Determination of the $^6Li \rightarrow \alpha + d$ vertex constant (asymptotic coefficient) from the $^4He + d$ phase-shift analysis

L. D. Blokhintsev, V. I. Kukulin,* A. A. Sakharuk, and D. A. Savin
Institute of Nuclear Physics, Moscow State University, Moscow 119899, Russia

E. V. Kuznetsova
Institute of Nuclear Researches, the Academy of Sciences, Moscow, Russia
(Received 24 July 1992; revised manuscript received 15 April 1993)

The $^6Li(1^+0) \rightarrow {}^4He + d$ virtual decay vertex constant G_0 and the respective asymptotic coefficient C_0 of the ^6Li wave function in the $^4He + d$ channel are found using the analytic continuation of the solution of a novel energy-dependent phase-shift analysis of elastic d-^4He scattering to the pole corresponding to the ^6Li ground state. The reliability and accuracy of the method used have been corroborated independently by three other ways: by direct solving the inverse problem for d-^4He scattering and by two different methods for finding a solution for the three-body $(\alpha + n + p)$ problem. The values $G_0^2 = 0.42 \pm 0.02$ fm and $C_0 = 2.93 \pm 0.15$ have been found, which seem to be the most accurate and reliable among the values obtained so far.

PACS number(s): 23.90.+w

I. INTRODUCTION

The nuclear vertex constants (NVC's) $G(a \rightarrow b + c)$ coincide up to kinematical factors with the amplitudes of virtual or real decay (or fusion) of a nucleus into two fragments b and c and belong to the fundamental nuclear constants, like binding energies and magnetic moments. The NVC's are of great importance in nuclear reaction theory and define the asymptotical behavior of nuclear wave functions in appropriate channels, which is determined in turn by the dynamics of strong interactions (see the detailed reviews on NVC's, for example [1,2,3]).

Copious NVC data have been accumulated, first of all for few-nucleon systems and light nuclei. The NVC's for the deuteron $(d \rightarrow n + p)$ [4] and for three-nucleon nuclei are now well known with a high accuracy. The NVC's for the $^4He \rightarrow {}^3He(^3H) + n(p)$, and $^4He \rightarrow d + d$ vertices are known now to within a lower accuracy. Among the NVC's for light nuclei, the S- and D-wave NVC's G_L $(L = 0,2)$ for the $^6Li \rightarrow {}^4He + d$ decay are particularly notable because they are of great importance in the theory of nuclear reactions induced by ^6Li ions, especially in view of the future advent of high quality beams of polarized ^6Li ions and an extensive use of ^6Li as target nuclei.

The NVC's $G_L (L = 0,2)$ for $^6Li \rightarrow {}^4He + d$ decay are related to the dimensionless asymptotic coefficients $C_L^{\alpha d}$ of the ^6Li wave function in the $\alpha + d$ channel [1,2] as

$$G_L = -i\sqrt{\pi N}\, e^{i\pi\eta/2} [\hbar/(\mu_{\alpha d} c)] C_L^{\alpha d} \sqrt{2\kappa}\ , \qquad \kappa = \sqrt{2\mu_{\alpha d}\varepsilon_{\alpha d}}, L = 0,2 \tag{1}$$

where $\mu_{\alpha d}$ is the reduced mass in the $^4He + d$ channel; $\varepsilon_{\alpha d}$ is the ^6Li binding energy relative to breakup into an α particle and a deuteron; $\eta = 2e^2\mu_{\alpha d}/(\hbar\kappa)$ is the Coulomb parameter for the $^6Li \rightarrow \alpha + d$ vertex; N is a combinatorial factor due to the nucleon identity [2] [in the case of the $^6Li \rightarrow {}^4He + d$ vertex $N = 1$ if the three-body $(n + p + \alpha)$ model wave function is used]. It should be noted that the dimensional asymptotic coefficients in [1] differ by an additional factor of $(2\kappa)^{1/2}$ from those used here and in [2].

