are kept, one handles a coupled channel problem between the ΛNN and ΣNN systems, which incorporates that three-body force but also leads to a modification of the YN force due to the presence of a second nucleon, called the dispersive effect. We shall treat the hypertriton in the coupled channel form.

The hypertriton has been studied by many authors over the years, even including heavier systems to better constrain the YN forces. Variational techniques [1,2] explored ranges and strengths for repulsive and attractive pieces of local parametrizations of the YN forces, including spin dependencies and even three-body force effects [2]; separable forces have been handled in various Faddeev calculations with increasing complexities in their dynamical assumptions: spin-dependent forces without tensor forces [3–5]; inclusion of tensor forces in the NN sector [6] as well as in the YN sector [7,8], and the allowance for Λ-Σ conversion [5,7–9]. Some of those potential parameterizations were chosen *ad hoc,* some others were chosen to mimic properties of meson-theoretical forces. Also, the Faddeev equations have been solved for local forces [10], not using any separable approximation. The resulting binding energy effects were quite diversified, reflecting the many *ad hoc* assumptions about the forces. Clearly, better and more experimental data on the YN systems are needed to reduce the great spread in the model force assumptions. Though the many binding energy results cannot be considered to lead to definite conclusions, general qualitative features of all these studies remain valuable.

On the other hand, the theoretical insights into meson-induced forces have developed over the years [11,12] and it now appears worthwhile to test the resulting YN interaction in the hypertriton. We shall use the very recent Jülich hyperon-nucleon interaction in a one boson exchange (OBE) potential parametrization [13] combined with various realistic NN interactions. To the best of our knowledge, this is the first time that this meson-theoretical interaction has been used in a hypertriton calculation.

It will not be the aim of the present study to adjust force parameters such that the hypertriton binding energy reproduces an experiment, but we shall take the YN forces as they are predicted by theory [13] in conjunction with the few YN data. The agreement or disagreement of theoretical and experimental hypertriton binding energies will then tell us whether the present day meson-theoretical (energy independent) baryon-baryon forces are adequate to describe that system or whether they possibly should be modified. In the second case, one has to, of course, keep in mind that, on top of the ΛNN three-body force combined with the dispersive effect, which is induced by the Λ-Σ conversion and which we shall take fully into account, one expects additional three-body forces [14]. Thus, an adjustment of the YN forces to the hypertriton might be a doubtful strategy. If, in the near future, precise four- and five-baryon calculations including one hyperon appear, then tests of meson-theoretical YN forces and the question of additional three-body forces will be better posed, since larger binding energies will be involved and two- and three-body forces will be probed under different conditions.

The formalism of coupled Λ-Σ channels in the context of the Faddeev equations will be displayed in Sec. II. This is not new and at least some of the equations have certainly appeared in the literature before (see for instance [5,8]). Nevertheless, since we shall solve the Faddeev equations including Λ-Σ conversion and the full complexities of baryon-baryon forces, not relying on separable structures, we find it justified to display the short derivation leading to the coupled set (2.22)–(2.25) below, which, to the best of our knowledge, has not been solved before in that generality. Previous studies, even on the formal level, have used either the separable nature of the forces at quite an early state, or, as in [10], did not include Λ-Σ conversion and therefore did not show that set in this form which we use directly. The technicalities to handle the set numerically are presented in Sec. II and our results in Sec. IV. We conclude in Sec. V.

II. FORMALISM

We shall keep both states for the hyperons, Λ and Σ, explicitly. Therefore, the wave function for the hypertriton has two orthogonal parts

$$|\Psi\rangle = |NN\Lambda\rangle\Psi_{NN\Lambda} + |NN\Sigma\rangle\Psi_{NN\Sigma} . \quad (2.1)$$

After choosing a three-body Hamiltonian, the energy eigenvalue equation reads

$$\langle NN\Lambda|H|NN\Lambda\rangle\Psi_{NN\Lambda} + \langle NN\Lambda|H|NN\Sigma\rangle\Psi_{NN\Sigma} = E\Psi_{NN\Lambda} , \quad (2.2)$$

$$\langle NN\Sigma|H|NN\Lambda\rangle\Psi_{NN\Lambda} + \langle NN\Sigma|H|NN\Sigma\rangle\Psi_{NN\Sigma} = E\Psi_{NN\Sigma} .$$

More explicitly, assuming only two-body (transition) potentials this set is in obvious notation

$$[H_0(N_1N_2\Lambda) + V_{N_1N_2} + V_{N_1\Lambda} + V_{N_2\Lambda}]\Psi_{N_1N_2\Lambda} + (V_{N_1\Lambda,N_1\Sigma} + V_{N_2\Lambda,N_2\Sigma})\Psi_{N_1N_2\Sigma} = E\Psi_{N_1N_2\Lambda} , \quad (2.3)$$

$$[H_0(N_1N_2\Sigma) + V_{N_1N_2} + V_{N_1\Sigma} + V_{N_2\Sigma}]\Psi_{N_1N_2\Sigma} + (V_{N_1,\Sigma,N_1\Lambda} + V_{N_2\Sigma,N_2\Lambda})\Psi_{N_1N_2\Lambda} = E\Psi_{N_1N_2\Sigma} .$$

We denote the two nucleons as $N_1$and N_2. For the kinetic energies H_0 we use nonrelativistic forms, which will be displayed below.

It is useful now to switch to a compact matrix notation

$$(E - \underline{H}_0)\underline{\Psi} = \underline{V}\underline{\Psi} , \quad (2.4)$$

where

$$\underline{H}_0 = \begin{bmatrix} H_0(NN\Lambda) & 0 \\ 0 & H_0(NN\Sigma) \end{bmatrix} , \quad (2.5)$$

$$\underline{V} = \begin{bmatrix} V_{N_1N_2} & 0 \\ 0 & V_{N_1N_2} \end{bmatrix} + \begin{bmatrix} V_{N_1\Lambda} & V_{N_1\Lambda,N_1\Sigma} \\ V_{N_1\Sigma,N_1\Lambda} & V_{N_1\Sigma} \end{bmatrix} + \begin{bmatrix} V_{N_2\Lambda} & V_{N_2\Lambda,N_2\Sigma} \\ V_{N_2\Sigma,N_2\Lambda} & V_{N_2\Sigma} \end{bmatrix} \equiv \underline{V}_{12} + \underline{V}_{13} + \underline{V}_{23} , \quad (2.6)$$

and

$$\underline{\Psi} \begin{bmatrix} \Psi_{N_1N_2\Lambda} \\ \Psi_{N_1N_2\Sigma} \end{bmatrix} . \quad (2.7)$$

We number the two nucleons by 1 and 2 and the Λ or Σ by 3.

