Photodisintegration of ^{4}He in the integrodifferential equation approach

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The photodisintegration of ^{4}He into $n+{}^3$He or $p+{}^3$H is calculated by employing the integrodifferential equation approach for the determination of the three- and four-body bound states involved, and by incorporating the final state interaction via an optical potential treatment. The results of this comparatively simple model reproduce the trends found within the integral equation approach, and are in a rather good agreement with the latest experimental data.

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I. INTRODUCTION

A reliable basis for the theoretical treatment of ^{4}He photodisintegration is provided by the exact set of integral equations [1–3] derived by Casel and Sandhas on the basis of the four-nucleon Alt-Grassberger-Sandhas formalism [4]. First applications of this technique [1,5] showed a considerable flattening of the apparent low-energy resonance peak seen in early measurements and indicated by shell model [6–8] and resonating group calculations [9]. Replacing the separable interactions used in Refs. [1] and [5] by the semirealistic Malfliet-Tjon (MT I+III) potential, an even more pronounced flattening was found in this energy region, consistent with a lowering of the theoretical values at higher energies [10]. Over the years a similar trend has been encountered experimentally [11] so that the latest data [12–15] are by now in almost perfect agreement with the theoretical results obtained for the MT I+III potential [16].

The complexity of the integral equation approach, on the other hand, makes its application so cumbersome that a full solution of the underlying equations could be performed up until now only below the three-fragment breakup threshold at 26.3 MeV. Beyond this threshold the plane wave approximation had to be employed, a step justified at higher energies, say above 50 MeV, but definitely not allowed in the intermediate region. Some further approximations, moreover, were used which one could avoid only by a rather demanding improvement of the present technique. A further shortcoming of the integral equation approach is its practical limitation to low particle numbers.

Under these circumstances it appears quite reasonable to study, beside the exact formalism, a model based on two simplifications. Instead of employing homogeneous integral equations for the determination of the three- and four-nucleon bound states involved, the much simpler techniques of the integrodifferential equation approach (IDEA) [17] are used (as in Ref. [10] the MT I+III potential is employed in these bound state calculations). The second simplification consists in taking into account the final state interaction (FSI) via an optical potential treatment of the relative movement of the outgoing clusters. In this way we avoid the highly demanding full solution of the four-body integral equations.

In Sec. II we present our formalism while in Sec. III we present our results and discussion.

II. FORMALISM

Before going into a more detailed discussion of approximation techniques and numerical results, it appears useful to recall some basic relations of the problem. The properly antisymmetrized amplitude for the photodisintegration of the ^{4}He bound state $|\psi_{\rm IV}\rangle$ into a three-body bound state $|\psi_{\rm III}\rangle$ and a nucleon of relative momentum $\mathbf{q}$ is given by

$$M^{\lambda}(\mathbf{q})=2^{(-)}\langle \mathbf{q};\psi_{\rm III}|H^{\lambda}_{\rm em}|\psi_{\rm IV}\rangle \ . \tag{1}$$

Here $|\mathbf{q};\psi_{\rm III}\rangle^{(-)}$ denotes the full outgoing scattering state associated with the channel state

$$|\mathbf{q};\psi_{\rm III}\rangle=|\mathbf{q}\rangle|\psi_{\rm III}\rangle \ . \tag{2}$$

In the plane wave (Born) approximation, the amplitude (1), therefore, reads

$$B^{\lambda}(\mathbf{q})=2\langle \mathbf{q}|\langle \psi_{\rm III}|H^{\lambda}_{\rm em}|\psi_{\rm IV}\rangle \ . \tag{3}$$