Unfortunately, the values of even the dominant S-wave NVC $G_0^{\alpha d}$ found by different techniques prove to be very different (see Table I), so that the constant has been known within a great uncertainty (it should be noted that the D-wave NVC $G_2^{\alpha d}$ for the $^6Li \rightarrow \alpha + d$ decay is several orders smaller than $G_0^{\alpha d}$). Therefore, we have made an attempt to determine the NVC G_0 more thoroughly and reliably using new methods. The general philosophy of our approach is partly discussed in the monograph [6] and in papers [7]; some tentative results are mentioned in [8,9].

II. SHORT DESCRIPTION OF METHODS USED

In the present work, three methods are used to find the S-wave NVC G_0. The first method is supported by a novel approach to the energy-dependent phase-shift analysis [6,8,10] on the basis of the statistical Padé-approximation technique [6] or, in other words, the type-III Padé approximation ($PA - III$). The approach yields immediately the S-matrix representation in such analytic rational form which can be continued directly to the unphysical region in energy to find the poles corresponding to bound states [the $^6Li(1^+0)$ ground state in our case] and to determine the respective residues through which the NVC's are expressed.

The second method is based on solving the inverse scattering problem [11] and makes use of the S matrix (i.e., the partial phase shifts) to carry out the inversion

*Electronic address: kukulin@compnet.msu.su

TABLE I. The NVC $|G_0|^2$ values found for the ^{6}Li(1^+0)$\rightarrow^4$He$+d$ vertex by different methods (only the values obtained with the account of the Coulomb d-^{4}He interaction are presented). More detailed information and the relevant references can be found in the reviews [1–3]. The value in the bottom line is from [5].

Method for finding NVC	$\lvert G_0\rvert^2$ (fm)
Phenomenological cluster wave function	0.72
Fit to the ^{6}Li charge form factor	0.3–0.4
Fit to the charge distribution in ^{6}Li	0.27
Forward dispersion relation for ^{4}He$+d$ scattering	0.22±0.01
Method of molecular wave function for α-^{6}Li scattering	0.58±0.06
Peripheral model for d-^{6}Li scattering	0.83±0.11
DWBA, ^{6}Li$(p,^3$He$)^4$He reaction	0.42
Impulse approximation with cutoff for ^{6}Li$(p,pd)^4$He reaction	0.18–0.68
Impulse approximation with cutoff for ^{6}Li$(d,2d)^4$He reaction	0.54–1.41
Peripheral model, ^{6}Li$(\alpha,2\alpha)^2$H reaction	0.75–2.5
Peripheral model, ^{6}Li$(d,^6$Li$)^2$H reaction	0.41±0.06[a]

[a]This value was found in Ref. [5] by averaging over many values with a large spread.

and to construct an effective two-body d-^{4}He potential which would describe the d-^{4}He scattering data. After that, the bound state in the potential [the ^{6}Li(1^+0) ground state] is determined. Having found the bound state, the asymptotic coefficient of the respective bound-state wave function determines, according to (1), the sought vertex constant.

The third method is based on direct solving the Faddeev equation in terms of the $\alpha+p+n$ model directly for the vertex function. We have attained a very good agreement among the results of the three independent methods, which indicates a high reliability of the G_0 value obtained.

The input data for the first method include the results of the energy-dependent phase shift analysis for the d-^{4}He scattering [9,10] at incident deuteron energies $E_d=0.8-5.5$ MeV, supplemented with the data of the recent energy-independent phase shifts analysis [12] at $E_d=6-10$ MeV. The Padé approximation and the subsequent analytic continuation were made using the Coulomb-modified scattering function $g_l^c(E)$, which is of the following form in the case of an arbitrary orbital momentum l [6,7]:

$$g_l^c(E)=p^{2l+1}B_l^2[B_0^2(\cot\delta_l^{c\,-N}-i)+2\eta H(\eta)]/B_0^2 ,$$

$$p=\sqrt{2\mu_{\alpha d}E} \quad (2)$$

where $\delta_l^{c\,-N}$ is a Coulomb-modified nuclear phase shift, η is the Coulomb parameter, and $B_l(\eta)$ is the Coulomb penetration factor for the lth partial wave,

$$H(\eta)=\Psi(i\eta)+\frac{1}{2i\eta}\ln[-i\eta\,\mathrm{sgn}(-Z_1Z_2)]$$

where $\Psi(X)$ is the digamma function [13]. At the real energies ($E>0$), the expression for $H(\eta)$ goes to [2,6]

$$H(\eta)=-C-\frac{1}{-i\eta}+i\eta\sum_{n=0}^{\infty}[n(n+i\eta)]^{-1}$$

where $C=0.5247\ldots$ is the Euler constant; E is kinetic energy in the c.m. system.