The integral form of that energy eigenvalue problem (2.4) provides the Faddeev decomposition

$$\underline{\Psi} = \frac{1}{E - \underline{H}_0}\underline{V}\underline{\Psi} \equiv \underline{\psi}^{(12)} + \underline{\psi}^{(13)} + \underline{\psi}^{(23)} \quad (2.8)$$

with

$$\underline{\psi}^{(ij)} \equiv \frac{1}{E - \underline{H}_0}\underline{V}_{ij}\underline{\Psi} . \quad (2.9)$$

Inserting the decomposition again in the right-hand side and summing up $\underline{V}_{ij}$ within the pair ij to infinite order yields the matrix Faddeev equations

$$\underline{\psi}^{(ij)} = \frac{1}{E - \underline{H}_0}\underline{T}_{ij}\sum_{kl\neq ij}\underline{\psi}^{(kl)} . \quad (2.10)$$

Thereby the two-body T matrices obey the (matrix) Lippmann-Schwinger equations

$$\underline{T}_{ij} = \underline{V}_{ij} + \underline{V}_{ij}\frac{1}{E - \underline{H}_0}\underline{T}_{ij} , \quad ij,kl = 12,13,23 \quad (2.11)$$

We assume $\underline{\Psi}$ to be antisymmetrized in the two nucleons numbered by 1 and 2. Then it follows simply from (2.9) that

$$P_{12}\underline{\psi}^{(13)} = -\underline{\psi}^{(23)} . \quad (2.12)$$

Therefore it suffices to regard only two coupled (matrix) Faddeev equations:

$$\underline{\psi}^{(12)} = \frac{1}{E - \underline{H}_0}\underline{T}_{12}(1 - P_{12})\underline{\psi}^{(13)} \quad (2.13)$$

and

$$\underline{\psi}^{(13)} = \frac{1}{E - \underline{H}_0}\underline{T}_{13}(\underline{\psi}^{(12)} - P_{12}\underline{\psi}^{(13)}) . \quad (2.14)$$

Each of the two Faddeev amplitudes has two components

$$\underline{\psi}^{(ij)} = \begin{bmatrix} \psi_\Lambda^{(ij)} \\ \psi_\Sigma^{(ij)} \end{bmatrix} \quad (2.15)$$

and the free propagator

$$\frac{1}{E = \underline{H}_0} = \begin{bmatrix} \frac{1}{E - H_0(NN\Lambda)} & 0 \\ 0 & \frac{1}{E - H_0(NN\Sigma)} \end{bmatrix} \equiv \begin{bmatrix} G_0^\Lambda & 0 \\ 0 & G_0^\Sigma \end{bmatrix} \quad (2.16)$$

as well as $\underline{T}$ operators are 2×2 matrices. In case of the two-nucleon $\underline{T}$ operator, the Lippmann-Schwinger equation decouples:

$$\underline{T}_{12} \equiv \begin{bmatrix} t_{12}^\Lambda & 0 \\ 0 & t_{12}^\Sigma \end{bmatrix} \quad (2.17)$$

with

$$t_{12}^\Lambda = V_{12} + V_{12}G_0^\Lambda t_{12}^\Lambda \quad (2.18)$$

$$t_{12}^\Sigma = V_{12} + V_{12}G_0^\Sigma t_{12}^\Sigma . \quad (2.19)$$

In case of the hyperon-nucleon $\underline{T}$ operator

$$\underline{T}_{13} \equiv \begin{bmatrix} t_{13}^{\Lambda\Lambda} & t_{13}^{\Lambda\Sigma} \\ t_{13}^{\Sigma\Lambda} & T_{13}^{\Sigma\Sigma} \end{bmatrix} , \quad (2.20)$$

one faces coupled Lippmann-Schwinger equations

$$\begin{bmatrix} t_{13}^{\Lambda\Lambda} & t_{13}^{\Lambda\Sigma} \\ t_{13}^{\Sigma\Lambda} & t_{13}^{\Sigma\Sigma} \end{bmatrix} = \begin{bmatrix} V_{13}^{\Lambda\Lambda} & V_{13}^{\Lambda\Sigma} \\ V_{13}^{\Sigma\Lambda} & V_{13}^{\Sigma\Sigma} \end{bmatrix} + \begin{bmatrix} V_{13}^{\Lambda\Lambda} & V_{13}^{\Lambda\Sigma} \\ V_{13}^{\Sigma\Lambda} & V_{13}^{\Sigma\Sigma} \end{bmatrix} \begin{bmatrix} G_0^{\Lambda} & 0 \\ 0 & G_0^{\Sigma} \end{bmatrix} \begin{bmatrix} t_{13}^{\Lambda\Lambda} & t_{13}^{\Lambda\Sigma} \\ t_{13}^{\Sigma\Lambda} & t_{13}^{\Sigma\Sigma} \end{bmatrix} . \tag{2.21}$$

Explicitly the Faddeev equations read

$$\psi_{\Lambda}^{(12)} = G_0^{\Lambda} t_{12}^{\Lambda} (1-P_{12}) \psi_{\Lambda}^{(13)} , \tag{2.22}$$

$$\psi_{\Sigma}^{(12)} = G_0^{\Sigma} t_{12}^{\Sigma} (1-P_{12}) \psi_{\Sigma}^{(13)} , \tag{2.23}$$

$$\psi_{\Lambda}^{(13)} = G_0^{\Lambda} t_{13}^{\Lambda\Lambda} (\psi_{\Lambda}^{(12)} - P_{12}\psi_{\Lambda}^{(13)}) + G_0^{\Lambda} t_{13}^{\Lambda\Sigma} (\psi_{\Sigma}^{(12)} - P_{12}\psi_{\Sigma}^{(13)}) , \tag{2.24}$$

$$\psi_{\Sigma}^{(13)} = G_0^{\Sigma} t_{13}^{\Sigma\Lambda} (\psi_{\Lambda}^{(12)} - P_{12}\psi_{\Lambda}^{(13)}) + G_0^{\Sigma} t_{13}^{\Sigma\Sigma} (\psi_{\Sigma}^{(12)} - P_{12}\psi_{\Sigma}^{(13)}) . \tag{2.25}$$

