Virtual particles versus superconductive vacuum polarizations in interacting systems

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The structure of the reference vacuum state plays a very important role in a theoretical dynamic description of interacting particles. This structure is generated by the residual interaction acting between the valence particles and, in systems under extreme high temperature and consequently high pressure, cannot be treated in the framework of the perturbation theory. In this paper we elaborate a nonperturbative approximation to include the vacuum-polarization effects of superconductive type in the calculation of the many-body dynamics. In this proposed model, the wave functions of the system are characterized by the strong coupling of the valence particles with the intrinsic vacuum states. These are associated with (a) superconductive vacuum polarizations, and (b) superconductive virtual particles formed coupling the vacuum excitations to the non-normal parity states of the system. The coupling of the valence particles with the vacuum excitations (a) and (b), in this paper, is generated within the equations of motion methods which, in the limit of factorization methods defined to compute the matrix elements of the nuclear Hamiltonian in the resulting complex states, and with the use of energy-dependent linearization approximations, introduced to generate the superconductive collective modes, are of easy application in the low-energy domain of the interacting systems. The energy-dependent linearization approximations define collective modes which are loose and freely moving in this low-energy domain, while strongly interacting with increasing energy.

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I. INTRODUCTION

The structure of the vacuum plays a crucial role in the study of the dynamics of interacting systems. Starting from a model in which the vacuum is assumed to consist of pairs of particles and/or particle holes coupled to $J=0$, we investigate nonperturbative approximations to include the structure of the vacuum in the dynamics of interacting particles. This structure modifies the valence particle dynamics generating the coupling of the valence states with the intrinsic-superconductive-vacuum states (ISVS's) which are formed by vector coupling of (a) valence particles and superconductive vacuum polarizations formed exciting, from the vacuum, particle-particle, hole-hole, and particle-hole pairs, and (b) valence particle and vacuum pairs coupled to non-normal parity to reproduce mesons in nuclear physics and gluons in the dynamics of quarks. The coupling with the ISVS's, especially in a system subjected to very high temperatures, is not suitable to be treated perturbatively, the perturbation approximation being poorly convergent.

In this paper we investigate a model to include ISVS coupling effects in the calculation of the many-body dynamics within a nonperturbative approximation. Evidence of the importance of nonperturbative methods in the study of the nonsuperconductive coupling of the valence particles with the (particle-hole) vacuum excitations has been already pointed out in Refs. [1–3] in the investigation of the dynamics of nuclei and in Ref. [4] in the interacting field dynamics. In Refs. [1,2] we have treated consistently, within a nonperturbative approximation, the coupling of the valence to the core degrees of freedom, under full consideration of the Pauli principle in the intrinsic vacuum states, which is essential for a good model reproduction of experimental energies, magnetic moments, electromagnetic transitions, and Gammow-Teller strengths of the low-lying states in the $A\pm1$ nuclei [5,22]. Via equations of motion methods, which, with the introduction of linearization approximations, generate eigenvalue equations, we were able to describe, nonperturbatively, the low-energy domain of the interacting systems. In Ref. [4], on the basis of formal solutions for the nuclear field in Walecka's effective meson theory [6], a nonperturbative approximation has been derived, to describe the propagation of particles correlated to particle-hole excitations.

New mathematical conditions are, however, necessary to generalize the model of Refs. [1,2,4] so that the valence particles could be coupled to the additional breaking of the (particle-particle) and (hole-hole) pairs from the vacuum to form, with the vacuum excitations studied in Refs. [1,2], the ISVS states. New linearization approximations are indeed necessary to include this new coupling mechanism in the theory of interacting systems. In terms of these linearizations, the dynamics of the valence particles is described by eigenvalue equations in which particle and antiparticle degrees of freedom, as proposed in Ref. [7] for the nuclear case, are coupled with the excitations of the model vacuum to form quasiparticle (q.p.) states. Two linearization procedures which consist in introducing the vacuum expectation values (aa) $\langle 0|p_i h_j|0\rangle=\delta_{ij}$ and (bb) $\langle 0|p_i p_j|0\rangle=\delta_{ij}$ are therefore investigated in this work.

Approximation (aa), introduced by Lane [8], has been elaborated in Refs. [1,2]. The linearization procedure (bb) [9] assumes the existence of superconductive vacuum

structure components. Within this approximation we generate the Bogoliubov-Valatin [10] quasiparticle transformation in first-order linearization, which is achieved by neglecting the explicit introduction of the IVSV's in the model space. The breaking of both vacuum symmetries within the higher-order linearization approximations define the ISVS's of the model.

In the low-energy domain, few quasiparticles characterize the physical properties of the system, so that we can restore (linearizing of the commutator chain) vacuum symmetries within the promotion of few q.p. states to valence level. The resulting eigenvalue equations characterizing the low-energy domain are simply solvable within a generalization of the recursive methods introduced to calculate matrix elements in Refs. [1,2].

In the high-energy part of the spectrum, the resulting collective states are strongly interacting.

The two types of vacuum-polarization effects (a) and (b) (see Ref. [11] for nonsuperconductive virtual particles) find a unified treatment within these approximations. In the zero-order (no active vacuum pairs) linearization approximation, which connects particles to antiparticles, we recognize the results of Ref. [15] for interacting quarks as we discuss in Sec. II A. In our higher-order linearization approximations, however, we are going beyond those results (Ref. [15]), allowing the interaction of the valence particles with the polarizations of the reference vacuum, terms that break the chiral symmetry introducing the mass matrix.

In Sec. II we remark that the dynamic evolution of the system introduces in the model the vacuum degrees of freedom. The linearization approximations (aa) and (bb) restore the vacuum pair symmetries, generating finite systems of eigenvalue equations. In Sec. II A the formalism is applied to describe the closed-shell polarizations in the $(A+1)$ systems. We show that the matrix elements of the many-body interaction in the configuration mixing wave functions (CMWF's), components of the ISVS's, are suitable to be calculated within recursive procedures. Within these recursive procedures, we relate the matrix elements of the interaction calculated with CMWF's of the $\{n\}$ kind with those of the $\{n-1\}$ kind being the kind of complexity equal to the number of particle-hole and/or particle-particle pairs in the model wave functions. In Sec. II B the model is applied to describe the dynamic evolution of the valence quasiquark.

In Sec. III we calculate this functional dependence introducing an expansion for the quasiparticle wave functions of the $\{n\}$th kind.

In Sec. IV we apply the model to the superallowed β decay.

In Sec. V we define the collective quasiparticle Hamiltonian for the low- and high-energy domains.

II. LINEARIZATION-APPROXIMATION METHODS

In the Heisenberg picture, the time evolution of a system is given in terms of the commutators of the Hamilton's operator with the creation operators of the valence particles. A perturbation approximation can be derived in terms of the Green's functions method [12], which is then truncated up to low orders in the most practical calculations. In systems of particles interacting via strong forces, this method is, however, purely convergent.

In this paper we overcome this type of approximation reconsidering the equations of motion methods. With the introduction of linearization approximations we generate, from the hierarchy of the commutator equations, systems of eigenvalue equations which give a powerful nonperturbative description of the dynamics of interacting particles. The amplitudes of the model modes are associated to the Green's functions of the dynamic theory. In order to reduce the dimension of the derived eigenvalue equations for these modes, we introduce an energy-dependent parameter which separates the low-energy domain from the high one.

The low-energy domain is characterized by valence particles coupled to a few ISVS components and described in terms of a degree of linearization with the following characteristics: (i) first-order linearization, where the symmetries of the vacuum are conserved; (ii) second- and higher-order linearization, where the commutator chain is linearized including explicitly in the model space the vacuum-polarization terms. Within this degree of linearization, the symmetries of the vacuum are restored after having promoted the symmetry breaking terms of high complexity (ISVS's) to valence character.