As shown in numerous works (see, for example, [7]), the scattering function defined by Eq. (2) is analytic in the upper-half k plane and can, therefore, be represented in some circle around the origin in the form of the Padé approximant

$$g_0^{(c)}\simeq P_N(E)/Q_M(E) , \quad (3)$$

where $P_N(E)$ and $Q_M(E)$ are polynomials of degrees N and M, respectively. The coefficients of PA's (3) in our case have been found in the course of the energy-dependent phase-shift analysis [7].

In the case of the S wave, the Coulomb-nuclear scattering amplitude $f_0(E)$ corresponding to (3) is found from the relation

$$f_0(E)=\frac{pB_0^2(\eta)}{P_N(E)/Q_M(E)-2ipB_0^2(\eta)} . \quad (4)$$

Here, the amplitude $f_l(E)$ has been normalized so that $f_l(E)=(e^{2i\delta_l^{c\,-N}}-1)/(2ip)$ for pure elastic scattering.

The *stable* pole of the amplitude on the negative semiaxis E (on the first energy sheet), i.e., the zero of the denominator which varies little with changing the PA order, corresponds to the bound state of the system, while the residue at the pole gives the NVC G_0 and the corresponding asymptotic constant C_0. It must be noted that, in the general case of $b+c\rightarrow b+c$ elastic scattering with nonconserved orbital momentum of channel L and with spin of channel S, the amplitude (T-matrix) residue at the pole corresponding to the compound nucleus $a\rightarrow b+c$ is related to the NVC $G_{LS}^{(a\rightarrow b+c)}$ and to the asymptotic constant $C_{LS}^{(a\rightarrow b+c)}$

$$\mathrm{Res}\langle L'S'|T^J(E)|LS\rangle|_{E=-\varepsilon_{bc}}=G_{L'S'}^{*(a\rightarrow b+c)}G_{LS}^{(a\rightarrow b+c)}=i^{L+L'}\pi N_{bc}e^{i\pi\eta/2}[\hbar/(\mu_{bc}c)]^2 2\kappa_{bc}C_{L'S'}^{(a\rightarrow b+c)}C_{LS}^{(a\rightarrow b+c)} \quad (5)$$

where μ_{bc}, κ_{bc}, η, and N_{bc} have the same meaning as in Eq. (1). Here, the amplitude normalization is such that at $L=L', S=S'$

$$\langle LS|T^J(E)|LS\rangle = -\frac{2\pi}{\mu p} e^{2i\delta^J_{LS}} \sin\delta^J_{LS}$$

when absorption effects do not occur. For the S-wave d-^{4}He scattering $L=L'=0$, $S=S'=1$, $g_0^{(c)}(E)$ was Padé parametrized using the diagonal ($N=M$) Padé approximants.

Table II presents the values of the constants $|G_0|^2$ and of the corresponding binding energies $\varepsilon_{\alpha d}$ obtained by analytic continuation of the elastic d-^{4}He $^3S_1$ partial amplitude for different orders of the Padé approximant. The very good value found by us for the binding energy $\varepsilon_{\alpha d}$ in the α-d channel is also a criterion of analytic continuation stability (apart from the Padé-approximant convergence). It should be stressed here that it is in a contrast with the conventional methods for finding NVC on the basis of analytic continuation of transfer-reaction differential cross section to the pole point with respect to $\cos\Theta$ (Θ is the scattering angle) where the position of the pole is supposed to be known beforehand (and to be set *manually).*