At low energies the process under consideration should take place primarily via an electric dipole transition. Thus, taking into account the exchange currents via Siegert's theorem [18,19], we are led to the following choice of the electromagnetic interaction

$$H^{\lambda}_{\rm em}=\frac{e}{2i\hbar c}E_{\gamma}\sum_{j=1}^{4}\hat{\epsilon}_{\lambda}\cdot\mathbf{x}_j(1+\tau_z^j) \ , \tag{4}$$

where E_γ represents the energy and $\hat{\epsilon}_\lambda$ the two polarization directions of the incident photon. This expression can be written as a sum of three terms, two of them acting within $|\psi_{\rm III}\rangle$, the third one depending on the coordinate $\mathbf{r}$ canonically conjugate to the relative momentum $\mathbf{q}$ between the nucleon and the center of mass of $|\psi_{\rm III}\rangle$. Since we employ an s-wave projected (Malfliet-Tjon) po-

tential, ψ_{III} is an s-wave state in the relative two-body variable $\xi_{ij}=\mathbf{x}_i-\mathbf{x}_j$ and, belonging to the orbital angular momentum L=0, it is an s wave also in the other Jacobi coordinate $\eta_k=\mathbf{x}_k-(\mathbf{x}_i+\mathbf{x}_j)/2$. For ψ_{IV} the same holds true with respect to ξ_{ij}, but in η_k this state has, in general, p-wave contributions, which, however, are not included in our IDEA construction. The two internal terms in the odd-parity operator H^{λ}_{em}, thus, vanish between $|\psi_{III}\rangle$ and $|\psi_{IV}\rangle$ so that H^{λ}_{em} reduces in Eq. (3) to

$$H^{\lambda}_{em}=\frac{e}{2i\hbar c}E_{\gamma}\hat{\epsilon}_{\lambda}\cdot\mathbf{r}\left[\frac{1}{3}(\tau^1_z+\tau^2_z+\tau^3_2)-\tau^4_z\right] . \tag{5}$$

Note that the computation of expression (3) with this interaction is most conveniently done for coordinate space wave functions as provided by the integrodifferential equation approach.

We finally recall that the cross section for an unpolarized incident photon beam is given by

$$\frac{d\sigma}{d\Omega}=\frac{\mu q}{2\pi\hbar^2 k_{\gamma}}\frac{1}{2}\sum_{\lambda=1}^{2}|M^{\lambda}(\mathbf{q})|^2 . \tag{6}$$

Here, μ denotes the reduced mass of the two outgoing fragments and k_{γ} is the photon momentum. If only the protons or the neutrons are counted experimentally, the above cross section has to be divided by two.

For the calculation of $|\psi_{III}\rangle$ and $|\psi_{IV}\rangle$ entering Eq. (3) and also the approximate full amplitude (13) considered below, the integrodifferential equation approach (IDEA) is used. The IDEA employs the potential harmonic basis, which is complete for two-body correlations generated by a two-body potential. Consequently, it only requires the solution of two-variable integrodifferential equations of the Faddeev type, not only for ^{3}H, but also for ^{4}He and larger nuclei.

In this method the A-nucleon wave function $\Psi(\mathbf{x})$ is written as a sum of amplitudes

$$\Psi(\mathbf{x})=\sum_{i<j\le A}F(\mathbf{r}_{ij},\rho)$$

obeying the Faddeev-type equations

$$\left[T+\frac{A(A-1)}{2}V_0(\rho)-E\right]F(\mathbf{r}_{ij},\rho)$$
$$=-[V(\mathbf{r}_{ij})-V_0(\rho)]\sum_{i<j\le A}F(\mathbf{r}_{ij},\rho) , \tag{7}$$

where ρ is the hyperradius and $V_0(\rho)$ the hypercentral potential representing the first term of the potential harmonic expansion of the interaction. The latter takes, to a good approximation, the correlations stemming from the coupling of the spectator particle's relative orbitals to those of the ij pair into account. T represents the A-body kinetic energy operator and $V(\mathbf{r}_{ij})$ the two-body potential which we take as the Malfliet-Tjon I-III potential in this paper. Equation (7) can be solved by projecting it into the r_{ij} space which results—in the case of a spin-dependent nucleon potential—in two coupled, two-variable integrodifferential equations:

$$\left[\frac{\hbar^2}{m}\Delta_0^2+\frac{A(A-1)}{2}V_0(r)+E\right]P^s_0(z,\rho)$$
$$=\left[\frac{V^{1+}+V^{3+}}{2}-V_0(\rho)\right]\Pi^s_0(z,\rho)$$
$$+\left[\frac{V^{1+}-V^{3+}}{2}\right]\Pi^{s'}_0(z,\rho) \tag{8}$$

and

$$\left[\frac{\hbar^2}{m}\Delta_0^2+\frac{A(A-1)}{2}V_0(r)+E\right]P^{s'}_0(z,\rho)$$
$$=\left[\frac{V^{1+}+V^{3+}}{2}-V_0(\rho)\right]\Pi^{s'}_0(z,\rho)$$
$$+\left[\frac{V^{1+}-V^{3+}}{2}\right]\Pi^{s}_0(z,\rho) , \tag{9}$$

where

$$\Pi^i(z,\rho)=P^i(z,\rho)+\int_{-1}^{+1}f_{(0)}(z,z')P^i(z',\rho)dz', \quad i=s,s' . \tag{10}$$

The Laplacian operator Δ_0^2 is given by

$$\Delta_0^2=\frac{\partial^2}{\partial\rho^2}-\frac{\mathcal{L}_0(\mathcal{L}_0+1)}{\rho^2}+\frac{4}{\rho^2}\frac{1}{W_0(z)}\frac{\partial}{\partial z}(1-z^2)W_0(z)\frac{\partial}{\partial z} , \tag{11}$$

where $\mathcal{L}_0=(D-3)/2$ and $D=3(A-1)$, while $z=2r_{ij}^2/\rho^2-1$. The weight function $W_0(z)$ is given by $W_0(z)=(1-z)^{\alpha}(1+z)^{1/2}$, with $\alpha=(D-5)/2$. The potentials V^{1+} and V^{3+} are the singlet and triplet even components of the Malfliet-Tjon potential, while $F(r_{ij},\rho)=\rho^{-(D-1)/2}P(z,\rho)$. This definition is employed to introduce the fully and mixed symmetric wave components $P^s(z,\rho)$ and $P^{s'}(z,\rho)$ respectively in Eqs. (8)–(10). The projection function $f_{[0]}(z,z')$, which is required to project Eq. (7) on the r_{ij} space, is defined in Ref. [17] where more details concerning the IDEA can be found.

It has recently been shown that the IDEA produces better results for ^{4}He than the extended shell model, even in its most sophisticated form, at a minute fraction of the computational effort [20]. Results obtained via the IDEA and the exact Faddeev-Yakubovsky equations, with Malfliet-Tjon forces like the ones we employ here, are in close agreement [21]. The same was found when comparing the IDEA with calculations based on more realistic forces and alternatives methods [22]. It is, therefore, not surprising that employing the IDEA does not lead to any noticeable deviations from the integral equation results. This will be verified by comparing the corresponding Born approximation results at higher energies where the FSI plays no role.

The most drastic, but particularly simplifying modification of our model approach consists in taking into account the FSI by replacing the plane wave $|\mathbf{q}\rangle$ in Eqs. (2) or (3) by scattering states $|\mathbf{q}\rangle^{(-)}$ generated by an optical n-^{3}He or p-^{3}H potential. In other words, instead

of solving a four-body integral equation providing us with the full scattering state $|\mathbf{q},\psi_{\rm III}\rangle^{(-)}$, we use the approximation

$$|\mathbf{q},\psi_{\rm III}\rangle^{(-)}\sim|\mathbf{q}\rangle^{(-)}|\psi_{\rm III}\rangle\ , \tag{12}$$

and thus

$$M^{\lambda}(\mathbf{q})\sim 2^{(-)}\langle\mathbf{q}|\langle\psi_{\rm III}|H^{\lambda}_{\rm em}|\psi_{\rm IV}\rangle\ . \tag{13}$$