In this section the linearization approximations of types (aa) and (bb) and the linearization parameter are discussed in connection with the dynamic evolution of the $[mp-(m-1)h]$ system (see the application section for the definition of the introduced indices). We write for CMWF's of the $\{n\}$th kind, introducing the superscript $\{n\}$ of the N and the subscript $\{n\}$ of the J's to characterize the kind of complexity [number of (particle-hole) and(particle-particle) pairs]:

$$|\Phi_{JM}[mp-(m-1)h]\rangle = \sum_{j_1 j_2 j_3 \cdots j_{2m-2} j_{2m-1} J_1 J_2 \cdots J^n J_i^1 J_i^2 \cdots J_i^{n-2}} N^n_{j_1(j_2 j_3)J_1 \cdots (j_{2m-2} j_{2m-1})J_n J} [a^\dagger_{j_1} \otimes (a^\dagger_{j_2} \otimes a_{j_3})^{J_1} \cdots \otimes (a^\dagger_{j_{2m-2}} \otimes a_{j_{2m-1}})^{J_n}]^J_M |0\rangle \tag{2.1}$$

[the $a^\dagger_{jm}$'s and the $(-1)^{j-m}a_{j-m}$'s are the creation and destruction operators], which in the notations introduced in the Appendix [see (A2)], takes the form

$$|\Phi_{JM}[mp-(m-1)h]\rangle=\sum_{\alpha_n J_1 J_2\cdots J_n} N^n_{\alpha_n J_1 J_2\cdots J_n J}|\,A^\dagger_n(\alpha_n J_1 J_2\cdots J_n;JM)|0\rangle\ . \tag{2.2}$$

In the first-order linearization approximation, the model vacuum state $|0\rangle$ consists of pairs of (particle-hole) and (particle-particle), coupled to the $J=0$, added to the true vacuum, and bears good symmetries. The evolution of the system is described by the equation

$$[H,A^\dagger_n(\alpha_n J_1 J_2\cdots J_n;J)]=\sum_{\alpha'_n J'_1 J'_2\cdots J'_n}\langle\,A_n(\alpha_n J_1 J_2\cdots J_n;J)\|H\|\,A^\dagger_n(\alpha'_n J'_1 J'_2\cdots J'_n;J)\rangle\,A^\dagger_n(\alpha'_n J'_1 J'_2\cdots J'_n;J) \tag{2.3}$$

for the linearization (aa) and by the equations

$$\begin{aligned}&[H,A^\dagger_n(\alpha_n J_1 J_2\cdots J_n;J)]\\&\quad=\sum_{\epsilon_n J'_1 J'_2\cdots J'_n}\langle\,A_n(\alpha_n J_1 J_2\cdots J_n;J)\|H\|P^\dagger_n(\epsilon_n J'_1 J'_2\cdots J'_n;J)\rangle P^\dagger_n(\epsilon_n J'_1 J'_2\cdots J'_n;J)\\&\quad+\sum_{\vartheta_n J'_1 J'_2\cdots J'_n}\langle 0\|H\|\,A^\dagger_n(\alpha_n J_1 J_2\cdots J_n;J)P^\dagger_n(\vartheta_n J'_1 J'_2\cdots J'_n;J)\rangle P_n(\vartheta_n J'_1 J'_2\cdots J'_n;J)\end{aligned} \tag{2.4}$$

for the linearizations (aa) and (bb). In Eq. (2.4) we have introduced the operators

$$\begin{aligned}P^\dagger_n(\epsilon_n J'_1 J'_2\cdots J'_n;J)&=[\,A^\dagger_n(\beta_n J'_1 J'_2\cdots J'_n;J)+G^\dagger_n(\hat\beta_n J'_1 J'_2\cdots J'_n;J)+\cdots\,]+B^\dagger_n(\eta_n J'_1 J'_2\cdots J'_n;J)\\&=N^n_{j'_1(j'_2 j'_3)J'_1\cdots(j'_{2m-2}j'_{2m-1})J'_n J}\{\,[a^\dagger_{j'_1}\otimes(a^\dagger_{j'_2}\otimes a_{j'_3})^{J'_1}\cdots(a^\dagger_{j'_{2m-2}}\otimes a_{j'_{2m-1}})^{J'_n}]^J\\&\quad+[a^\dagger_{j'_1}\otimes(a^\dagger_{j'_2}\otimes a_{j'_3})^{J'_1}\cdots(a^\dagger_{j'_{2m-1}}\otimes a_{j'_{2m-2}})J'_n]^J\\&\quad+[a^\dagger_{j'_1}\otimes(a^\dagger_{j'_2}\otimes a_{j'_3})^{J'_1}\cdots\otimes(a^\dagger_{j'_{2m-2}}\otimes a^\dagger_{j'_{2m-1}})^{J'_n}]^J\\&\quad+[a^\dagger_{j'_1}\otimes(a^\dagger_{j'_2}\otimes a_{j'_3})^{J'_1}\cdots\otimes(a_{j'_{2m-2}}\otimes a_{j'_{2m-1}})^{J'_n}]^J\}\end{aligned} \tag{2.5}$$

+ the operators $B^\dagger_n(\eta_n J'_1 J'_2\cdots J'_n;J)$ obtained replacing in $[\,A^\dagger_n(\beta_n J'_1 J'_2\cdots J'_n;J)+\cdots\,]$ the $(m-1)$ destruction with the corresponding creation operators. The additional commutator $[H,A_n(\rho_n J_1 J_2\cdots J_n;J)]$ has been omitted for simplicity.

The $P_n(\vartheta_n J'_1 J'_2\cdots J'_n;J)$ are the adjoints of the $P^\dagger_n(\epsilon_n J'_1 J'_n\cdots J'_n;J)$ operators, obtained replacing the first single-particle operator in $[\,A^\dagger_n(\alpha_n J'_1 J'_2\cdots J'_n;J)+\cdots\,]$ with the corresponding adjoint and the $B_n(\mu_n J'_1 J'_2\cdots J'_n;J)$ with $B^\dagger_n(\eta_n J'_1 J'_2\cdots J'_n;J)$.

We have to note that Eqs. (2.3) and (2.4) result from the full calculation of the commutator on the left side of the equations. No analogous expressions have until now been reported in the most quoted literature.

The linearization approximations (aa) and (bb), respectively, applied to the nonlinear terms, generate the interaction matrix elements. It is important to remark that within the linearization conditions (aa) and (bb) the system of interacting particles is superconductive also in the first-order linearization approximation.

The two-body interaction excites one or more pairs of particles or of (particles holes) which contribute to the formation of the intrinsic superconductive vacuum states (ISVS's). In the following we include explicitly this symmetries breaking term in the model wave functions (high-order linearizations).

The evolution of the system is then described, modifying Eqs. (2.3) and (2.4), by the commutator equations

$$\begin{aligned}&[H,A^\dagger_n(\alpha_n J_1 J_2\cdots J_n;J)]\\&\quad=\sum_{\epsilon_n J'_1 J'_2\cdots J'_n}\langle\,A_n(\alpha_n J_1 J_2\cdots J_n;J)\|H\|P^\dagger_n(\epsilon_n J'_1 J'_2\cdots J'_n;J)\rangle P^\dagger_n(\epsilon_n J'_1 J'_2\cdots J'_n;J)\\&\quad+\sum_{\vartheta_n J'_1 J'_2\cdots J'_n}\langle 0\|H\|\,A^\dagger_n(\alpha_n J_1 J_2\cdots J'_N;J)P^\dagger_n(\vartheta_n J'_1 J'_2\cdots J'_n;J)\rangle P_n(\vartheta_n J'_1 J'_2\cdots J'_n;J)\\&\quad+\sum_{\epsilon_{n+1} J'_1 J'_2\cdots J'_{n+1}}\langle\,A_n(\alpha_n J_1 J_2\cdots J_n;J)\|V\|P^\dagger_{n+1}(\epsilon_{n+1} J'_1 J'_2\cdots J'_{n+1};J)\rangle P^\dagger_{n+1}(\epsilon_{n+1} J'_1 J'_2\cdots J'_{n+1};J)\\&\quad+\sum_{\vartheta_{n+1} J'_1 J'_2\cdots J'_{n+1}}\langle 0\|V\|\,A^\dagger_n(\alpha_n J_1 J_2\cdots J_n;J)P^\dagger_{n+1}(\vartheta_{n+1} J'_1 J'_2\cdots J'_{n+1};J)\rangle P_{n+1}(\vartheta_{n+1} J'_1 J'_2\cdots J'_{n+1};J)\end{aligned} \tag{2.6}$$

