Approximate ways to treat the nucleon-nucleon tensor force in the four-nucleon bound state

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Several approximation methods are tested in the four-nucleon bound-state problem as a means to understand how the two-nucleon tensor force propagates through the underlying (2)+(2) and (3)+1 subsystem amplitudes to yield a final four-nucleon binding energy. The aim is to show how to include as much of the nucleon-nucleon tensor force as possible in the dominant S-state component of the four-nucleon wave function for the purpose of optimizing the starting point in the iterative solution of the full problem.

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Until the initial work of Fonseca [1] and the more recent calculations of Glöckle and Kamada [2], the two-nucleon tensor force in the four-nucleon system had always been treated approximately [3–6], leading to four nucleon bound-state equations where all relevant orbital angular momenta are zero. As a consequence, the resulting four-nucleon wave function is reduced to the $\mathcal{S}=0$, $\mathcal{L}=0$ components with symmetric and mixed-symmetric spatial configurations. At the time, the question was: "How close to the exact four-nucleon binding energy can these approximate methods lead us?"

In order to be more specific, let us consider a simple problem where nucleons interact through rank-one interactions in channels $^1S_0$ and $^3S_1$-$^3D_1$. It is well known that in the triplet channel V_{00} ($V_{ll'}$) is a poor starting point for the calculation of t_{00} and an even poorer approximation in the calculation of the triton binding energy. In earlier calculations by Tjon [4], he proposed use of the full two-body t matrix truncated to the $l=l'=0$ element which leads to a triton wave function that carries no D-state component. Calculations by Fonseca [1] using Tjon's approach give the binding energies shown in Table I in the column labeled t_{00}. They are compared with the exact results obtained from the solution of the Faddeev equations for the wave-function components $\psi^j_{\nu SL}(Q)$, where

$$|\psi^j_{\nu SL}\rangle=\sum_{\nu'S'L'}\langle \nu SL|B^j|\nu'S'L'\rangle\tau_{\nu'}|\psi^j_{\nu'S'L'}\rangle\ , \tag{1}$$

where B^j is the well-known one-nucleon-exchange driving mechanism whose matrix elements may be found in Ref. [7]. Here we follow the traditional notation where ν denotes a singlet or triplet two-nucleon pair, S is the channel spin, and L is the particle-pair orbital angular momentum. The values of S and L are displayed in Table II for $j=\frac{1}{2}^+$.

Although the t_{00} approximation looks reasonable for the binding energy, it is by no means the only way one may reduce the problem to $L=0$ components alone. If one truncates Eq. (1) to $(\nu,S,L)=(1,\frac{1}{2},0)$ and $(2,\frac{1}{2},0)$ components and solves the resulting 2×2 integral equation, the results obtained are shown under I_{00} in Table I. (This approach was called the "truncated t-matrix" approximation and denoted "t_{00}" by Gibson and Lehman in the last of Refs. [3], while Fonseca in Ref. [1] called it the "truncated $\frac{1}{2}^+$" approximation.) Despite the fact that both the t_{00} and I_{00} wave functions have no D-state components, both are equally good starting points for an iterative solution of the full (tensor) Faddeev equations for the wave function and binding energy, and both are significantly better than a typical arbitrary starting function.

If we now proceed to the four-nucleon problem, it is evident that there is additional freedom present for the choice of the approximation methods. For rank-one two-nucleon interactions [3], the four-nucleon equations read

$$|R^{13}_{\nu c}\rangle=\sum_{\nu'c'}\tau_\nu\langle \nu c|X|\nu'c'\rangle\{|R^{13}_{\nu'c'}\rangle+|R^{22}_{\nu'c'}\rangle\} \tag{2a}$$

and

TABLE I. Three-nucleon binding energies in MeV for different potential models and approximation methods used to include tensor-force effects.

$P_D(\%)$	t_{00}[a]	Exact[b]	I_{00}[b]
4	8.657	8.573	8.191
5.5	8.078	8.040	7.560
7	7.560	7.588	7.051

[a]Reference [1].
[b]Last of Refs. [3].

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TABLE II. Values of S and L for $j=\frac{1}{2}^+$. $\nu=1$ denotes a spin-singlet nucleon-nucleon pair while $\nu=2$ denotes a spin-triplet pair.

ν	S	L
1	$\frac{1}{2}$	0
2	$\frac{1}{2}$	0
2	$\frac{3}{2}$	2

$$|R^{22}_{\nu c}\rangle=\sum_{\nu' c'} 2\tau_\nu\langle \nu c|Y|\nu' c'\rangle|R^{13}_{\nu' c'}\rangle \ . \tag{2b}$$

The superscripts 13 and 22 denote the (3)+1 and (2)+(2) components, respectively, and c gives the corresponding channels shown in Table III, where Σ is the four-nucleon channel spin and Δ the relative (3)+1 or (2)+(2) orbital angular momentum. In Eq. (2a), X is the three-nucleon Alt-Grassberger-Sandhas (AGS) [8] operator for $n+\nu\to n+\nu'$ scattering and Y the AGS operator for the scattering of two noninteracting pairs ν and ν'. It should be noted that X exists on-shell, but Y only takes place off-shell embedded in the four-nucleon system. The equations for X are given by

$$\langle \nu'S'L'j'|X|\nu SLj\rangle=\delta_{j'j}\langle \nu'S'L'|X^j|\nu SL\rangle \tag{3}$$

and

$$\langle \nu'S'L'|X^j|\nu SL\rangle=\langle \nu'S'L'|B^j|\nu SL\rangle + \sum_{\nu''S''L''}\langle \nu'S'L'|B^j|\nu''S''L''\rangle\tau_{\nu''} \times\langle \nu''S''L''|X^j|\nu SL\rangle \ , \tag{4}$$