We found that our $|G_0|$ values obtained in the extrapolation procedure proved to be linearly dependent on the corresponding bound-state energy $\varepsilon_{\alpha d}$ to within a high accuracy. The similar linear dependence of $|G_0|$ (or C_0) upon the energy was found also by the authors of [14]. Thus, by making linear extrapolation to the experimental $\varepsilon_{\alpha d}$ value 1.47 MeV, we obtain $|G_0|^2=0.41$ fm and the respective asymptotic constant $C_0=2.93$. Then the $G_0^{\alpha d}$ value obtained can be compared with the results of microscopic calculations of the ^{6}Li nucleus in terms of a three-body model [15–18] and using the refined resonating group method (RGM) [19]. In particular, the $G_0^{\alpha d}$ values which are very close to the present results were obtained in [16,19], making allowance for the correction for binding energy, thereby yielding a good independent corroboration of the NVC G_0 values found here.

In the second approach, we found the G_0 value using the method for solving the inverse scattering problem [11] to infer the effective two-particle d-^{4}He interaction potential $V_{d\alpha}(r)$, including the Coulomb interaction, from the "experimental" d-^{4}He phase shifts. In the approach we used the same phase-shift analysis results [9,10] as in the former approach. Generally, the short-range part of the potential has been found to be complex and energy-independent, but depends weakly on L; namely, it contains central and spin-orbit components, either of which is presented as a series expansion in a certain orthonormalized basis (in our case we used the convenient harmonic oscillator basis [11]). The linear coefficients of the expansion and the scale parameters of the basis are inferred from the requirement that the experimental d-^{4}He phase shifts should be described best in the χ^2 sense. Then, these coefficients (in fact, the interaction potential) are sought from the very convenient linearized iteration procedure [11]. The solution for the inverse problem has to be stabilized using both $^3S_1$ and ^3D_j partial phase shifts. In this case four terms in the central and spin-orbital components each are sufficient to take in the series expansion of the potential on harmonic oscillator basis (see Ref. [11] for the values of the potential parameters).

After that, by solving the Scrhödinger equation with the d-^{4}He interaction potential found, we calculate the binding energy $\varepsilon_{\alpha d}$ and the ^{6}Li ground-state wave function. The asymptotic part of the latter gives $G_0^{\alpha d}$ and C_0 immediately. The ^{6}Li binding energy $\varepsilon_{\alpha d}^{\text{theor}}$ calculated with the ^{4}He-d interaction potential was rather close to its experimental value, $\varepsilon_{\alpha d}^{\text{theor}}=1.55$ MeV. And, again, using the well-established linear extrapolation of $|G_0|$ to the experimental value of $\varepsilon_{\alpha d}^{\text{expt}}\cong 1.47$ MeV we find the values

$$|G_0|^2=0.42 \text{ fm and } C_0^{\alpha d}=2.98$$

which have proved to be very close to the respective values obtained in terms of the first approach on the basis of Padé approximation for the phase-shift solution and presented above. If the given method disregards the Coulomb interaction $V^q_{d\alpha}$, we obtain $G_0^2=0.33$ fm (see the discussion below for the comparison).

We would like to stress that to obtain $C_0^{\alpha d}$ and G_0 within that two-body potential approach we use the ^{6}Li ground-state wave function normalized to unity. On the other hand, this wave function is an effective approximation to the "exact" many-body overlap integral $\langle \psi_d \psi_\alpha | \psi 6_{\text{Li}} \rangle$, ψ_i being the internal wave function of a composite system i. This overlap integral is normalized not to unity but to the so called amount of αd clusterization (or spectroscopic factor) $S_{\alpha d}$ which, by definition, is less than unity. Hence the question arises whether the values of $C_0^{\alpha d}$ and G_0 obtained by our second (potential) approach should be multiplied by $S_{\alpha d}^{1/2}$. The answer to this question is not trivial. We believe the normalization of the effective two-body wave function should depend on the way it is used. For example, if it is used to calculate vertex parts of a pole graph corresponding to some transfer reaction, then one should normalize the wave function to the spectroscopic factor to obtain correct absolute values of the amplitude of this graph. On the other hand, if the wave function satisfies the two-body Schrödinger equation with a certain potential (as it does in the discussed approach) then the general relation between the asymptotic coefficient and the residue of a scattering amplitude calculated with the same potential [Eq. (1)] assumes that the wave function is normalized to unity. This point of view is supported by Ref. [20] in

TABLE II. Binding energy $\varepsilon_{\alpha d}$, values of squared NVC $|G_0|^2$ and of asymptotic normalization constants $C_0^{\alpha d}$ for different orders of diagonal Padé approximants in Eq. (3).