and by the ones involving the commutators of H with $A_n(\rho_n J_1 J_2 \cdots J_n;J)$, omitted for simplicity. The $P^\dagger_{n+1}(\epsilon_{n+1}J'_1J'_2\cdots J'_{n+1};J)$ and $P_{n+1}(\vartheta_{n+1}J'_1J'_2\cdots J'_{n+1};J)$ operators create the intrinsic vacuum components which are obtained by vector-coupling selected components of $P^\dagger_n(\epsilon_n J'_1J'_2\cdots J'_n;J)$ and $P_n(\vartheta_n J'_1J'_2\cdots J'_n;J)$ with one (particle-hole), one (hole-particle), one (particle-particle), or one (hole-hole) pair. These terms were linearized in the $\{n\}$-order linearization approximations.

Higher-order linearization approximations modify Eq. (2.6) introducing ISVS's of $\{n+2\}$, $\{n+3\}$, etc., kind. Neglecting these terms and taking the expectation values of the commutator equations between the vacuum and polarized states, we derive the eigenvalue equations that describe the coupled system (valence states and polarization states of the $\{n+1\}$ kind). The resulting eigenvectors obtained from the diagonalization form the basis in which we expand the wave functions that characterize the symmetry breaking terms of higher order, important in the high-energy domain.

In Secs. II A and II B we apply the formalism to describe (a) the superconductive polarizations of the closed shells interacting with a valence nucleon and (b) the dynamic evolution of a quark state.

A. Closed-shell polarizations

In this subsection we apply the formal equation (2.6) to a system characterized by a single nucleon outside a closed shell. We write, for the wave functions,

$$|\Phi^0_{j_1m_1}(\alpha_0)\rangle = X^0_{\alpha_0}A^\dagger_0(\alpha_0;j_1m_1)|0\rangle = X^0_{j_1}a^\dagger_{j_1m_1}|0\rangle ,$$

with $\alpha_0=\{j_1\}$.

Within the linearization approximations (aa) and (bb), the commutator (2.6) is reduced to

$$\begin{aligned}[H, A^\dagger_0(\alpha_0;J)] = &\sum_{\beta_0}\epsilon_J A^\dagger_0(\beta_0;J) + \langle 0\|V\|A^\dagger_0(\beta_0;J)A^\dagger(\rho_0;J)\rangle A_0(\rho_0;J) + \sum_{\beta_1J'_1}\langle A_0(\alpha_0;J)\|V\|A^\dagger_1(\beta_1J'_1;J)\rangle A^\dagger_1(\beta_1J'_1;J)\\ &+\sum_{\rho_1J'_1}\langle 0\|V\|A^\dagger_0(\alpha_0;J)A^\dagger_1(\rho_1J'_1;J)\rangle A_1(\rho_1J'_1;J) + \sum_{\hat\beta_{1,1}J'_1}\langle A_0(\alpha_0;J)\|V\|G^\dagger_{1,1}(\hat\beta_{1,1}J'_1;J)\rangle G^\dagger_{1,1}(\hat\beta_{1,1}J'_1;J)\\ &+\sum_{\hat\rho_{1,1}J'_1}\langle 0\|V\|A^\dagger_0(\alpha_0;J)G^\dagger_{1,1}(\hat\rho_{1,1}J'_1;J)\rangle G_{1,1}(\hat\rho_{1,1}J'_1;J) , \end{aligned} \tag{2.7}$$

where ϵ_J is the single-particle energy, $\rho_0=\{j_1^{-1}\}$ the coordinates of a hole, and where $A^\dagger_1(\beta_1J'_1;J)$ and $G^\dagger_{1,1}(\hat\beta_{1,1}J'_1;J)$ create the two-particle–one-hole (2p-1h) intrinsic vacuum components formed by vector-coupling one particle to the one (particle-hole) vacuum pair and the time-reversal ones where the particle is coupled to one (hole-particle) pair. The caret indicates that the time of the (particle-hole) pair has been reversed, and the $\{1\}$ has been introduced to classify the time-reversal (TR) components. The commutator relations (2.7) and those for the $A_0(\rho_0;J)$ operator linearized as not to include in the model the operators of the vacuum structure components (3q.p.) reproduce the quasiparticle transformation of Bogoliubov [10] and Valatin [10].

The dynamic evolution introduces in the calculation the three quasiparticle states for which Eq. (2.6) takes the form

$$\begin{aligned}[H, A^\dagger_1(\alpha_1,J'_1;J)] = &\sum_{\beta_0}\langle A_1(\alpha_1J'_1;J)\|V\|A^\dagger_0(\beta_0;J)\rangle A^\dagger_0(\beta_0;J) + \sum_{\rho_0}\langle 0\|V\|A^\dagger_1(\alpha_1J'_1;J)A^\dagger_0(\rho_0;J)\rangle A_0(\rho_0J)\\ &+\sum_{\epsilon_1J''_1}\langle A_1(\alpha_1J_1;J)\|H\|P^\dagger_1(\epsilon_1J''_1;J)\rangle P^\dagger_1(\epsilon_1J''_1J) + \sum_{\vartheta_1J''_1}\langle 0\|H\|A^\dagger_1(\alpha_1J_1J)P^\dagger_1(\vartheta_1J''_1J)\rangle P_1(\vartheta_1J''_1;J)\\ &+\sum_{\epsilon_2J''_1J''_2}\langle A_1(\alpha_1,J'_1;J)\|V\|P^\dagger_2(\epsilon_2J''_1J''_2;J)\rangle P^\dagger_2(\epsilon_2J''_1J''_2;J)\\ &+\sum_{\vartheta_2J''_1J''_2}\langle 0\|V\|A^\dagger_1(\alpha_1J'_1;J)P^\dagger_2(\vartheta_2J''_1J''_2;J)\rangle P_2(\vartheta_2J''_1J''_2;J) . \end{aligned} \tag{2.8}$$

The commutators of the nuclear Hamiltonian with the other operators on the right side of Eq. (2.8) are explicitly not given.

The expectation value of the linearized commutators $[H, A^\dagger_1(\alpha_1J'_1;J)]$, $[H, A_1(\rho_1,J'_1;J)]$, . . . , is taken between the ground and excited modes of the system to define the energy and wave functions of a single valence nucleon in a medium in terms of the matrix elements of the two-body nuclear interaction. These matrix elements, calculated using the recoupling technique of Ref. [13], have been considered as the starting point of the recursive expansion we introduce for the five quasiparticle states.

In the no-coupling limit, this expansion is exactly given in Fig. 1 for the matrix elements, calculated with direct components and, in Fig. 2, for those calculated in the adjoint components. For the additional matrix elements that characterize the commutator relations with the TR components, we elaborate the expansions illustrated in Fig. 3 and for the matrix elements involving the adjoint TR operators those of Fig. 4. For the matrix elements of the nuclear interaction characterizing the new linearize-

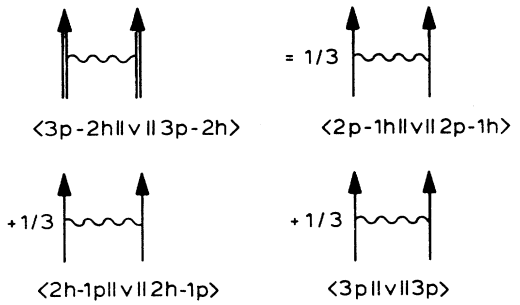

FIG. 1. Exact decomposition of the matrix elements of the second kind in terms of the first kind (direct components).