The results obtained on this basis are, of course, expected to depend on the choice of the optical potential. In turn, this means that the present procedure should represent a sensitive tool for testing the quality of such potentials. In what follows we choose the conventional form

$$V_{\rm opt}(E,r)=-V_0(E)f(r,R_0,a_0)-iW_V(E)f(r,R_I,a_I) \tag{14}$$

with

$$f(f,R_x,a_x)=\left[1+\exp\left[\frac{r-R_x}{a_x}\right]\right]^{-1},\quad R_x=r_x A^{1/3} \tag{15}$$

employed successfully by Podmore and Sherif in interpreting the n-^{3}He and p-^{3}H collisions [23]. For different sets of parameters proposed by these authors we found rather large differences in the present photonuclear case which means that these different choices of $V_{\rm opt}$ cannot be phase equivalent. In fact, calculating the corresponding phase shifts directly, we got quite different results for $^{01}\delta_1$, i.e., for the p wave which is the only one relevant in $E1$ transitions.

This observation suggested a new fit of the parameters in Eqs. (14) and (15). We adjusted the potential depth as a function of energy, without varying the geometry parameters, to reproduce the p-wave phase shifts of Tombrello [24] over the whole energy region. Note that these phase shifts are consistent with the ones obtained in other analysis [25] and in the integral equation calculations by Tjon [26] and Fonseca [27]. The differences of the fit to the data are less than 0.1°.

III. RESULTS AND DISCUSSION

The (γ,p) and (γ,n) photodisintegration cross sections obtained within this approach are shown in Fig. 1. The (γ,p) results were obtained by including the Coulomb interaction to the scattering states $|\mathbf{q}\rangle^{(-)}$. We also mention that the experimental thresholds have been used in the calculations. Since for the MT I+III potential employed the three- and four-nucleon binding energies ($E_T=8.54$ MeV, $E_\alpha=29.74$ MeV) are close to the experimental values, this is a minor point in our case. The agreement with most recent ^{4}He$(\gamma,p)^3$H data [12,13] is remarkable. This is not only a rather satisfactory result in itself, but shows that the elastic n-^{3}He or p-^{3}H scattering and the corresponding photodisintegration processes are treated consistently in our model. We, moreover, note that the ratio $\sigma_T(\gamma,p)/\sigma_T(\gamma,n)$ is of the order of 1.05, i.e., no charge symmetry breaking is observed. This is in agreement with the latest (γ,p) data which are close to the (γ,n) data.

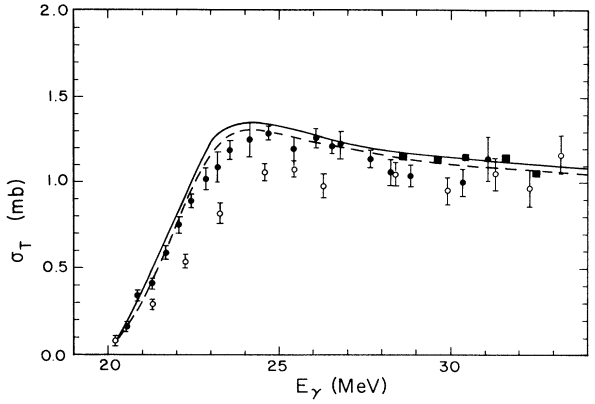

FIG. 1. Photodisintegration cross section obtained in our model for proton (——) and neutron (— — —). The data are from Ref. [12] ⊢ (p), Ref. [13] ■ (p), and Ref. [11] ⊢ (n).