Padé approximant order $[N,M]$	$\varepsilon_{\alpha d}$ (MeV)	$\lvert G_0\rvert^2$(fm)	$C_0^{\alpha d}$
[1,1]	1.3491	0.3672	2.83
[2,2]	1.4186	0.4055	2.93
[3,3]	1.3884	0.3877	2.88
Values adopted at present	1.47617	0.2–2.5[a]	2–7[a]

[a]The extremes have been inferred from the review [1].

which the vertex constant $t\rightarrow d+n$ was correctly calculated using the *two-body* triton wave function normalized to *unity*.

By the way the spectroscopic factor $S_{\alpha d}$ in the ^{6}Li case is very close to unity [19] and multiplying G_0 and $C_0^{\alpha d}$ by $S_{\alpha d}^{1/2}$ practically does not change their values.

Note also that, in contrast to vertex constants and asymptotic coefficients, spectroscopic factors cannot be extracted directly from experiment and can only be determined by calculations involving model assumptions.

To independently test the above-proposed methods for deriving the S-matrix poles and residues from experimental data, we calculated the NVC $G_L(^6\text{Li}\rightarrow\alpha+d)$ using the Faddeev equations in terms of the three-body $(\alpha+n+p)$ model for ^{6}Li nucleus (see Table III). Use was made of the same pair interactions as in [15], namely, the one-term separable potentials in the $^3S_1$ channel of NN system and in the $S_{1/2}$, $P_{3/2}$, and $P_{1/2}$ channels of $N\alpha$ system. The Coulomb p-α interaction was neglected. The $N\alpha$ potential in $S_{1/2}$ state was taken to be of two different forms, namely, (i) in the form of pure *repulsive* potential (version A in [15]); (ii) in the form of *attractive* potential leading to the deep-lying $N\alpha$ 0s state (version B in [15]) which corresponds to the (Pauli-forbidden) totally symmetric five-nucleon configuration $|s^5[5],L=0\rangle$ and whose contribution has been excluded from the three-body solution using the orthogonal projection technique [21].

To find the NVC's G_0 and G_2, the kernels of the modified (to account of the Pauli-forbidden states) Faddeev integral equations were continued to the unphysical region of imaginary momenta, and after this the three-body equations were solved directly for the S- and D-wave vertex functions. As a result, we found $|G_0|^2=0.325$ fm in the case of the $N\alpha$ interaction in the form (i). Then the theoretical G_0 value was found by the new solution of the inverse scattering problem [11] using as input the theoretical three-body d-^{4}He S-wave phase shifts and inversion method described above. In this case, the Coulomb interaction can be conveniently allowed for by adding the d-^{4}He Coulomb potential $V_{d\alpha}^{\text{Coul}}$ to the nuclear d-^{4}He potential $V_{d\alpha}^{N}$ inverted from theoretical three-body d-^{4}He phase shifts. The method of effective potentials, disregarding (allowing for) the Coulomb interaction, leads to $|G_0|^2=0.317$ fm (0.413 fm) in version (i) and to $|G_0|^2=0.321$ fm (0.419 fm) in version (ii).

Tables IV(a) and IV(b) present the NVC G_0^2 values obtained by the above mentioned techniques and show also the respective values of the asymptotic coefficient $C_0^{\alpha d}$. From the tables it is evident that the different methods yield very similar G_0^2 values. Indeed, all the results obtained, making allowance for the Coulomb+nuclear d-^{4}He interaction (both by analyzing experimental data and by using the inverse scattering method), belong to the range

$$|G_0|^2=0.42\pm0.02 \text{ fm}$$

while the theoretical three-body results obtained by different techniques and for different $N\alpha$ potentials disregarding the Coulomb interaction belong to the range $|G_0|^2=0.33\pm0.01$ fm. It should be noted that the theoretical $C_0^{\alpha d}$ values in Table IV(a) are rather close to the values $C_0^{\alpha d}=2.03-2.25$ obtained by Lehman *et al.* [18] by solving the Faddeev equations for the α-n-p system.