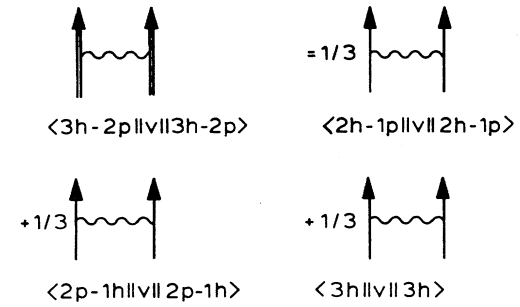

FIG. 2. Decomposition of matrix elements calculated with the adjoint direct components.

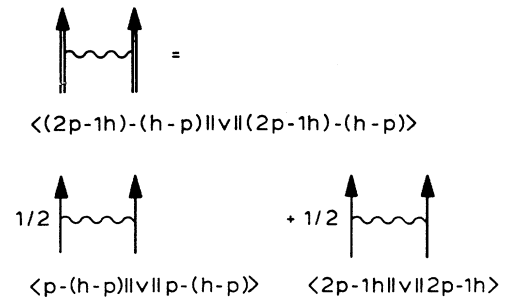

FIG. 3. Factorization method used for the TR components.

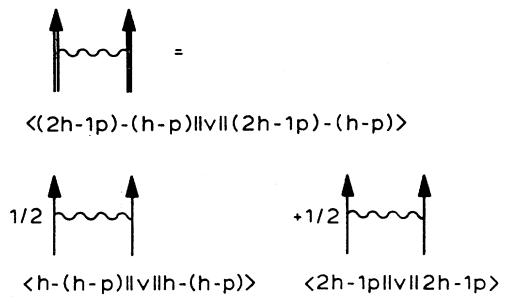

FIG. 4. As in Fig. 3 for the adjoint components.

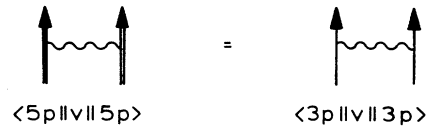

FIG. 5. Matrix elements of the five particle components.

tion method, we have the expansions of Figs. 5 and 6. The commutator hierarchy [Eq. (2.2) and (2.8)] is linearized restoring the vacuum symmetries within CMWF's of the nth kind, components of the $(2m+1)$ quasiparticle states.

Calculation of the matrix elements of the many-body Hamiltonian that characterizes the commutators of the Hamiltonian with the complex creation operators of the ISVS's is done by generalizing the expansions defined in the figures for CMWF's of the second kind. To calculate these expansion coefficients (Sec. III), we introduce the algebra of projection operators connected with the algebra of unit tensor operators in the work of Racah [14]. The linearization approximations used to restore the symmetry of the vacuum after the promotion of vacuum pairs to valence character define collective quasiparticles and the terms beyond the linearization approximation introduce their four-point, six-point, etc., interaction sections (Sec. V). A practical application of this formalism to light nuclei is presented in Sec. IV.

B. Dynamic evolution of a quark state

In this section we apply Eq. (2.6) to study the dynamic evolution of a single-quark state.

We define the many-quark Hamiltonian

$$H=\sum_{\alpha\beta} t_{\alpha\beta}a^{\dagger}_{\alpha}a_{\beta}+\tfrac{1}{2}\sum_{\alpha\beta\gamma\delta}(\alpha\beta|V^{\text{eff}}|\gamma\delta)a^{\dagger}_{\alpha}a^{\dagger}_{\beta}a_{\delta}a_{\gamma}\ , \tag{2.9}$$

where the greek letters denote the quark quantum numbers $\{\alpha\}\equiv\{lsjc\}$ and where

$$V^{\text{eff}}_{ij}=V_0 f(r_{ij}\sigma_{ij}S_{ij})(\lambda^c_i\cdot\lambda^c_j)\ , \tag{2.10}$$

where $f(r_{ij}\sigma_{ij}S_{ij})$, being a function of the quark relative distance, spin, and tensor operators, reproduces the quark-quark interaction. The λ^c_i are the generators of the

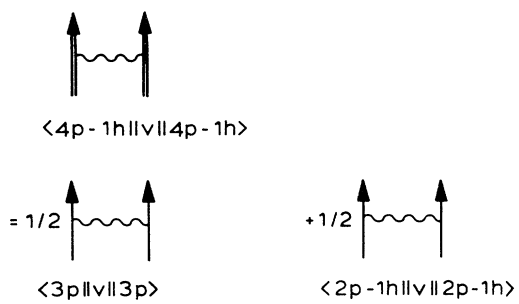

FIG. 6. As in Fig. 5 for the (4p-1h) particle components.

SU_3 color group.

We apply the equations of motion method (2.6) to the single-quark creation operator $a_\alpha^\dagger$ to determine the functional dependence of the quark-quark potential. With the linearization (aa), the quark and antiquark states are not connected by the equation of motion method (good chiral symmetry), and the set of commutator equations

$$[H,a_\alpha^\dagger]=\epsilon_\alpha a_\alpha^\dagger, \quad [H,a_\rho]=\bar{\epsilon}_\rho a_\rho \tag{2.11}$$

are disconnected. In this approximation the average potentials, for the quark and antiquark, respectively, can be approximated in a nonrelativistic limit with two harmonic oscillator potentials, where the energies ϵ_α and $\bar{\epsilon}_\rho$ are the quark's and antiquark's mean-field energies (Hartree-Fock) (the definition of the single-particle energies is associated with the introduction of a cutoff parameter). Taking the expectation values of Eq. (2.11) between the model states, we calculate the wave functions of the quark and antiquark. These are eigenvalues of a collective Hamiltonian that can be associated with the Lagrangian introduced by Nambu–Jona-Lasinio [15].

The corresponding quark wave functions are

$$|\Phi_\alpha\rangle \equiv \begin{bmatrix} a_\alpha^\dagger \\ a_\rho \end{bmatrix} |0\rangle \ . \tag{2.12}$$

The linearization approximation (bb) breaks the chiral symmetry, mixing the quark and antiquark degrees of freedom.

The commutator relations that define the model states are

$$[H,a_\alpha^\dagger]=\epsilon_\alpha a_\alpha^\dagger+\Xi_\rho a_\rho, \quad [H,a_\rho]=\bar{\epsilon}_\rho a_\rho+\bar{\Xi}_\alpha a_\alpha^\dagger \ , \tag{2.13}$$

and the corresponding quark wave functions take the form

$$|\Phi_\alpha\rangle \equiv \begin{bmatrix} a_\alpha^\dagger + a_\rho \\ a_\alpha + a_\rho^\dagger \end{bmatrix} |0\rangle \ ; \tag{2.14}$$

i.e., with both linearization approximations, the two potentials generate a common sector where the quark and antiquark can coexist. The linearization of the system with the implicit inclusion of the symmetry breaking terms, however, does not reproduce the flavor masses. In order to introduce the quark flavor masses, we have to linearize the commutators including explicitly in the model space the vacuum structure components as recognized in Ref. [16], and given in Eq. (2.6). To calculate the matrix elements of the nuclear interaction in Eq. (2.6), we need to introduce the color recoupling algebra. This can be done using as in Ref. [17] the two SU(2) representations of the $SU_c(3)$ group. To solve in this approximation the dynamic equation Eq. (2.6), we take the expectation values between the vacuum, the one quark-q, and the three quasiquark components and diagonalize the resulting matrix.

The quark acquires a flavor mass and is characterized by the wave functions

$$|q\rangle = c_1|q\rangle + c_2|\bar{q}\rangle + c_3|qq\bar{q}\rangle + c_4|\bar{q}q\bar{q}\rangle$$
$$+ c_5|qqq\rangle + c_6|\bar{q}\,\bar{q}\,\bar{q}\rangle \ , \tag{2.15}$$

with $\{q\}\equiv\{u,s,d\}$. The eigenvectors defined in (2.15) are eigenvectors of a collective highly nonlinear Hamiltonian.

