Asymmetry versus symmetric quadrupole deformation in even-even nuclei with $94 \leq A \leq 192$

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From experimental $E2$ matrix elements, the asymmetry shape parameter $\delta_{\rm eff}$ (corresponding to Bohr's model parameter γ) is derived for the ground states of some fifty even-even nuclei with $94 \leq A \leq 192$ ($42 \leq Z \leq 76$) by the sum-rule method. For the nuclei studied, an overall correlation between asymmetry and quadrupole deformation emerges.

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The basic properties of the nuclear shape are described [1] by the quadrupole deformation β and the asymmetry angle γ. The experimental value of the parameter β of an even-even nucleus in its ground state can be derived from the transition rate $B(E2, 0_1^+ \rightarrow 2_1^+)$ and is already well known for most nuclei spectroscopically studied [2]. Much more limited is the experimental information on the asymmetry. In the special case where the angular momentum projection K is a good quantum number and γ is small, it is determined by the ratio of the two intrinsic quadrupole moments (Eq. 4-246 in Ref. [1]). The interest in electromagnetic properties of asymmetric nuclei has considerably increased in recent years in view of the expected triaxial shapes at higher angular momenta (cf. e.g., Refs. [3–5] and references therein). The parameter γ is usually fitted in model calculations of nuclear properties compared to nuclear spectroscopic data. Values of γ obtained in this way are thus strongly model dependent.

An approach for a model-independent determination of the β-γ shape distribution is offered by the sum-rule method outlined in Refs. [6,7]. From invariant products of the $E2$ operator it is possible to define two intrinsic-frame quadrupole parameters Q_J and δ_J for each nuclear state J. The expectation values $\langle Q_J^2 \rangle$ and $\langle \cos 3\delta_J \rangle$ can be derived [6,7] from an expansion over the experimental reduced $E2$ matrix elements associated with the state J. The quadrupole parameter $\langle Q^2 \rangle$ is a measure of quadrupole deformation including static and dynamic contributions, the latter resulting from fluctuations around an equilibrium shape [$\beta_{\rm rms} = (4\pi/3ZR_0^2)\sqrt{\langle Q^2 \rangle}$]. The value of δ is closely related [6,7] (up to higher-order terms) with the collective-model parameter γ.

The expressions for $\langle Q_J^2 \rangle$ and $\langle \cos 3\delta \rangle$ are exactly valid if a complete set of experimental reduced $E2$ matrix elements is available. Therefore, in practical cases the question of reasonable convergence becomes important. For instance, as argued in Ref. [6], the sum-rule considerations are supposed to retain their validity without the explicit inclusion of the $E2$ strength from the giant $E2$ resonance. A large set of experimental $E2$ matrix elements between low-lying states could be at present extracted from advanced Coulomb excitation (CE) investigations [6,9–13]. There, the shape parameters Q (and sometimes δ) are determined for ground and several excited states facilitating a study of the shape evolution with increasing excitation energy and spin. Complex investigations of this type, however, have been published so far for a limited number of nuclei.

The most simple case in the application of the sum-rule method occurs with the ground state of an even-even nucleus ($J=0$). There, the expression for $\langle Q^2 \rangle$ reduces to a sum over the squared reduced $E2$ matrix elements ($\langle 0^+ \| E2 \| 2^+ \rangle = \langle 2^+ \| E2 \| 0^+ \rangle$) linking this state with all the $J^\pi = 2^+$ levels:

$$\langle Q_{\rm g.s.}^2 \rangle = \sum_r |\langle 0^+ \| E2 \| 2_r^+ \rangle|^2 , \qquad (1)$$

where $r = 1, 2, 3, \ldots$. It can be easily checked that in the mass region $90 < A < 190$ the above sum is practically exhausted by the first two terms ($r = 1, 2$) including the first (2_1^+) and second (2_2^+) excited states with $J^\pi = 2^+$. Where known (e.g., in ^{104}Ru [8], $^{106-110}$Pd [6,9]), higher-order terms with $r \geq 3$ are found to contribute less than 1% to this sum. Therefore,

$$\langle Q_{\rm g.s.}^2 \rangle \approx |\langle 0^+ \| E2 \| 2_1^+ \rangle|^2 + |\langle 0^+ \| E2 \| 2_2^+ \rangle|^2 . \qquad (2)$$