The two parts of the total wave function in (2.1) are finally given as

$$\Psi_{NN\Lambda} = \psi_{\Lambda}^{(12)} + (1-P_{12}) \psi_{\Lambda}^{(13)} , \tag{2.26}$$

$$\Psi_{NN\Sigma} = \psi_{\Sigma}^{(12)} + (1-P_{12}) \psi_{\Sigma}^{(13)} . \tag{2.27}$$

Obviously, by choosing $\psi_{\Sigma,\Lambda}^{(12)}$ to be antisymmetric in 1 and 2, the total wave function is antisymmetric in the two nucleons, as it should be.

III. TECHNICALITIES

We work in momentum space and in a partial wave representation. Jacobi momenta for arbitrary masses are

$$\mathbf{p}_{\alpha} = \frac{m_{\gamma}\mathbf{k}_{\beta} - m_{\beta}\mathbf{k}_{\gamma}}{m_{\gamma}+m_{\beta}} \tag{3.1}$$

$$\mathbf{q}_{\alpha} = \frac{(m_{\beta}+m_{\gamma})\mathbf{k}_{\alpha} - m_{\alpha}(\mathbf{k}_{\beta}+\mathbf{k}_{\gamma})}{m_{\alpha}+m_{\beta}+m_{\gamma}} , \tag{3.2}$$

where the **k**'s denote the individual momenta. Then the kinetic energy H_0 in the center-of-mass system is

$$H_0 = \frac{1}{2} \frac{m_{\beta}+m_{\gamma}}{m_{\beta}m_{\gamma}} \mathbf{p}_{\alpha}^2 + \frac{1}{2} \frac{m_{\beta}+m_{\gamma}+m_{\alpha}}{m_{\alpha}(m_{\beta}+m_{\gamma})} \mathbf{q}_{\alpha}^2 + m_{\alpha}+m_{\beta}+m_{\gamma} . \tag{3.3}$$

We have to keep the rest masses due to the Λ-Σ transitions.

As above, particle 1 and 2 are nucleons and particle 3 is either Λ or Σ. Also, we use the convenient "odd man out" notation. Thus, denoting the hyperon masses by m_r $(r=\Lambda,\Sigma)$, we get for $\alpha\beta\gamma=312$

$$\mathbf{p}_3 = \tfrac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2) , \tag{3.4}$$

$$\mathbf{q}_3 = \frac{2m_N\mathbf{k}_3 - m_r(\mathbf{k}_1+\mathbf{k}_2)}{2m_N+m_r} , \tag{3.5}$$

$$H_0 = \frac{\mathbf{p}_3^2}{m_N} + \frac{\mathbf{q}_3^2}{2} \frac{2m_N+m_r}{2m_Nm_r} + 2m_N + m_r , \tag{3.6}$$

and for $\alpha\beta\gamma=231$

$$\mathbf{p}_2 = \frac{m_N\mathbf{k}_3 - m_r\mathbf{k}_1}{m_N+m_r} , \tag{3.7}$$

$$\mathbf{q}_2 = \frac{(m_N+m_r)\mathbf{k}_2 - m_N(\mathbf{k}_3+\mathbf{k}_1)}{2m_N+m_r} , \tag{3.8}$$

$$H_0 = \frac{\mathbf{p}_2^2}{2} \frac{m_N+m_r}{m_Nm_r} + \frac{\mathbf{q}_2^2}{2} \frac{2m_N+m_r}{m_N(m_N+m_r)} + 2m_N + m_r . \tag{3.9}$$

It follows that the free propagators $G_0^{\Lambda,\Sigma}$ are either

$$G_0^{\Lambda,\Sigma} = \frac{1}{E - 2m_N - m_{\Lambda,\Sigma} - \frac{\mathbf{p}_3^2}{m_N} - \frac{\mathbf{q}_3^2}{2} \frac{2m_N+m_{\Lambda,\Sigma}}{2m_Nm_{\Lambda,\Sigma}}} \tag{3.10}$$

or

$$G_0^{\Lambda,\Sigma} = \frac{1}{E - 2m_N - m_{\Lambda,\Sigma} - \frac{\mathbf{p}_2^2}{2} \frac{m_N+m_{\Lambda,\Sigma}}{m_Nm_{\Lambda,\Sigma}} - \frac{\mathbf{q}_2^2}{2} \frac{2m_N+m_{\Lambda,\Sigma}}{m_N(m_N+m_{\Lambda,\Sigma})}} \tag{3.11}$$

depending on the choice of Jacobi momenta. Clearly the first choice is suitable for the Faddeev amplitude $\psi_{\Lambda,\Sigma}^{(12)}$ and the second for $\psi_{\Lambda,\Sigma}^{(13)}$.