We have solved the Faddeev equations to find not only S-wave NVC $G_0(^6\text{Li}\rightarrow\alpha+d)$, but also the D-wave NVC $G_2(^6\text{Li}\rightarrow\alpha+d)$ and the respective asymptotic coefficient $C_2^{\alpha d}$. The resultant values

$$|G_2|^2=0.31\times10^{-4} \text{ fm and } C_2^{\alpha d}=0.022 \text{ [in version (i)]}$$

agree with the well-known assertion that the D-wave component of the ^{6}Li ground-state wave function is

TABLE III. The same as in Table II but obtained from the Faddeev phase shifts as input (the Coulomb interaction disregarded).

Padé approximant order $[N,M]$	$\varepsilon(^6\text{Li}\rightarrow{}^4\text{He}+d)$ (MeV)	$\lvert G_0\rvert^2$ (fm)	C_0
[1,1]	1.9572	0.1865	1.83
[2,2]	2.4424	0.3381	2.34
[3,3]	2.4729	0.3518	2.37
Values obtained by interpolating to exact binding energy	2.409	0.323	2.28
Direct solution of Faddeev equations for the vertex function	2.409	0.325	2.29

TABLE IV. The values of squared NVC $|G_0|^2$ and of asymptotic normalization constants $C_0^{\alpha d}$ for different methods and different versions of two-particle potentials: (a) the Coulomb interaction disregarded; (b) the Coulomb interaction included.

Method	$\lvert G_0\rvert^2$ (fm)	C_0
(a)		
PA,[a] theor. three-body phase shifts [version (i)]	0.34	2.34
Solving inverse problem (SIP) expt. phase shifts[b]	0.33	2.32
SIP, theor. three-body phase shifts [version (i)]	0.32	2.28
SIP, theor. three-body phase shifts [version (ii)]	0.32	2.27
Direct solution of Faddeev equations for the vertex function [version (i)]	0.33	2.30
(b)		
PA, expt. phase shifts	0.41	2.93
SIP,[c] expt. phase shifts	0.42	2.96
SIP,[c] theor. phase shifts [version (i)]	0.41	2.93
SIP,[c] theor. phase shifts [version (ii)]	0.42	2.96

[a]Padé approximation; the results for PA [2,2] are presented.
[b]After switching off the Coulomb interaction.
[c]Solution of ^{4}He-d inverse scattering problem.

small. It should be emphasized that not only the absolute value but even the sign of $G_2(^6\mathrm{Li}\rightarrow\alpha+d)$ [with respect to $G_0(^6\mathrm{Li}\rightarrow\alpha+d)$] cannot be considered at present as established firmly (see [18,22]). It would be of great interest and importance, therefore, to find G_2 (and $C_2^{\alpha d}$) by analyzing the experimental data on the elastic d-^{4}He scattering using a procedure similar to that applied above to G_0 but generalized to two-channel case. To that end, probably, it is necessary that more accurate phase-shift analysis data (especially on the mixing parameter ϵ_1) should be obtained and more sophisticated method of analytical continuation from the data should be used.

III. CONCLUSION

In conclusion, the principal results of the work will be outlined.

(i) The energy-dependent phase-shift analysis based on the Padé-approximant representation of the S matrix (or of the scattering function) makes it possible to obtain a precise analytic parametrization of the S matrix in not only the physical but also unphysical regions, thereby permitting the S-matrix poles corresponding to the bound state, as well as the residues in the poles (i.e., NVC's), to be found by *direct* analytic continuation.

(ii) Having obtained reliable phase-shift values, one can construct the accurate effective two-particle interaction potential in the given channel from which the NVC values can be derived. This inverse scattering approach is a new alternative method for finding the NVC's.