In Ref. [18] it was already assumed that the pions are associated with a collective superposition of quark-antiquark pairs moving in the confining potential under the additional influence of the pseudoscalar isovector part of the effective interaction. In our model, however, pions are collective states of quark-antiquark coupled with the intrinsic vacuum structure components. These components play an important role in the determination of the magnetic properties of hadrons [19].

To describe the heavier quarks, we use this base and we introduce the vacuum structure components of higher complexity such as four-, six-, etc., point interactions acting between the collective quarks (see Sec. V).

III. TRANSFORMATION COEFFICIENTS

The recursive expansions introduced in the previous section simplify the calculation of the matrix elements of the nuclear Hamiltonian in the quasiparticle states that characterize the dynamical evolution of the valence system. In this section the transformation coefficients associated with these expansions are calculated connecting them with matrix elements of unit tensor operators [20] generators of the $SU_{2J+1}(n)$ group (where $\{n\}$ specifies the kind of complexity of the CMWF's).

We apply this expansion method to the four main subcases that characterize the main components of the quasiparticle states of the $\{n\}$th kind.

A. Transformation coefficients for the direct terms

The matrix elements of the nuclear Hamiltonian, calculated with respect to the direct components of CMWF's of the $\{n\}$th kind, have been expanded, as illustrated in Fig. 1 for $n=2$, in terms of the matrix elements calculated with respect to the CMWF's of the $\{n-1\}$th kind.

We introduce, for the direct components of CMWF's of the $\{n\}$th kind, a linear combination of the form

$$
\begin{aligned}
&|\Phi^n_{JM}(\alpha_n J_1 J_2 \cdots J_n)\rangle \\
&= \sum_{\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\vartheta_1^n} {}^{2n+1}T_J(\alpha_n J_1 J_2 \cdots J_n | \}\alpha_{n-1} J_r \bar{\alpha}_{n-1} J_s)[|\Phi^{n-1}_{J_r}(\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\Phi^0_{J_s}(\bar{\alpha}_{n-1} J_s)\rangle]^J_M \\
&+ \sum_{\rho_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\vartheta_2^n} {}^{2n+1}Z_J(\alpha_n J_1 J_2 \cdots J_n | \}\rho_{n-1} J_c \bar{\rho}_{n-1} J_s)[|\bar{\Psi}^{n-1}_{J_r}(\rho_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\Lambda^0_{J_s}(\bar{\rho}_{n-1} J_s)\rangle]^J_M \\
&+ \sum_{\eta_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\vartheta_3^n} {}^{2n+1}V_J(\alpha_n J_1 J_2 \cdots J_n | \}\eta_{n-1} J_r \bar{\eta}_{n-1} J_s)[|\Lambda^{n-1}_{J_r}(\eta_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |M^0_{J_s}(\bar{\eta}_{n-1} J_s)\rangle]^J_M ,
\end{aligned}
\tag{3.1}
$$

where $\Phi^0_{J_s}(\bar{\alpha}_{n-1} J_s)$, $\Lambda^0_{J_s}(\bar{\rho}_{n-1} J_s)$, and $M^0_{J_s}(\bar{\eta}_{n-1} J_s)$ denote the (particle-hole), (particle-particle), and (hole-hole) wave functions, respectively, and ϑ_1^n, ϑ_2^n, and ϑ_3^n give the number of different $\{\alpha_{n-1}\}$, $\{\rho_{n-1}\}$, and $\{\eta_{n-1}\}$ active combinations we can form with the $\{\alpha_n\}$ coordinates. In the linear combination (3.1), CMWF's of the $\{n\}$ kind characterized by the quantum number $\{\alpha_n\}$ are defined in terms of CMWF's of the $\{n-1\}$ kind characterized by the quantum numbers $\{\alpha_{n-1}\}$, $\{\rho_{n-1}\}$, and $\{\eta_{n-1}\}$, and therefore are obtained projecting from the total antisymmetry states $\{\alpha_n\}$ a (particle-hole), a (particle-particle), and a (hole-hole) pair, respectively. The $\{\bar{\alpha}_{n-1}\}$, $\{\bar{\rho}_{u+1}\}$, and $\{\bar{\eta}_{n-1}\}$ are complementary coordinates as defined in Refs. [1,2].

The transformation coefficients

$$
{}^{2n+1}T_J(\alpha_n J_1 J_2 \cdots J_n | \}\alpha_{n-1} J_r \bar{\alpha}_{n-1} J_s) ,
$$

$$
{}^{2n+1}Z_J(\alpha_n J_1 J_2 \cdots J_n | \}\rho_{n-1} J_r \bar{\rho}_{n-1} J_s) ,
$$

and

$$
{}^{2n+1}V_J(\alpha_n J_1 J_2 \cdots J_n | \}\eta_{n-1} J_r \bar{\eta}_{n-1} J_s)
$$

are calculated by defining the unit tensor operators [20]

$$
u^k(n) = [U^\dagger_{\bar{\alpha}_n}(J_i) \otimes U_{\bar{\alpha}_n}(J'_i)]^k , \tag{3.2}
$$

$$
\tilde{h}^k(n) = [H^\dagger_{\bar{\rho}_n}(J_i) \otimes H_{\bar{\rho}_n}(J'_i)]^k , \tag{3.3}
$$

$$
p^k(n) = [P^\dagger_{\bar{\eta}_n}(J_i) \otimes P_{\bar{\eta}_n}(J'_i)]^k , \tag{3.4}
$$

where $U_{\bar{\alpha}_n}(J'_i)$, $H_{\bar{\rho}_n}(J'_i)$, and $P_{\bar{\eta}_n}(J'_i)$ destroy a (particle-hole), a (hole-hole), and a (particle-particle) pair, respectively, when applied on CMWF's of the $\{n\}$th kind and reducing the $SU_{2J+1}(n)$ [14] representations carried by the wave functions on the right side of Eq. (3.1).

The ${}^{2n+1}T_J(\alpha_n J_1 J_2 \cdots J_n | \}\alpha_{n-1} J_r \bar{\alpha}_{n-1} J_s)$ are the eigenvalues of the matrix with

$$
\begin{aligned}
&({}^{2n+1}T_J(\alpha_n J_1 J_2 \cdots J_n | \}\alpha_{n-1} J_r \bar{\alpha}_{n-1} J_s))^\dagger \, {}^{2n+1}T_J(\alpha_n J_1 J_2 \cdots J_n | \}\alpha_{n-1} J_r \bar{\alpha}_{n-1} J_s) \\
&= \sum_{\substack{k J_i J'_i \\ J^2_r J^3_r}} (-1)^{J_i + J'_i + J^2_r + J^3_r + J_s + J'_s + 1} \hat{k}^{1/2} \hat{J}_r \hat{J}'_r
\begin{Bmatrix} J_r & J & J_s \\ J & J_r & k \end{Bmatrix}
\begin{Bmatrix} J'_r & J & J'_s \\ J & J'_r & k \end{Bmatrix}
\begin{Bmatrix} J_i & J_r & J^2_r \\ J_r & J_i & k \end{Bmatrix}
\begin{Bmatrix} J'_i & J'_r & J^3_r \\ J'_r & J'_i & k \end{Bmatrix} \\
&\times [({}^{2n-1}T^k_{J_r}(\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} | \}\alpha_{n-2} J_i \bar{\alpha}_{n-2} J^2_r))^\dagger \, {}^{2n-1}T^k_{J_r}(\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} | \}\alpha_{n-2} J_i \bar{\alpha}_{n-2} J^2_r)] \\
&\times [({}^{2n-1}T^k_{J'_r}(\beta_{n-1} J'_{k_1} J'_{k_2} \cdots J'_{k_{n-1}} | \}\beta_{n-2} J'_i \bar{\beta}_{n-2} J^3_r))^\dagger \, {}^{2n-1}T^k_{J'_r}(\beta_{n-1} J'_{k_1} J'_{k_2} \cdots J'_{k_{n-1}} | \}\beta_{n-2} J'_i \bar{\beta}_{n-2} J^3_r)]
\end{aligned}
\tag{3.5}
$$

as matrix elements. The procedure defines the transformation coefficients of the $\{n\}$th kind in terms of the transformation coefficients of the $\{n-1\}$th kind. Analogous expressions hold for the ${}^{2n+1}Z_J(\alpha_n J_1 J_2 \cdots J_n | \}\rho_{n-1} J_r \bar{\rho}_{n-1} J_s)$ and the ${}^{2n+1}V_J(\alpha_n J_1 J_2 \cdots J_n | \}\epsilon_{n-1} J_r \bar{\epsilon}_{n-1} J_s)$ transformation coefficients.