We use now the technique described, for instance, in [15,16]. If Λ or Σ is the "spectator" particle, the partial wave projected states are

$$|p_3q_3\alpha r\rangle \equiv |p_3q_3(ls)j(\lambda\tfrac{1}{2})I(jI)J(tt_r)T\rangle . \tag{3.12}$$

Here $(ls)j$ denote the orbital angular momentum, total spin, and total angular momentum of the $2N$ subsystem, while $(\lambda\frac{1}{2})I$ are the orbital angular momentum, spin, and total angular momentum of the third particle ("spectator") which is either $r=\Lambda$ or $r=\Sigma$. The total three-body angular momentum J is coupled from j and I. Finally, t and t_r are the $2N$ isospin and the isospin of the hyperon, respectively, which are coupled to total isospin T. The brackets denote angular momentum coupling. All possible values of these quantum numbers coupled to $J^{\pi}=\frac{1}{2}^{+}$

and $T=0$ of the hypertriton have to be taken into account in order to have a complete basis. Those basis states (3.12) are chosen to be antisymmetric under exchange of the two nucleons, which are guaranteed by $(-1)^{l+s+t}=-1$. On the other hand, in the case that one of the nucleons is the spectator, the basis states are

$$|p_2q_2\beta r\rangle \equiv |p_2q_2(ls)j(\lambda\tfrac{1}{2})I(jI)J[(t_r\tfrac{1}{2})t\tfrac{1}{2}]T\rangle \ , \tag{3.13}$$

where the two-body isospin t $(=\frac{1}{2})$ results from the coupling of the hyperon isospin t_r and the nucleon isospin $\frac{1}{2}$, which is coupled with the other nucleon isospin $\frac{1}{2}$ to the total isospin T. ($t=\frac{3}{2}$ is excluded because of $T=0$). The other definitions are obvious, as in (3.12). There is no symmetry requirement for the two-body subsystem of those basis states (3.13).

Now we are ready to write down the representation of the set (2.22)–(2.25) with respect to the basis (3.12) and (3.13):

$$\langle p_3q_3\alpha r|\psi_r^{(12)}\rangle = \frac{1}{E-2m_N-m_r-\dfrac{p_3^2}{m_N}-\dfrac{1}{2}\dfrac{2m_N+m_r}{2m_Nm_r}q_3^2} \times \sum_{\alpha' r'}\int_0^\infty dp_3' p_3'^2 \int_0^\infty dq_3' q_3'^2 \langle p_3q_3\alpha r|t_{12}^r|p_3'q_3'\alpha' r'\rangle\langle p_3'q_3'\alpha' r'|(1-P_{12})|\psi_{r'}^{(13)}\rangle \ . \tag{3.14}$$

The two-nucleon t matrix t_{12}^r is clearly diagonal in the spectator quantum numbers; thus

$$\langle p_3q_3\alpha r|t_{12}^r|p_3'q_3'\alpha' r'\rangle = \frac{\delta(q_3-q_3')}{q_3q_3'}\delta_{\lambda\lambda'}\delta_{II'}\delta_{rr'}t_{l,l'}^{sjtr}\left[p_3,p_3',E-2m_N-m_r-\frac{1}{2}\frac{2m_N+m_r}{2m_Nm_r}q_3^2\right] , \tag{3.15}$$

where $t_{l,l'}^{sjtr}$ is determined by the Lippmann-Schwinger equations (2.18) and (2.19). The permutation operator P_{12} can be applied to the left and yields a factor -1. Thus, we are left with the intermediate result

$$\langle p_3q_3\alpha r|\psi_r^{(12)}\rangle = \frac{2}{E-2m_N-m_r-\dfrac{p_3^2}{m_N}-\dfrac{1}{2}\dfrac{2m_N+m_r}{2m_Nm_r}q_3^2} \times \sum_{l'}\int_0^\infty dp_3' p_3'^2 t_{ll'}^{sjtr}\left[p_3,p_3',E-2m_N-m_r-\frac{1}{2}\frac{2m_N+m_r}{2m_Nm_r}q_3^2\right]\langle p_3'q_3\alpha(l')r|\psi_r^{(13)}\rangle \ . \tag{3.16}$$

Here $\alpha(l')$ denotes the set of discrete quantum numbers α except for l, which is l' and can change due to the action of the tensor force. The Faddeev amplitudes $\psi_r^{(13)}$ are naturally represented by the second type of basis states (3.13), as is obvious from Eqs. (2.34) and (2.25). This requires the recoupling

$$\langle p_3'q_3\alpha' r|p_2q_2\beta r\rangle = \int_{-1}^{1} dx\, G_{\alpha'\beta}^r(q_3q_2x)\frac{\delta(p_3'-\pi')}{p_3'^{l'+2}}\frac{\delta(p_2-\pi)}{p_2^{l+2}} \tag{3.17}$$

with

$$\pi' = \sqrt{q_2^2+\rho_3^2q_3^2+2\rho_3q_2q_3x} \ , \tag{3.18}$$

$$\pi = \sqrt{q_3^2+\rho_2^2q_2^2+2\rho_2q_2q_3x} \ , \tag{3.19}$$

where

$$\rho_3=\tfrac{1}{2} \ , \quad \rho_2=\frac{m_r}{m_N+m_r} \ . \tag{3.20}$$

The geometrical quantity $G_{\alpha'\beta}^r$ is given in the Appendix. Its evaluation follows the same lines as in [16]. Thus, we end up with

$$\langle p_3q_3\alpha r|\psi_r^{(12)}\rangle = \frac{2}{E-2m_N-m_r-\dfrac{p_3^2}{m_N}-\dfrac{1}{2}\dfrac{2m_N+m_r}{2m_Nm_r}q_3^2} \times \sum_{l'}\int_0^\infty dq_2q_2^2\int_{-1}^{1}dx\frac{t_{ll'}^{sjtr}\left[p_3,\pi',E-2m_N-m_r-\dfrac{1}{2}\dfrac{2m_N+m_r}{2m_Nm_r}q_3^2\right]}{\pi'^{l'}} \times \sum_\beta G_{\alpha(l')\beta}^r(q_3q_2x)\frac{\langle \pi q_2\beta r|\psi_r^{(13)}\rangle}{\pi^{l_\beta}} \tag{3.21}$$

for $r=\Lambda$ and Σ.