where B^j is the same as in Eq. (1) and (νSL) run over the channels listed in Table II for $j=\frac{1}{2}^+$. Likewise, the equation for Y which involves the pairs ν' and ν has the form

$$\langle \nu'|Y|\nu\rangle=\bar\delta_{\nu'\nu}\langle \nu'|\bar B|\nu\rangle + \sum_{\nu''}\langle \nu'|\bar B|\nu''\rangle\bar\delta_{\nu''\nu}\tau_{\nu''}\langle \nu''|Y|\nu\rangle \ , \tag{5}$$

where $\bar\delta_{\nu'\nu}=1-\delta_{\nu'\nu}$, ν'' runs over ν and ν', and

$$\langle \nu'|\bar B|\nu\rangle=\sum_{\substack{sl\\ s'l'}}|f^\nu_{sl}\rangle G_0\langle f^{\nu'}_{s'l'}| \ . \tag{6}$$

In Eq. (6), G_0 is the four-particle propagator, $|f^\nu\rangle$ is the two-nucleon form factor that comes from the corresponding rank one potential in channel ν, and (sl) gives the pair spin and relative-orbital angular momenta that are consistent with the total angular momentum and parity of channel ν. If $\nu'=\nu=1$ ($^1S_0$), then $s=0$ and $l=0$, but for $\nu'=\nu=2$ ($^3S_1$-$^3D_1$), we have $s=1$ with $l=0$ and 2. Therefore, the off-shell scattering of noninteracting triplet pairs involves a summation with $l=0$ and $l=2$, while for two singlet pairs only the $l=0$ term contributes.

If we now attempt to truncate the four-nucleon equations to $\Delta=0$ channels alone, different possible options arise, some of which may lead to overbinding, while others underbind. We investigate six alternatives based on different truncations at the two-body, (3)+1, and (2)+(2) levels.

At the two-body level, we can start with t_{00} alone for singlet and triplet pairs. This leads to $l=0$ terms alone in the equation for Y [Eq. (5)] and 2×2 equations for X^j, all as if the tensor force were absent, except in the nucleon-nucleon propagator τ_ν. This we denote as the t_{00}/t_{00} approximation.

Beginning at the (3)+1 level, we may derive two options denoted as the I_{00} and X_{00} approximations. The I_{00} approximation amounts to truncating Eq. (4) to a 2×2 equation while for the X_{00} approximation we solve Eq. (4) exactly, but only use the $L=0$ components in the solution of the four-nucleon equations.

Finally, at the (2)+(2) level, one has a similar option named Y_{00} which involves the exact solution of Eq. (5) for two triplet pairs, but retaining only the $l=l'=0$ component of the Y amplitude in the four-nucleon problem.

In Table IV, we show the results of our calculations for different combinations of two-body, (3)+1, and (2)+(2) approximations together with the exact result Ref. [1]. Although the accuracy of each approximation depends on the strength of the two-nucleon tensor force, one can easily conclude that t_{00}/t_{00} and X_{00}/Y_{00} are the best starting points for a full iterative solution of the four-

TABLE III. Four-nucleon channels for total angular momentum $J=0^+$. The three-body subsystem has been limited to $j=\frac{1}{2}^+$.

		(3)+1 channels				(2)+(2) channels		
ν	S	L	j	Σ	Δ	ν'	Σ	Δ
1	$\frac{1}{2}$	0	$\frac{1}{2}^+$	0	0	1	0	0
2	$\frac{1}{2}$	0	$\frac{1}{2}^+$	0	0	2	0	0
2	$\frac{3}{2}$	2	$\frac{1}{2}^+$	0	0	2	2	2

TABLE IV. Four-nucleon binding energies. Exact and various approximations.

Treatment of tensor force	Percentage D state in the deuteron 4%	5.5%	7%
I_{00}/t_{00}	30.39	26.56	23.61
I_{00}/Y_{00}[a]	31.68	28.15	25.41
X_{00}/t_{00}	31.66	28.09	25.27
t_{00}/t_{00}	32.85	29.24	26.15
Exact	32.34	29.10	26.56
X_{00}/Y_{00}[a]	32.74	29.56	27.03
t_{00}/Y_{00}[a]	34.01	30.79	28.02

[a]Also obtained by convolution method to same number of significant figures.

nucleon bound-state problem. It is also clear that going from t_{00} to X_{00} at the (3)+1 level, while keeping t_{00} at the (2)+(2) level, decreases the binding energy by $\simeq 2.5$ MeV. On the other hand, if t_{00} is changed to Y_{00} at the (2)+(2) level, while retaining t_{00} at the (3)+1 level, the binding energy increase ranges from 1.2 to 1.9 MeV depending on the strength of the nucleon-nucleon tensor force. Given its simplicity, t_{00}/t_{00} therefore becomes the most recommended approximation.

Finally, it is worth mentioning that if we solve the four-body equations using the convolution method [9] together with t_{00}, I_{00}, or X_{00}, for the (3)+1 subamplitudes, the t_{00}/Y_{00}, I_{00}/Y_{00}, and X_{00}/Y_{00} results are naturally recovered as indicated in Table IV, but never any of the approximations based on t_{00} at the (2)+(2) level. Given that the convolution method is based, as the name suggests, on the convolution of two noninteracting-pair propagators $\tau_{\nu'}$ and τ_{ν}, it automatically includes, in intermediate states, all possible combinations of s and d waves in both pairs, which, in Eq. (5), can only be reached through an exact solution. In other words, the convolution method precludes a complete set of states for both ν' and ν, while in Eq. (5), one may truncate ν' and ν independently.

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