B. Hole conjugation

The direct components of the one-hole conjugation states are expanded according to the formula

$$
\begin{aligned}
&|\tilde{\Psi}^n_{JM}(\rho_n J_1 J_2 \cdots J_n)\rangle \\
&\quad = \sum_{\rho_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\eta_1^n} {}^{2n+1}Z_J(\rho_n J_1 J_2 \cdots J_n | \}\rho_{n-1} J_r \bar{\rho}_{n-1} J_s)[|\tilde{\Psi}^{n-1}_{J_r}(\rho_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes \Phi^0_{J_s}(\bar{\rho}_{n-1} J_s)\rangle]^J_M \\
&\quad + \sum_{\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\eta_2^n} {}^{2n+1}T_J(\rho_n J_1 J_2 \cdots J_{n-1} | \}\alpha_{n-1} J_r \bar{\alpha}_{n-1} J_s) \\
&\qquad\qquad \times [|\Phi^{n-1}_{J_r}(\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |M^0_{J_s}(\bar{\alpha}_{n-1} J_s)\rangle]^J_M \\
&\quad + \sum_{\mu_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s}^{\eta_3^n} {}^{2n+1}C_J(\rho_{n-1} J_1 J_2 \cdots J_n | \}\mu_{n-1} J_r \bar{\mu}_{n-1} J_s) \\
&\qquad\qquad \times [|M^{n-1}_{J_r}(\mu_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\Lambda^0_{J_s}(\bar{\mu}_{n-1} J_s)\rangle]^J_M \ , \qquad (3.6)
\end{aligned}
$$

where

$$
({}^{2n+1}C_J(\rho_n J_1 J_2 \cdots J_n | \}\mu_{n-1} J_r \bar{\mu}_{n-1} J_s))^\dagger = \langle \tilde{\Psi}^n_J(\rho_n J_1 J_2 \cdots J_n) \| D_{\bar{\mu}_{n-1}}(J_s) \| M^{n-1}_{J_r}(\mu_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \ , \qquad (3.7)
$$

and where η_1^n, η_2^n, and η_3^n give the number of different $\{\rho_{n-1}\}$, $\{\alpha_{n-1}\}$, and $\{\mu_{n-1}\}$ combinations that we can form with the $\{\rho_n\}$ coordinates and the

$$
s^k_{m_k}(n) = [\bar{D}^\dagger_{\bar{\mu}_n}(J_i) \otimes \bar{D}_{\mu_n}(J_i')]^k_{m_k} = \sum_{n_{n-1}} s^k_{m_k}(n-1, \mu_{n-1})
$$

are the unit tensor operators as defined in Ref. [1]. The transformation coefficients are eigenvalues of a matrix similar to that of Eq. (3.5).

C. Transformation coefficients for the time-reversal states

For the time-reversal components, we define the expansion

$$
\begin{aligned}
&|\hat{\tilde{\Phi}}^n_{i,JM}(\hat{\alpha}_{n,i} J_1 J_2 \cdots J_n)\rangle \\
&\quad = \sum_{\substack{\hat{\alpha}_{n-1,j} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} \\ J_r J_s}}^{\vartheta_4^n} {}^{2n+1}\hat{T}_{i,j}(\hat{\alpha}_{n,i} J_1 J_2 \cdots J_n | \}\hat{\alpha}_{n-1,j} J_r \hat{\bar{\alpha}}_{n-1,j} J_s) \\
&\qquad\qquad \times [|\hat{\tilde{\Psi}}^{n-1}_{j,J_r}(\hat{\alpha}_{n-1,j} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\hat{\tilde{\Phi}}^0_{J_s}(\hat{\bar{\alpha}}_{n-1,j} J_s)\rangle]^J_M \\
&\quad + \sum_{\substack{\hat{\alpha}_{n-1,i} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} \\ J_r J_s}}^{\vartheta_5^n} {}^{2n+1}\hat{P}_{i,J}(\hat{\alpha}_{n,i} J_1 J_2 \cdots J_n | \}\hat{\alpha}_{n-1,i} J_r \hat{\bar{\alpha}}_{n-1,i} J_s) \\
&\qquad\qquad \times [|\tilde{\hat{\Phi}}^{n-1}_{i,J_r}(\hat{\alpha}_{n-1,i} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\tilde{\Phi}^0_{J_s}(\hat{\bar{\alpha}}_{n-1,i} J_s)\rangle]^J_M \ , \qquad (3.8)
\end{aligned}
$$

with $i = 1, \ldots, n_x$ [number of (h-p)-core pairs]; $j = i-1$, where ϑ_4^n and ϑ_5^n give the number of the different partitions we can form with the $\{\hat{\alpha}_{n,i}\}$ coordinates and where

$$
\hat{\tilde{\Psi}}^{n-1}_{j,J_r}(\hat{\alpha}_{n-1,j} J_{k_1} J_{k_2} \cdots J_{k_{n-1}}) = \begin{cases} \tilde{\Phi}^{n-1}_{J_r}(\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}}) & \text{for } i = 1 \ , \\ \hat{\tilde{\Phi}}^{n-1}_{j,J_r}(\hat{\alpha}_{n-1,j} J_{k_1} J_{k_2} \cdots J_{k_{n-1}}) & \text{for } 2 \le i \le n_x \ . \end{cases}
$$

The transformation coefficients are calculated in terms of the matrix elements of the unit tensor operators

$$
\tilde{\tilde{\mathcal{U}}}^k(n) = \tilde{\tilde{u}}^k(n) + \hat{\tilde{\tilde{u}}}^k(n) = [\bar{U}^\dagger_{\bar{\alpha}_{n,i}}(J_i) \otimes \bar{U}_{\bar{\alpha}_{n,i}}(J_i')]^k + [\hat{\bar{U}}^\dagger_{\hat{\bar{\alpha}}_{n,i}}(J_i) \otimes \hat{\bar{U}}_{\hat{\bar{\alpha}}_{n,i}}(J_i')]^k \ , \qquad (3.9)
$$

where $\bar{U}_{\bar{\alpha}_{n,i}}(J_i')$ and $\hat{\bar{U}}_{\hat{\bar{\alpha}}_{n,i}}(J_i')$ destroy a (particle-hole) and a (hole-particle) pair, respectively.

The expressions we get for the transformation coefficients of Eq. (3.8) are of the form given in Eq. (3.5). The same holds for the transformation coefficients of adjoint CMWF's.