The remaining two equations (2.24) and (2.25) are treated analogously:

$$\langle p_2q_2\beta r|\psi_r^{(13)}\rangle=\frac{1}{E-2m_N-m_r-\frac{p_2^2}{2}\frac{m_N+m_r}{m_Nm_r}-\frac{q_2^2}{2}\frac{2m_N+m_r}{m_N(m_N+m_r)}}\times\sum_{l's'r'}\int_0^\infty dp_2'p_2'^2t_{lsrl's'r'}^{jt}\left[p_2p_2',E-2m_N-m_r-\frac{q_2^2}{2}\frac{2m_N+m_r}{m_N(m_N+m_r)}\right]\times\langle p_2'q_2\beta(l')r'|\psi_{r'}^{(12)}-P_{12}\psi_{r'}^{(13)}\rangle\ . \tag{3.22}$$

Again one has $\beta(l')=\beta$ except for l'. Now the hyperon-nucleon t matrix obeys the coupled set (2.21), which allows transitions for Λ to Σ, l to l', and s to s', but conserves total angular momentum j and total isospin t. Again, on the right-hand side recoupling coefficients are required. For the first term

$$\langle p_2'q_2\beta'r|p_3q_3\alpha r\rangle=\int_{-1}^1dxG_{\beta'\alpha}^r(q_2q_3x)\frac{\delta(p_2'-\nu')}{p_2'^{l'+2}}\frac{\delta(p_3-\nu)}{p_3^{l+2}} \tag{3.23}$$

with

$$\nu'=\sqrt{q_3^2+\rho_2^2q_2^2+2\rho_2q_2q_3x}\ , \tag{3.24}$$

$$\nu=\sqrt{q_2^2+\rho_3^2q_3^2+2\rho_3q_2q_3x}\ , \tag{3.25}$$

where ρ_2 and ρ_3 are given in (3.20). The quantity $G_{\beta'\alpha}^r$ is related to $G_{\alpha'\beta}^r$ in (3.17) as

$$G_{\beta'\alpha}^r(q_2q_3x)=G_{\alpha\beta'}^r(q_3q_2x)\ . \tag{3.26}$$

For the second term

$$\langle p_2'q_2\beta'r|P_{12}|p_2q_2''\beta r\rangle=\int_{-1}^1dx\ G_{\beta'\beta}^r(q_2q_2''x)\frac{\delta(p_2'-\mu')}{p_2'^{l'+2}}\frac{\delta(p_2-\mu)}{p_2^{l+2}} \tag{3.27}$$

with

$$\mu'=\sqrt{q_2''^2+\rho_2^2q_2^2+2\rho_2q_2q_2''x}\ , \tag{3.28}$$

$$\mu=\sqrt{q_2^2+\rho_2^2q_2''^2+2\rho_2q_2q_2''x}\ , \tag{3.29}$$

and $G_{\beta'\beta}^r$ is given in the Appendix. Thus, we arrive at

$$\langle p_2q_2\beta r|\psi_r^{(13)}\rangle=\frac{1}{E-2m_N-m_r-\frac{p_2^2}{2}\frac{m_N+m_r}{m_Nm_r}-\frac{q_2^2}{2}\frac{2m_N+m_r}{m_N(m_N+m_r)}}\times\left[\sum_{l's'r'}\int_{-1}^1dx\int_0^\infty dq_3q_3^2\frac{t_{lsrl's'r'}^{jt}\left(p_2\nu',E-2m_N-m_r-\frac{q_2^2}{2}\frac{2m_N+m_r}{m_N(m_N+m_r)}\right)}{\nu'^{l'}}\times\sum_\alpha G_{\beta(l')\alpha}^{r'}(q_2q_3x)\frac{\langle\nu q_3\alpha r'|\psi_{r'}^{(12)}\rangle}{\nu^{l_\alpha}}-\sum_{l's'r'}\int_{-1}^1dx\int_0^\infty dq_2''q_2''^2\frac{t_{lsrl's'r'}^{jt}\left[p_2\mu',E-2m_N-m_r-\frac{q_2^2}{2}\frac{2m_N+m_r}{m_N(m_N+m_r)}\right]}{\mu'^{l'}}\times\sum_{\beta''}G_{\beta(l')\beta''}^{r'}(q_2q_2''x)\frac{\langle\mu q_2''\beta''r'|\psi_{r'}^{(13)}\rangle}{\mu^{l''}}\right]\ . \tag{3.30}$$

TABLE I. Examples of three-body channel quantum numbers α and β which are defined in the basis states (3.12) and (3.13), respectively. For both the ΛNN and ΣNN systems ($r=\Lambda$ and $r=\Sigma$), α and β are specified. The quantum numbers (l,s,j,t) describe the two-body subsystem, while (λ,I) belong to the spectator. In the case of $\alpha(\beta)$, the two-body subsystem consists of $2N$ (hyperon $+N$) and the spectator is a hyperon (N). For other quantum numbers, see (3.12) and (3.13). The most important combinations of (l,s,j,λ,I,t) coupled to $J^{\pi}=\frac{1}{2}^{+}$ and $T=0$ of the hypertriton are given. The upper half of the table shows quantum numbers α for $r=\Lambda$ and $r=\Sigma$, and the lower half shows quantum numbers β for $r=\Lambda$ and $r=\Sigma$.

Channel	l	s	j	λ	I	t
1	0	1	1	0	$\frac{1}{2}$	0
2	2	1	1	0	$\frac{1}{2}$	0
3	0	1	1	2	$\frac{3}{2}$	0
4	2	1	1	2	$\frac{3}{2}$	0
5	0	0	0	0	$\frac{1}{2}$	1
6,11	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$
7,12	0	1	1	0	$\frac{1}{2}$	$\frac{1}{2}$
8,13	2	1	1	0	$\frac{1}{2}$	$\frac{1}{2}$
9,14	0	1	1	2	$\frac{3}{2}$	$\frac{1}{2}$
10,15	2	1	1	2	$\frac{3}{2}$	$\frac{1}{2}$

This coupled set (3.21) and (3.30) is solved numerically using the techniques as described in [16] and [17]. The use of orthonormalized states, which we obtain by applying the kernel of the coupled set to some starting initial state (in principal, arbitrary) is an especially efficient method [18] and has already been used very successfully, even for much larger systems [19]. Typical numbers for the discretized momenta p and q are $Np=40$ and $Nq=23$. Sufficient cutoff momenta are $p_{\max}=60\ \mathrm{fm}^{-1}$ and $q_{\max}=10\ \mathrm{fm}^{-1}$. The x integrations can be handled by about eight points. The set (3.21) and (3.30) is solved under the assumption that the two-body forces act only up to certain total two-body angular momenta $j_{\max}$ for the NN and YN subsystems. This leads to corresponding numbers of channel quantum numbers α and β, as exemplified in Table I.