For the same 0^+ ground state, one obtains

$$\langle \cos 3\delta_{\rm g.s.} \rangle = -\left[\frac{7}{10}\right]^{1/2} \langle Q_{\rm g.s.}^2 \rangle^{-3/2} \times \sum_{r,t} \langle 0^+ \| E2 \| 2_r^+ \rangle \langle 2_r^+ \| E2 \| 2_t^+ \rangle \times \langle 2_t^+ \| E2 \| 0^+ \rangle . \qquad (3)$$

From careful empirical analysis of the contribution of each product of matrix elements in all available cases we arrive at the conclusion that the sum on the right hand side (r.h.s.) of Eq. (3) can be approximated with

$$\sum_{r,t} \langle 0^+ \| E2 \| 2_r^+ \rangle \langle 2_r^+ \| E2 \| 2_t^+ \rangle \langle 2_t^+ \| E2 \| 0^+ \rangle \approx \langle 0^+ || E2 || 2_1^+ \rangle^2 \langle 2_1^+ || E2 || 2_1^+ \rangle + 2 \langle 0^+ \| E2 \| 2_1^+ \rangle \langle 2_1^+ \| E2 \| 2_2^+ \rangle \langle 2_2^+ \| E2 \| 0^+ \rangle .$$

Thereby, the term $\langle 0^+ \| E2 \| 2_2^+ \rangle^2 \langle 2_2^+ \| E2 \| 2_2^+ \rangle$ as well as terms including higher-lying 2_r^+ states with $r \geq 3$ can be neglected. Generally speaking, this approximation will hold as long as $|\langle 0_1^+\|E2\|2_1^+\rangle| \gg |\langle 0_1^+\|E2\|2_r^+\rangle|$ and $|\langle 2_1^+\|E2\|2_1^+\rangle| \geq |\langle 2_r^+\|E2\|2_r^+\rangle|$ for $r \geq 2$. This is certainly the case in the nuclei considered in this work. With this approximation, Eq. (3) reduces to

$$\langle \cos 3\delta_{\rm g.s.}\rangle \approx -\left[\frac{7}{10}\right]^{1/2} \langle Q^2_{\rm g.s.}\rangle^{-3/2}[\langle 0^+\|E2\|2_1^+\rangle^2\langle 2_1^+\|E2\|2_1^+\rangle + 2\langle 0^+\|E2\|2_1^+\rangle\langle 2_1^+\|E2\|2_2^+\rangle\langle 2_2^+\|E2\|0^+\rangle] \ . \quad (4)$$

Further, one may deduce an effective value for the asymmetry parameter

$$\delta_{\rm eff} = \tfrac{1}{3}\arccos(\langle \cos 3\delta_{\rm g.s.}\rangle) \ .$$

In all cases where a large set of $E2$ matrix elements has been published (CE in ^{104}Ru [8], $^{106-110}$Pd [9], ^{114}Cd [10], ^{166}Er [11], ^{168}Er [12], ^{172}Yb [13], ^{182,184}W [19], i.e., in nuclei with different shape), values of $\delta_{\rm eff}$ derived from Eqs. (1) and (3) and Eqs. (2) and (4), respectively, have been compared. The differences $\Delta\delta_{\rm eff}$ hardly exceed 1°. Thus, one can be confident that the approximation made in Eq. (4) is justified. Moreover, it can be easily shown that the r.h.s. of Eq. (4) [and Eq. (2)] is predominantly determined by the ratio of static to transitional $E2$ moments

$$(\langle 2_1^+\|E2\|2_1^+\rangle / \langle 0_1^+\|E2\|2_1^+\rangle) \ .$$

Indeed, the asymmetry parameter δ increases when the absolute value of this ratio decreases. This is exactly in the spirit of model calculations [3,14] of $E2$ moments performed with variation of the parameter γ.

The validity of Eq. (4) facilitates the derivation and the systematics of the asymmetry shape parameters $\delta_{\rm eff}$ over a wide range of even-even nuclei. Data on quadrupole moments $Q(2_1^+)$ in many nuclei have been recently [15] summarized

$$[Q(2_1^+) = 0.758\langle 2_1^+\|E2\|2_1^+\rangle] \ .$$

Off-diagonal matrix elements $\langle 0_1^+\|E2\|2_{1,2}^+\rangle$ and $\langle 2_1^+\|E2\|2_2^+\rangle$ are commonly available (e.g., see Ref. [2], Nuclear Data Sheets, and references quoted in Ref. [15]).