D. Transformation coefficients for $(n+1)$ particles and the $(n+1)$ holes states coupled with (particle-hole)-core pairs

The CMWF's for $(n+1)$ particles or $(n+1)$ holes coupled with the (particle-hole)-core pairs are expanded according to the number of active (p-h)-core pairs $\{m_x\}$. We distinguish between three different subcases for which we write the expansions. (a) For the $m_x=0$ subcase,

$$|\Lambda^n_{JM}(\eta_n J_1 J_2 \cdots J_n)\rangle = \sum_{\eta_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}} J_r J_s} {}^{2n+1}W_J(\eta_n J_1 J_2 \cdots J_n | \} \eta_{n-1} J_r \bar{\eta}_{n-1} J_s)[|\Lambda^{n-1}_{J_r}(\eta_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\Lambda^0_{J_s}(\bar{\eta}_{n-1} J_s)\rangle]^J_M \,. \tag{3.10}$$

(b) For the $m_x=1$ subcase,

$$|\Lambda^n_{JM}(\eta_n J_1 J_2 \cdots J_n)\rangle = \sum_{\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}}}^{\vartheta^n_6} {}^{2n+1}U_J(\eta_n J_1 J_2 \cdots J_n | \} \alpha_{n-1} J_r \bar{\alpha}_{n-1} J_s)[|\Phi^{n-1}_{J_r}(\alpha_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\Lambda^0_{J_s}(\bar{\alpha}_{n-1} J_s)\rangle]^J_M + \sum_{\eta_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}}}^{\vartheta^n_7} {}^{2n+1}V_J(\eta_n J_1 J_2 \cdots J_n | \} \eta_{n-1} J_r \bar{\eta}_{n-1} J_s)[|\Lambda^{n-1}_{J_r}(\eta_{n-1} J_{k_1} J_{k_2} \cdots J_{k_{n-1}})\rangle \otimes |\Phi_{J_s}(\bar{\eta}_{n-1} J_s)\rangle]^J_M \,, \tag{3.11}$$

where ϑ^n_6 and ϑ^n_7 give the number of the allowed partitions. (c) For the $m_x \geq 2$ subcase, the expansion is that of Eq. (3.1) and is, therefore, not given explicitly.

The expansion coefficients

$${}^{2n+1}W_J(\eta_n J_1 J_2 \cdots J_n | \} \eta_{n-1} J_r \bar{\eta}_{n-1} J_s)$$

of subcase (a) are a generalization of the $\{n+1\} \rightarrow \{n-1\}$ coefficients of fractional percentage (CFP) elaborated in Ref. [14] for the special case of $\{n+1\}$ particles in the same $\{j\}$ shell. Analogous expansions hold for CMWF's with $(n+1)$ holes coupled with (particle-hole)-core pairs, as well as for CMWF's with $(n+1)$ particles or $(n+1)$ holes coupled with n_x (hole-particle) pairs.

IV. APPLICATION TO SUPERALLOWED BETA DECAY

In this section we apply successively the linearization approximations (aa) and (bb), which lead to a superconductive system of linear equations to describe dynamic valence particle systems, to the calculation of the superallowed beta decay of ^{9}Li to levels in ^{9}Be.

In Table I we present a comparison of the experimental feature of the Gammow-Teller beta decay of the ground state of ^{9}Li into the $J=\frac{3}{2}^-$, $T=\frac{1}{2}$ state of ^{9}Be with the prediction of the calculation done introducing step by step the linearization approximations (aa) and (bb). The value of the superallowed Gammow-Teller matrix elements is strongly increasing with the inclusion in the model of the superconductive ISVS's. From the value 2.8 obtained coupling of the valence particle with only (particle-hole) pairs coupled to normal parity states, we calculate a small increase to 2.99 with the additional consideration of pairs coupled to mesons (non-normal parity states). To approach the experimental value, we have, however, to include in the model the CMWF components resulting from the coupling of the valence particle with pair of particles coupled to $J \neq 0$. This is the result (3) of Table I. The experimental value has been performed by the Isolde collaboration [21].

Three types of two-body matrix elements appear in the calculation leading to diagonalization of eigenvalue matrices of the order of 300×300: (1) polarization matrix elements, (2) particle-hole matrix elements, and (3) particle-particle matrix elements. The analytical form of the particle-hole matrix elements and their parameters are those of Ref. [22].

The particle-particle matrix elements are calculated with the effective interaction of Ref. [23], which contains a strong tensor component. Matrix elements calculated with this interaction in the mass-16 system agree remarkably well with those quoted in Ref. [24], which we obtained from a solution of the Bethe-Golstone equation using the Paris potential.

TABLE I. Comparison of the experimental Gammow-Teller beta decay with the theoretical results obtained applying successively the different linearization approximations (aa) and (bb).

	B(GT)	
1	2.80	pure shell model
2	2.99	shell model+meson contributions
3	4.09	shell model+meson contributions +superconductive diagrams
Expt. [22]	5.6±1.2	

V. COLLECTIVE QUASIFERMIONS

In the previous sections, the commutator chain for the dynamic evolution of interacting particles has been reduced to a finite system of equations restoring the vacuum symmetries within the linearization approximations (aa) and (bb).

According to Refs. [2,25], the corresponding eigenvectors are interpreted as describing collective quasifermion states, eigenvectors of the collective Hamiltonian

$$H_{\text{coll}}=\sum_i E_i F^{\dagger}_{i,J}(\delta_n J_1 J_2 \cdots J_n) F_{i,J}(\delta_n J_1 J_2 \cdots J_n) , \tag{5.1}$$

where the operators

$$F^{\dagger}_{i,J}(\delta_n J_1 J_2 \cdots J_n)|0\rangle = [\, \xi^0_{\sigma_0}\Theta^{\dagger}_0(\sigma_0;J)+\xi^1_{\sigma_1 J_1 J}N^1_{\sigma_1 J_1 J}\Theta^{\dagger}_1(\sigma_1 J_1;J)+\cdots + \xi^n_{\sigma_n J_1 J_2 \cdots J_n J}\lambda^n_{\sigma_n J_1 J_2 \cdots J_n J}\Theta^{\dagger}_n(\sigma_n J_1 J_2 \cdots J_n;J)\,]|0\rangle \tag{5.2}$$

and

$$\Theta^{\dagger}_0(\sigma_0;J)=\begin{bmatrix} A^{\dagger}_0(\alpha_0;J) \\ A_0(\rho_0;J)\end{bmatrix} ,$$

$$\Theta^{\dagger}_n(\sigma_n J_1 J_2 \cdots J_n;J)=\begin{bmatrix} P^{\dagger}_n(\epsilon_n J_1 J_2 \cdots J_n J) \\ P_n(\vartheta_n J_1 J_2 \cdots J_n;J)\end{bmatrix}$$

create quasiparticle states of the nth kind. Considering that the states created by the $F^{\dagger}_{i,J}(\delta_n J_1 J_2 \cdots J_n)$ operators form an orthonormal basis, we can prove that their commutator with the collective Hamiltonian H_{coll} generates a system of equations similar to those linearized.

To extend the validity of the equation of motion method to the high-energy domain, we define an energy-dependent linearization approximation (see Sec. II) which consists in restoring vacuum symmetries within few ISVS's, and show that the higher-order terms, neglected by the linearization method, form four-, six-, etc., point interactions acting between the (5.2) states. We discuss the approximation only for the direct components, assuming that the model space is dominated by the (3q.p.) excitations and that the (5q.p.) states have been linearized to contribute to the formation of the mean-field potential for the collective quasifermions. The higher-order (7q.p.) and (9q.p.) terms are included in the collective Hamiltonian as four- and six-point interactions. This can be proved by writing the direct components of the (9q.p.) states in terms of sums with the operator structure

$$\Phi^1_{J_s'}(\bar{\alpha}_1' J_{k_4}')\Phi^1_{J_s''}(\bar{\alpha}_1'' J_{k_4}'')\tilde{\Psi}^1_{J_s'''}(\bar{\rho}_1''' J_{k_4}''') .$$

The (7q.p.) and the other quasiparticle components also can be cast as products of CMWF's defined by the linearizations.

In general, the terms neglected in linearizing the system within the (3q.p.) generate an interaction of the collective quasifermions, and the collective Hamiltonian (5.1) takes the form

$$H_{\text{coll}}=\sum_i E_i F^{\dagger}_{i,J}(\delta_n J_1 J_2 \cdots J_n) F_{i,J}(\delta_n J_1 J_2 \cdots J_n) + \tfrac{1}{2}\sum_{ijlk} W_{ijkl}(\delta_n \delta_n' \delta_n'' \delta_n''') F^{\dagger}_{i,J}(\delta_n J_1 J_2 \cdots J_n) F^{\dagger}_{j,J}(\delta_n' J_1' J_2' \cdots J_n') F_{l,J}(\delta_n'' J_1'' J_2'' \cdots J_n'') F_{k,J}(\delta_n''' J_1''' J_2''' \cdots J_n''') .$$

The validity of the introduced energy-dependent linearization can now be extended, without further complication, to components characterized by a higher degree of complexity.