IV. RESULTS

Before we embarked into the many channel calculations, we tested our code using the Stepien-Rudzka and Wycech separable potential [20], and repeated the calculations by Afnan and Gibson [8]. The potential parameters are given in Tables III and IV (4% deuteron d state) of Ref. [8]. Our result for an 11 channel calculation is 2.55 MeV, which agrees sufficiently well with 2.5447 MeV determined by Afnan [21]. Now we turn to the meson-theoretical Jülich hyperon-nucleon interaction model. We use an energy independent OBE parametrization (model $\tilde{A}$), as described in [13]. Between the baryons N, Λ, and Σ, the mesons π, K, ρ, ω, K^*, and the fictitious σ are thereby exchanged. Strict isospin conservation has been assumed. This model $\tilde{A}$ describes the very few experimental YN data. But, of course, those few data are, by far, not sufficient to fix the force model well. The situation is not comparable to the NN case, where there is only room left for fine tuning. For the NN potential we take the OBE Bonn B potential [22]. In order to explore the sensitivity to different choices of the NN force we also use the Reid [23] and Paris [24] potentials.

We can write the coupled set (3.21) and (3.30) of homogeneous integral equations formally as

$$\eta(E)\underline{\psi}=\underline{K}(E)\underline{\psi} \qquad (4.1)$$

with $\eta(E)=1$. Introducing the eigenvalue $\eta(E)$ allows one to study the coupled set at energies E in the neighborhood of the bound-state energy eigenvalue. The nonlinear energy eigenvalue problem is thus converted into a linear one for η. In case a bound state exists one has to search for its energy E below the Λd threshold at -2.225 MeV, such that $\eta(E)=1$. We have chosen the energy to be zero at the ΛNN threshold. The calculations have been performed using $m_N=938.919$ MeV, $m_\Lambda=1115.6$ MeV, $m_\Sigma=1193.1$ MeV, and $\hbar c=197.3271$ MeV fm. The eigenvalue η has been determined at three energies below the Λd threshold. The simplest dynamical assumption is to allow the YN and NN forces to act only in the states $^1S_0$ and $^3S_1$-$^3D_1$. As shown in Table I this leads to a 15-channel calculation. This table includes the most important orbital angular momenta for the two relative motions, and the spin-isospin states for the various two-body subsystems and the corresponding third particle. In the Faddeev language this characterizes the Faddeev components and therefore the wave function. Of course, this most simple calculation includes the tensor forces in the YN and NN systems and the Λ-Σ transitions. We display in Fig. 1 the η eigenvalue for three energies below the Λd threshold. For the convenience of possible later studies we also give η at $E=-2.40$ MeV in Table II. That calculation is based on the Bonn B potential, which provides the highest triton binding energy for all realistic NN potentials known (with the exception of the Bonn A version, which has an even smaller d state probability, but whose ε_1 values are definitely too small at high energies). Clearly the eigenvalues η are smaller than 1 and therefore the system is not bound. Now we include more force components, both for YN and NN: 30 channels for $j_{\max}=1$, 54 channels for $j_{\max}=2$, 78 channels for $j_{\max}=3$, and 102 channels for $j_{\max}=4$. The corresponding modifications for η are displayed in Fig. 1 and at $E=-2.40$ MeV in Table II. Though we find a slight increase in η, and obviously a perfect convergence, that hypertriton is not bound. We repeated those studies for the Paris potential and for the Reid potential, as shown in Table II. In no case is the hypertriton bound. The eigenvalues for the three potentials are similar in agreement with the expectation that the loosely bound hypertriton sees mainly the tails of the deuteron. Those tails are very similar for the three potentials as described by the asymptotic s-wave normalization A_s, the asymptotic d/s ratio, and, of course, the deuteron binding energy. Those three numbers are $A_s=0.8860$, 0.8869, 0.877 58; $d/s=0.0264$, 0.026 08, 0.026 223; and $E_d=2.224\,61$, 2.2249, 2.224 62 for Bonn B, Paris, and Reid, respectively. The d state

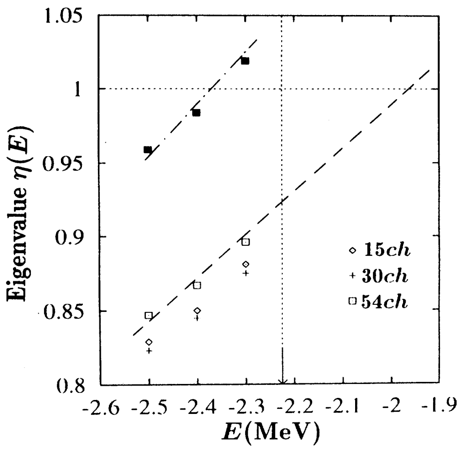

FIG. 1. Eigenvalues $\eta(E)$ of the Faddeev kernel defined in (4.1) which are calculated at three energies below the Λd threshold. The energy E is set to zero at the ΛNN threshold. The arrow indicates the Λd threshold. For the NN system, the Bonn B potential is used. The results for the various truncations of the YN and NN force components are shown: 15 channels ($^1S_0$ and $^3S_1$-$^3D_1$), 30 channels ($j_{max}=1$), and 54 channels ($j_{max}=2$). The results of the 78 channel ($j_{max}=3$) and 102 channel ($j_{max}=4$) calculations are at almost the same point as the 54 channel calculation. The lines are only to guide the eyes. The dashed line is extrapolated artificially above the ΛNN threshold. The dash-dotted line indicates the result of a 54 channel calculation in which the $^1S_0$ component of the Jülich YN potential is multiplied by a phenomenological enhancement factor 1.04.