(iii) Our NVC value $|G_0(^6\mathrm{Li}\rightarrow\alpha+d)|^2=0.42\pm0.02$ fm obtained by direct analytic continuation of the S matrix in the Padé-approximant form has been confirmed by the results of other approaches and seems to be sufficiently reliable.

(iv) The allowance for the Coulomb d-^{4}He interaction (made by pure theoretical and semiphenomenological methods) raised the NVC $|G_0|^2$ value by approximately 25%.

ACKNOWLEDGMENTS

We thank Doctor V. N. Pomerantsev and Doctor V. M. Krasnopol'sky for help and valuable discussions.

[1] L. D. Blokhintsev, E. I. Dolinsky, and A. M. Mukhamedzhanov, El. Part. Atom. Nucl. (ECHAYA) **8**, 1189 (1977).
[2] M. P. Locher and T. Mizutani, Phys. Rep. **46**, 43 (1978).
[3] L. D. Blokhintsev *et al.*, El. Part. Atom. Nuclei (ECHAYA) **15**, 1296 (1984).
[4] J. Horaček, J. Bok, V. M. Krasnopol'sky, and V. I. Kukulin, Phys. Lett. **172**, 1 (1980), and the references therein to earlier works.
[5] G. A. Avakov *et al.*, Yad. Fiz. **43**, 824 (1986) [Sov. J. Nucl. Phys. **43**, 524 (1986)].
[6] V. I. Kukulin, V. M. Krasnopol'sky, and J. Horaček, *Theory of Resonances, Principles and Applications* (Kluwer, Dordrecht, 1989), Chaps. I and VI.
[7] V. M. Krasnopol'sky, V. I. Kukulin, and J. Horaček, Czech. J. Phys. B **35**, 805 (1985); **39**, 593 (1989).
[8] V. I. Kukulin and E. V. Kuznetsova, presented at the XIIth International Conference on Few Body Problems in Physics, Vancouver, Canada, 1989 (unpublished).
[9] V. I. Kukulin, V. M. Krasnopol'sky, E. V. Kuznetsova, and J. Horaček, Czech. J. Phys. B **40**, 945 (1990).
[10] V. M. Krasnopol'sky, V. I. Kukulin, E. V. Kuznetsova, J. Horaček, and N. M. Queen, Phys. Rev. C **43**, 822 (1991); University of Birmingham, Report No. 15/90 1990.
[11] V. I. Kukulin and V. N. Pomerantsev, Yad. Fiz. **51**, 376 (1990) [Sov. J. Nucl. Phys. **51**, 240 (1990)].
[12] B. Jenny *et al.*, Nucl. Phys. **A397**, 61 (1983).
[13] *Handbook of Mathematical Functions,* edited by M. Abramovitz and I. Stegun (Dover, New York, 1964).
[14] D. R. Lehman and M. Rajan, Phys. Rev. C **25**, 2743 (1982).
[15] L. D. Blokhintsev, V. I. Kukulin, and D. A. Savin, Yad. Fiz. **51**, 1289 (1990) [Sov. J. Nucl. Phys. **51**, 819 (1990)].
[16] V. I. Kukulin *et al.*, Nucl. Phys. **A417**, 128 (1984); **A453**, 365 (1986).
[17] B. Bang and C. Gignoux, Nucl. Phys. **A313**, 119 (1979).
[18] W. C. Parke and D. R. Lehman, Phys. Rev. C **29**, 2319 (1984).
[19] R. G. Lovas *et al.*, Nucl. Phys. **A474**, 451 (1987).
[20] Yu. V. Orlov, N. M. Petrov, and G. N. Teneva, Yad. Fiz. **55**, 38 (1992) [Sov. J. Nucl. Phys. **55**, 24 (1992)].
[21] V. I. Kukulin and V. N. Pomerantsev, Ann. Phys. (N.Y.) **111**, 330 (1978).
[22] H. R. Weller and D. R. Lehman, Annu. Rev. Nucl. Part. Sci. **53**, 1325 (1988); A. M. Eiro, F. D. Santos, J. Phys. G **16**, 1139 (1990).