CE reorientation measurements generally provide two (normally quite different) values for the quadrupole moment $Q_\pm(2_1^+)$ depending on the usually unknown sign of the term

$$P_3 = \pm|\langle 0_1^+\|E2\|2_1^+\rangle\langle 2_1^+\|E2\|2_2^+\rangle\langle 2_2^+\|E2\|0_1^+\rangle| \ .$$

We have checked in several nuclei Eq. (4) with both alternatives and found values of $\delta_{\rm eff}^{(+)}$ (at positive interference) only by up to few degrees larger than those at negative interference $\delta_{\rm eff}^{(-)}$. In the analysis of CE reorientation measurements (Eqs. 34, 35, and 47 in Ref. [16]), the expression in the brackets [r.h.s. of Eq. (4)] appears again, only the factor in front of the second term (P_3) is usually different from 2. This similarity may be the origin of both values $\delta_{\rm eff}^{(+)}$ and $\delta_{\rm eff}^{(-)}$ lying relatively close. On the other hand, in the nuclei considered here the positive sign of the term P_3 is widely considered as more probable (e.g., Ref. [17]). For the above reasons, we apply Eq. (4) at a positive sign of P_3.

In this way, we have estimated according to Eq. (4) the expectation values of $\langle \cos 3\delta\rangle$ for the ground states of nearly fifty even-even nuclei with $94 \leq A \leq 192$ $(42 \leq Z \leq 76)$ for which data on $E2$ matrix elements are available. More accurate input data on matrix elements would change somewhat individual values of $\langle \cos 3\delta\rangle$ but the revealed trends will remain. Wherever information on $Q(2_2^+)$ is available, we additionally include the term

$$\langle 0_1^+\|E2\|2_2^+\rangle^2\langle 2_2^+\|E2\|2_2^+\rangle$$

in the sum [r.h.s. of Eq. (4)] for completeness although its influence on $\delta_{\rm eff}$ is not significant. In Fig. 1, the results on $\langle \cos 3\delta\rangle$ are presented versus the quadrupole deformation $\beta_{\rm rms}$ derived from the values of $\langle Q^2\rangle$ [Eq. (2)]. Indicated also (r.h.s.) is the corresponding asymmetry angle $\delta_{\rm eff}$. An overall correlation between the asymmetry parameter δ and quadrupole deformation emerges from this presentation.

Equations (3) and (4) provide a mean value of the asymmetry parameter $\delta_{\rm eff}$ but they give no information on the "softness" of the nuclear potential with respect to the γ degree of freedom. The softness in δ can be determined from sixth-order products of experimental $E2$ matrix elements available at present only in very few cases [6]. There are several hints in the literature that the asymmetry is generally dynamical, i.e., that nuclei with firmly established rigid triaxiality would hardly exist (e.g., Refs. [5,18] and references therein).

In Fig. 1, a value of $\delta_{\rm eff} \approx 30°$ is observed only in nuclei

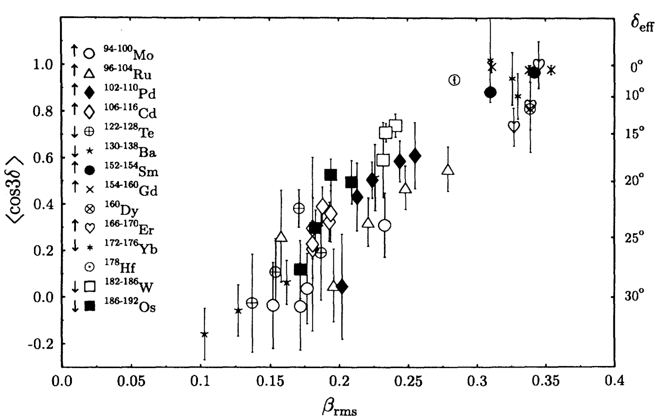

FIG. 1. Values of $\langle \cos 3\delta\rangle$ for ground states of even-even nuclei (indicated in the figure) derived from experimental $E2$ matrix elements versus the quadrupole deformation [cf. Eqs. (2) and (4) and text]. On the right-hand axis, the corresponding asymmetry parameter $\delta_{\rm eff}$ is given. Arrows beside the isotope symbol indicate that in the corresponding isotopic chain the mass number increases (↑) or decreases (↓) with increasing quadrupole deformation. Data are taken from Refs. [2,8–13,15,17,19], from some references quoted in Ref. [15] as well as from current issues of Nucl. Data Sheets.