probabilities of the deuteron P_d are, however, quite different, namely 4.99%, 5.77%, and 6.47% for Bonn B, Paris, and Reid, respectively. Also, it has been reported [1,25] that the deuteron gets significantly distorted in the hypertriton, an effect which will depend on the properties of the NN force. A detailed study of that effect, now with realistic NN forces, will be deferred to a forthcoming article. In any case, from [1, 25] and the different P_d's one might expect at least a slight dependence on the shorter range parts of the deuteron. We can see this by extrapolating the energy dependence of the eigenvalue η for the three potentials, as shown in the example of Fig. 1 and described below, and one finds a lack of binding energy which differs by about 30 keV for the three potentials. Thus, as in the triton, the smaller the P_d, the stronger the binding. Also, it has been reported [8] that there is a visible interplay of the tensor forces for the YN and NN systems, which should be checked by using different NN force models. Finally, as has been pointed out in several places [14,2], in case a three-body potential will be included on top of two-body potentials, the interplay of both potentials can depend quite sensitively on the type of forces chosen, in other words, on the correlations induced by the two-body forces at short distances. This is a well-known fact in three- and four-nucleon physics. For a recent study see [26]. The inclusion of three-body forces makes sense quantitatively only, if they are consistent with the accompanying two-body forces. The three-body forces used in our present study, induced by

TABLE II. Eigenvalues η of the Faddeev kernel defined in Eq. (4.1) which are calculated at 2.40 MeV below the ΛNN threshold for the Bonn B, Paris, and Reid soft core (RSC) NN potentials and the meson-exchange Jülich YN interaction (model $\tilde{A}$). The results for the various truncations of the YN and NN force components are given: 15 channels ($^1S_0$,$^3S_1$-$^3D_1$), 30 channels ($j_{max}=1$), 54 channels ($j_{max}=2$), 78 channels ($j_{max}=3$), and 102 channels ($j_{max}=4$).

Channels	Bonn B	Paris	RSC
15	0.850	0.837	0.830
30	0.845	0.834	0.827
54	0.867	0.856	0.847
78	0.868	0.858	
102	0.869	0.859	

Λ-Σ conversion, are consistent with the two-body forces in the case of Bonn B, since they are generated by the same sort of meson-exchange mechanism. In the case of the Paris or Reid potential, this cannot be claimed and the differences seen in Table II might be caused (at least partially) by the mismatch of three- and two-body forces.

As a very rough estimate for the lack of binding energy one may extrapolate the energy dependence of η to energies above the Λd threshold, as indicated by the dashed line in Fig. 1, and determine the energy at which η crosses the value 1. We read off a "lack of binding energy" of about 0.4 MeV. A better feeling for the amount of missing attraction in the YN interaction can be obtained from purely phenomenological enhancement factors for different force components required to lead to a bound hypertriton. We find that η depends quite sensitively on the $^1S_0$ strength and not so much on the $^3S_1$-$^3D_1$ strength. Known from many previous studies on the hypertriton (see for instance [6]), this is based on the simple fact that the averaged ΛN interaction is $\frac{3}{4}$ singlet and only $\frac{1}{4}$ triplet and therefore primarily sensitive to the YN singlet interaction. An increase by about 4% (multiplication of the $^1S_0$ YN potential by 1.04) leads to a bound hypertriton at about the correct binding energy. (The experimental Λ separation energy is 0.13 ± 0.05 MeV.) This is shown for the example of the Bonn B potential in Fig. 1 by the dash-dotted line. That curve corresponds to a 54 channel calculation. For the three NN potentials taken together with the 4% increased $^1S_0$ YN potential we find precisely the following binding energies: -2.36, -2.32, and -2.29 MeV for Bonn B, Paris, and Reid, respectively. Again, we see roughly the 30 keV difference, already mentioned above. It should be noted, however, that such an increase of the $^1S_0$ YN potential strength leads to a total ΛN cross section, which is too large in comparison to the existing data. Therefore, a possible modification of the YN interaction has to be more subtle, and it remains to be seen whether the physical potential parameters (cutoff values, coupling constants, σ-meson parameters) can be modified in such a manner that a bound hypertriton results. Such a study is in preparation.

Furthermore, we would like to add a remark on three-body force effects, which are included in our coupled channel calculation. The (intermediate) transitions from

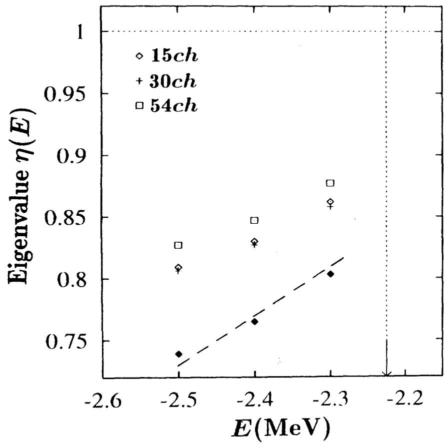

FIG. 2. Eigenvalues $\eta(E)$ using the Reid NN potential. Otherwise the notation of Fig. 1 holds. The dashed line connects η values, which demonstrate a three-body force effect for a 15 channel calculation (see text).

Λ to Σ induces a three-body force, together with a dispersive effect if one works in the ΛNN space only. The dispersive effect is a modification of the YN force due to the presence of the second nucleon. It simply arises due to the change in the intermediate hyperon-nucleon propagator, which in the hypertriton includes the kinetic energy of the second nucleon. Both effects, the proper three-body force effect and dispersive effect, should be called three-body force effects. They have been studied before quite intensively in the context of more simplified models [14,5,27,2,4,8,9]. We can see roughly the proper three-body force effect by turning off the $t_{N\Lambda,\Sigma N}$ matrix elements of the two-body t matrix in the three-body system, keeping them, however, in the two-body system. This excludes ΣNN states between consecutive t operations, and one stays therefore always in the space of ΛNN states. The result is that, for instance, at $E=-2.40$ MeV the eigenvalue for a 15 channel calculation $\eta=0.830$ drops to $\eta=0.765$, which is shown by the dashed line in Fig. 2. (The Reid NN potential is used.) Thus, that three-body force effect is attractive and quite significant. Not taking it into account would require that the ΛN forces would have to be even more enhanced and would have to simulate quite different and interesting physics.

From the study presented one has to expect that the experimental binding energy of the hypertriton as a small number will depend very sensitively on the interplay of NN and YN forces and that three-body force effects induced by Λ-Σ conversion will be very significant. Additional proper three-body forces [14] may also play a role, but should be consistent to the baryon-baryon-force model.