VI. CONCLUSION

In this paper we investigated the dynamic evolution of interacting particles under the assumption that vacuum symmetries are spontaneously broken. Vacuum symmetry breaking terms of superconductive character are explicitly included in the model space. The symmetries are restored within the n-pair vacuum excitations within the energy-dependent linearization approximations (aa) and (bb), which define systems of eigenvalue equations. The low-energy spectrum is characterized by free moving collective quasifermions, eigenvalues of the derived collective Hamiltonian. The model eigenvalue equations are solvable in terms of recursive expansions that relate the matrix elements of the one- and two-body operators in CMWF's of the $\{n\}$th kind to those of the $\{n-1\}$th kind. The transformation properties of CMWF's under special unitary groups enable us to derive the transformation coefficients of the recursive expansions. In the high-energy spectrum, the collective quasifermions interact via a four- and six-point interactions. The formalism is valid for a realistic two-body potential.

The breaking of pair symmetries corresponds to the in-

troduction of the massive Golstone bosons in a "fully" linearized theory (first-order linearization).

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APPENDIX

ISVS's for one valence particle. (1) First-order linearization of types (aa) and (bb):

$$|\Phi^0_{j_1}(j_1)\rangle = [X^0_{j_1} a^+_{j_1} + Y^0_{j_1} a_{j_1}]|0\rangle$$
$$= [X^0_{\alpha_0} a^+_{\alpha_0} + Y^0_{\rho_0} a_{\rho_0}]|0\rangle \ . \quad \text{(A1)}$$

The amplitudes $X^0_{\alpha_0}$ and $Y^0_{\rho_0}$ are the eigenvectors of the secular equation

$$\begin{vmatrix} \epsilon_{\alpha_0}+\Omega_1 & \Xi_1 \\ \tilde{\Xi}_1 & \tilde{\epsilon}_{\rho_0}+\tilde{\Omega}_1 \end{vmatrix} \begin{bmatrix} X^0_{\alpha_0} \\ Y^0_{\rho_0} \end{bmatrix} = 0 \ ,$$

with

$$\Omega_1 = \langle A_0(\alpha_0;j_1)\|H\|A^\dagger_0(\alpha_0;j_1)\rangle \ ,$$
$$\tilde{\Omega}_1 = \langle A^\dagger_0(\rho_0;j_1)\|H\|A_0(\rho_0;j_1)\rangle \ ,$$

and

$$\Xi_1 = \langle 0\|V\|A^\dagger_0(\alpha_0;j_1)A^\dagger_0(\rho_0;j_1)\rangle \ ,$$
$$\tilde{\Xi}_1 = \langle A_0(\rho_0;j_1)A_0(\alpha_0;j_1)\|V\|0\rangle \ .$$

(2) Second-order linearization of types (aa) and (bb) without the (A1) terms:

$$\begin{aligned}|\Phi^1_J(j_1(j_2j_3)J_1;JM)\rangle &= X^1_{j_1j_2j_3J_1J}[a^\dagger_{j_1}\otimes(a^\dagger_{j_2}\otimes a_{j_3})^{J_1}]^J + \chi^1_{1,j_1j_2j_3J_1J}[a^\dagger_{j_1}\otimes(a^\dagger_{j_3}\otimes a_{j_2})^{J_1}]^J \\ &+ Y^1_{j_1j_2j_3J_1J}[a_{j_1}\otimes(a^\dagger_{j_2}\otimes a_{j_3})^{J_1}]^J + \mathcal{Y}^1_{1,j_1j_2j_3J_1J}[a_{j_1}\otimes(a^\dagger_{j_3}\otimes a_{j_2})^{J_1}]^J \\ &+ Z^1_{j_1j_2j_3J_1J}[a^\dagger_{j_1}\otimes(a^\dagger_{j_2}\otimes a^\dagger_{j_3})^{J_1}]^J + W^1_{j_1j_2j_3J_1J}[a_{j_1}\otimes(a_{j_2}\otimes a_{j_3})^{J_1}]^J \\ &= [X^1_{\alpha_1J_1J}A^\dagger_1(\alpha_1J_1J) + \chi^1_{1,\hat{\alpha}_{1,1}J_1J}G^\dagger_{1,1}(\hat{\alpha}_{1,1}J_1J) \\ &+ Y^1_{\rho_1J_1J}A_1(\rho_1J_1J) + \mathcal{Y}^1_{1,\hat{\rho}_{1,1}J_1J}G_{1,1}(\hat{\rho}_{1,1}J_1J) + Z^1_{\eta_1J_1J}B^\dagger_1(\eta_1J_1J) + W^1_{\mu_1J_1J}B_1(\mu_1J_1J)]|0\rangle \ . \quad \text{(A2)}\end{aligned}$$

The amplitudes $X^1_{\alpha_1J_1J}$, $\chi^1_{1,\hat{\alpha}_{1,1}J_1J}$, $Y^1_{\rho_1J_1J}$, $\mathcal{Y}^1_{1,\hat{\rho}_{1,1}J_1J}$, $Z^1_{\eta_1J_1J}$, and $W^1_{\mu_1J_1J}$ are the eigenvectors of the secular equation

$$\begin{vmatrix} E_{\text{2p-1h}}+\Omega_1 & \Xi_1 & \Xi_2 & \Xi_3 & \Xi_4 & \Xi_5 \\ & \tilde{E}_{\text{2p-1h}}+\tilde{\Omega}_1 & \tilde{\Xi}_2 & \tilde{\Xi}_3 & \tilde{\Xi}_4 & \tilde{\Xi}_5 \\ & & E_{\text{2h-1p}}+\Omega_2 & \Sigma_3 & \Sigma_4 & \Sigma_5 \\ & & & \tilde{E}_{\text{2h-1p}}+\tilde{\Omega}_2 & \tilde{\Sigma}_4 & \tilde{\Sigma}_5 \\ & & & & E_{\text{3p}}+\Omega_3 & S_5 \\ & & & & & E_{\text{3h}}+\tilde{\Omega}_3 \end{vmatrix} \begin{bmatrix} X^1_{\alpha_1J_1J} \\ \chi^1_{1,\hat{\alpha}_{1,1}J_1J} \\ Y^1_{\rho_1J_1J} \\ \mathcal{Y}^1_{1,\hat{\rho}_{1,1}J_1J} \\ Z^1_{\eta_1J_1J} \\ W^1_{\mu_1J_1J} \end{bmatrix} = 0 \ , \quad \text{(A3)}$$

where

$$\Omega_1 = \langle A_1(\alpha'_1J'_1;J)\|H\|A^\dagger_1(\alpha_1J_1J)\rangle, \quad \tilde{\Omega}_1 = \langle G_{1,1}(\hat{\alpha}'_{1,1}J'_1;J)\|H\|G^\dagger_{1,1}(\hat{\alpha}_{1,1}J_1J)\rangle \ ,$$
$$\Omega_2 = \langle A^\dagger_1(\rho'_1J'_1;J)\|H\|A_1(\rho_1J_1;J)\rangle, \quad \tilde{\Omega}_2 = \langle G^\dagger_{1,1}(\hat{\rho}'_{1,1}J'_1;J)\|H\|G_{1,1}(\hat{\rho}_{1,1}J_1;J)\rangle \ ,$$
$$\Omega_3 = \langle B_1(\eta'_1J'_1;J)\|H\|B^\dagger_1(\eta_1J_1;J)\rangle, \quad \tilde{\Omega}_3 = \langle B^\dagger_1(\mu'_1J'_1;J)\|H\|B_1(\mu_1J_1;J)\rangle$$

are the diagonal matrix elements and where Ξ_i, $\tilde{\Xi}_i$, Σ_i, $\tilde{\Sigma}_i$, S_i, and $\tilde{S}_i$ are the corresponding off-diagonals. With the diagonalization of the matrix (A3), we define the superconductive collective fermions which, interacting, describe the higher part of the spectrum.

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