with $\beta_{rms} < 0.2$. Most of these nuclei are spherical with a potential which is completely γ unstable in the region $0° < \gamma < 60°$. This is certainly the case, e.g., in ^{138}Ba ($N = 82$) where the effective quadrupole deformation ($\beta_{rms} = 0.1$) is supposed to arise mainly from dynamical contributions. Strongly deformed nuclei ($A > 150$) with $\beta_{rms} > 0.3$ are axial with a small effective asymmetry of $\delta_{eff} < 15°$. (With the data from Ref. [11] we derive $\delta_{eff} = 14°$ instead of 18° as given [11] for the case of ^{166}Er.) Weakly deformed nuclei with $0.15 < \beta_{rms} < 0.3$ are asymmetric with $15° < \delta_{eff} < 30°$. This observation is a clear verification of earlier suggestions that the best prospects for γ instability are expected in weakly deformed systems [5].

A different view at the shape parameter distribution is presented in Fig. 2 where the quantity $q_2 = \beta_{rms} \sin\delta_{eff}$ is plotted versus the mass number A. The values of q_2 for a very broad region of nuclei are placed in the interval $0.06 < q_2 < 0.10$. These are notably the studied nuclei with $A < 140$ except for ^{138}Ba and those with $A \geq 184$. The well-deformed nuclei as classical axial rotors reveal an (expected) tendency for a decrease of q_2 but even there in some cases (^{166,170}Er, etc.) the q_2 values reach that interval. A quantity which illustrates the deviation from axial symmetry of an ellipsoid (symmetry axis 3) is the eccentricity e_3 (Eq. 5.75 in Ref. [20]) including the semiaxes $R_{1,2}$. Using the relations [1] for the asymmetric increase δR_κ of each axis ($\kappa = 1,2,3$) one obtains

$$e_3 = \frac{R_1^2 - R_2^2}{R_2^2} = \left[\frac{15}{\pi}\right]^{1/2} \beta \sin\gamma \ . \qquad (5)$$

Thus, the values $q_2 = \beta_{rms} \sin\delta_{eff}$ plotted in Fig. 2 turn out to be convenient to compare the deviations from axial symmetry in a broad range of nuclei characterized by different shape parameters β_{rms} and δ_{eff} (i.e., β and γ). Those deviations would arise from γ fluctuations and/or rigid triaxiality. The narrowness of the interval in which most of the q_2 values are lying is remarkable. Fluctuation contributions are certainly present in (most of) these nuclei. One would expect static contributions from rigid triaxiality of specific nuclei to result in q_2 values which are appreciably higher than the systematic trend. In Fig. 2, such q_2 values are hard to identify, a possible exception being indicated by the data points around $A = 100$.

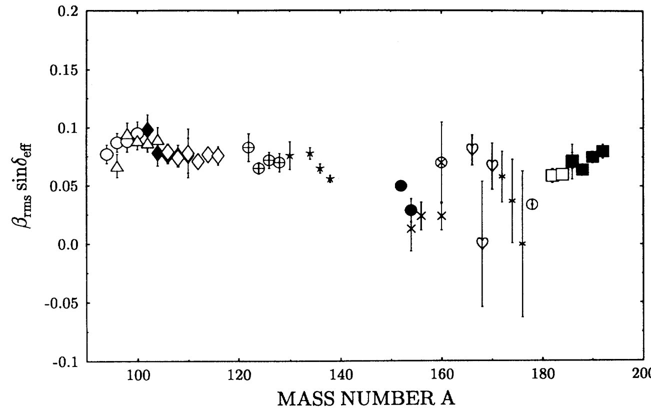

FIG. 2. The data from Fig. 1 presented as $q_2 = \beta_{rms} \sin\delta_{eff}$ versus mass number A. The same symbols are used as in Fig. 1. Several q_2 values characterizing nuclei with neighboring Z but the same A (e.g., Pd-Cd, ^{186}W-^{186}Os) are overlapping.

In conclusion, we have applied the sum-rule method [6,7] to ground states of even-even nuclei using an approximation which requires the knowledge of only four $E2$ matrix elements experimentally known for many nuclei. The asymmetry parameter δ_{eff} (which is closely related with the collective-model parameter γ) derived in this way for some fifty nuclei with $94 \leq A \leq 192$ ($42 \leq Z \leq 76$) reveals a pronounced tendency to gradually increase with decreasing quadrupole deformation. In comparison, the quantity $\sqrt{15/\pi}\beta_{rms} \sin\delta_{eff}$ characterizing the eccentricity of the nuclear ellipsoid shape about the symmetry axis fluctuates only weakly around 0.16 for transitional nuclei with a decreasing tendency for nuclei with strong quadrupole deformation.

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