V. OUTLOOK

The meson-theoretical Jülich hyperon-nucleon interaction in the form of the OBE parametrization, together with various realistic NN interactions, turns out not to bind the hypertriton. We included the Λ-Σ transitions in a coupled channel procedure and carried the partial wave expansion to convergence. It appears that the hyperon-nucleon potential is not yet settled enough due to the lack of data. These results cannot be too much of a surprise. We feel that there is still room for modifying the meson-theoretical YN force, without spoiling the agreement to the very few YN data and without coming into conflict with well established theoretical ingredients. Such a study and its application to the hypertriton is in preparation, in collaboration with the Jülich group. Finally, studying by the same rigorous techniques the four-body and possibly the five-body systems will magnify the effects and will be a very rewarding test for the dynamical assumptions.

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APPENDIX

The full expressions of the quantities $G^r_{\alpha'\beta}(q_3q_2x)$ and $G^r_{\beta'\beta}(q_2q_2''x)$ which appear in the recoupling coefficients (3.17) and (3.27), respectively, are given. First, we mention details for the basis states (3.12) and (3.13). In the states (3.12)

$$|p_3q_3\alpha r\rangle \equiv |p_3q_3(ls)j(\lambda\tfrac{1}{2})I(jI)J(tt_r)T\rangle$$

the spin $\frac{1}{2}$ of nucleon 1 is coupled with the spin $\frac{1}{2}$ of nucleon 2 to the 2N total spin s in this order. It is just the same with the isospin $\frac{1}{2}$ of nucleon 1, the isospin $\frac{1}{2}$ of nucleon 2, and the 2N total isospin t. Also in the other type of basis states (3.13)

$$|p_2q_2\beta r\rangle \equiv |p_2q_2(ls)j(\lambda\tfrac{1}{2})I(jI)J[(t_r\tfrac{1}{2})t\tfrac{1}{2}]T\rangle$$

the spin $\frac{1}{2}$ of the hyperon numbered by 3 is coupled with the spin $\frac{1}{2}$ of nucleon 1 to the two-body spin s in this order.

Then the quantity $G^r_{\alpha'\beta}$ is

$$G^r_{\alpha'\beta}(q'qx)=\tfrac{1}{2}\sum_k P_k(x)\sum_{l_a'+l_b'=l'}\sum_{l_a+l_b=l} q'^{\,l_b'+l_b}q^{\,l_a'+l_a}g^{kl_a'l_b'l_al_b}_{\alpha'\beta}h_{\alpha'\beta}\ , \qquad \text{(A1)}$$

where $P_k(x)$ are the Legendre polynominals. The purely geometrical factor $g^{kl_a'l_b'l_al_b}_{\alpha'\beta}$ is given as

$$g_{\alpha'\beta}^{kl'_a l'_b l_a l_b} = \sqrt{\hat{l}'\hat{s}'\hat{j}'\hat{\lambda}'\hat{I}'\,\hat{l}\hat{s}\hat{j}\hat{\lambda}\hat{I}}\,(-1)^{l+s}\hat{k}\rho'^{l'_b}\rho^{l_a}$$

$$\times\sum_{LS}\hat{L}\hat{S}\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & s'\\ \frac{1}{2} & S & s\end{Bmatrix}\begin{Bmatrix}l' & s' & j'\\ \lambda' & \frac{1}{2} & I'\\ L & S & J\end{Bmatrix}\begin{Bmatrix}l & s & j\\ \lambda & \frac{1}{2} & I\\ L & S & J\end{Bmatrix}\left[\frac{(2l'+1)!}{(2l'_a)!(2l'_b)!}\right]^{1/2}\left[\frac{(2l+1)!}{(2l_a)!(2l_b)!}\right]^{1/2} \tag{A2}$$

$$\times\sum_{ff'}\begin{Bmatrix}l'_a & l'_b & l'\\ \lambda' & L & f'\end{Bmatrix}\begin{pmatrix}l'_b & \lambda' & f'\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}l_b & l_a & l\\ \lambda & L & f\end{Bmatrix}\begin{pmatrix}l_a & \lambda & f\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}f' & l'_a & L\\ f & l_b & k\end{Bmatrix}\begin{pmatrix}k & l'_a & f\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}k & l_b & f'\\ 0 & 0 & 0\end{pmatrix}$$

where $\rho'=\frac{1}{2}$, $\rho=m_r/(m_N+m_r)$, and $\hat{x}=2x+1$. The isospin part $h_{\alpha'\beta}$ is

$$h_{\alpha'\beta}=(-1)^{1-t_r}\sqrt{2\hat{t}_r}\begin{Bmatrix}t_r & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 0 & t_r\end{Bmatrix}=1\ . \tag{A3}$$

Next, the quantity $G^r_{\beta'\beta}$ is

$$G^r_{\beta'\beta}(q'qx)=(-1)^{l+s+1+t_r}\tfrac{1}{2}\sum_k P_k(x)\sum_{l'_a+l'_b=l'}\ \sum_{l_a+l_b=l} q'^{l'_b+l_b}q^{l'_a+l_a}g_{\beta'\beta}^{kl'_a l'_b l_a l_b}h_{\beta'\beta}\ , \tag{A4}$$

where the geometrical factor $g_{\beta'\beta}^{kl'_a l'_b l_a l_b}$ is given by the same expression as in (A2), with the expectation of $\rho'=m_r/(m_N+m_r)$. The isospin recoupling coefficients $h_{\beta'\beta}$ are

$$h_{\beta'\beta}=(-1)^{t_r}2\begin{Bmatrix}\frac{1}{2} & t_r & \frac{1}{2}\\ \frac{1}{2} & 0 & \frac{1}{2}\end{Bmatrix}=-1\ . \tag{A5}$$

Notice that the expression (A4) differs from (A1) in an additional phase factor $(-1)^{l+s+1+t_r}$, which comes from the exchange of the hyperon and the nucleon 2 in the state $P_{12}|p_2q_2\beta r\rangle$.

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