It is quite instructive to see how sensitively the situation depends on the choice of the final state interaction. Adjusting the parameters in the optical potential [14] to the p-wave phase shifts of the resonating group method (RGM) calculations by Reichstein *et al.* [28], our approach yields the solid curve shown in Fig. 2, which agree with the direct RGM calculation by Wachter *et al.* [9] (dashed curve). Both these RGM calculations agree with early measurements (upper shaded area taken over from the compilation of Ref. [29]) but fail to reproduce the (γ,n) data of Ref. [29] (lower shaded area) and, in particular, the latest experimental (γ,p) results of Refs. [12] and [13]. It should be mentioned, however, that by fitting the optical model parameters to the phase shifts of Furutani *et al.* [29], obtained by means of the generator

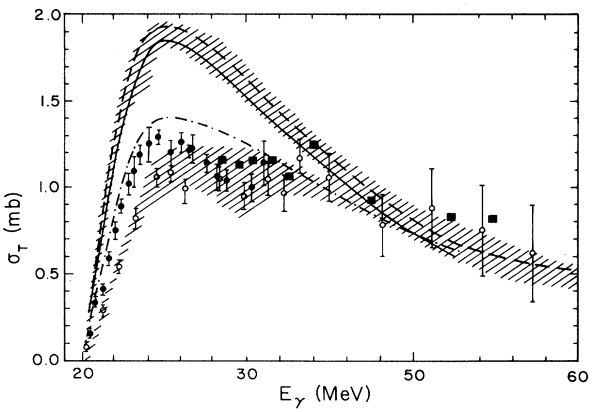

FIG. 2. Model treatment with optical potential parameters adjusted to the RGM phase shifts of Ref. [28] (——). Direct RGM results of Ref. [9] (— — —). Results for the optical potential adjusted to GCM phase shifts [30] (—·—·—·). The data are as in Fig. 1. Shaded areas are taken from the compilation of Ref. [29] and correspond to the (γ,p) (upper) and (γ,n) (lower) data.

coordinate method (GCM), a better agreement with the new data is achieved (dot-dashed curve).

These quite different results demonstrate most clearly the sensitivity of low-energy photodisintegration against the nuclear parameters, a sensitivity less pronounced in purely nuclear observables. This is due to the fact that only p waves enter the final state in our calculation of the photodisintegration amplitude, while a large number of partial waves play a role in purely nuclear cross sections.

At higher energies, say above 50 MeV, the final state corrections are found to be negligible. In other words, the Born approximation (3) appears to be fully justified. The only approximation in this region, thus, consists in determining the bound states $|\psi_{\rm III}\rangle$ and $|\psi_{\rm IV}\rangle$ by means of the integrodifferential equation approach (IDEA). This approximation is *a priori* expected to be of minor importance and indeed, comparing in Fig. 3 the Born results obtain in Ref. [10] via integral equation (dashed curve), with the corresponding Born results for the IDEA (solid curve), we see that both approaches provide practically identical results. Moreover, the agreement of these curves with the data of Ref. [14] and, in particular, of Ref. [15] is remarkable. Let us mention here that a preliminary presentation of these results appears in Refs. [31,32].

Summarizing, we conclude that both the integral equation technique and the above model lead to a very good agreement with recent experimental data, at least when using instead of simple separable interactions, the semirealistic MT I-III potential. In our model this agreement was achieved by choosing the optical potential parameters in accordance with Tombrello's p-wave phase shifts, which in turn are consistent with microscopic Faddeev-type calculations. In this sense the present approach leads not only to quite a satisfactory result, but reflects consistency between our nuclear and photonuclear observables.

IV. AN ADDITIONAL COMMENT

After submission of the present investigations, a paper by Unkelbach and Hoffman has appeared [33], in which the RGM results by Wachter *et al.* [9] are adjusted to the new experimental data. This is accomplished by replacing the experimental photon energy $E_\gamma = E_\gamma^{\rm ex}$, chosen in the previous calculations of the Siegert operator [cf. Eq. (5)], by a model energy E_γ^m consistent with the RGM binding energies. Due to the large deviations of the RGM energies from their real values, this replacement implies, in fact, a considerable modification of the energy dependence of the amplitude, leading from the former resonance shape to the recent flat behavior. Alternatively, Unkelbach and Hofmann show that a similar modification is achieved by changing the electromagnetic operator, but choosing, as in the previous calculations, the experimental photon energy $E_\gamma^{\rm ex}$ instead of E_γ^m. The

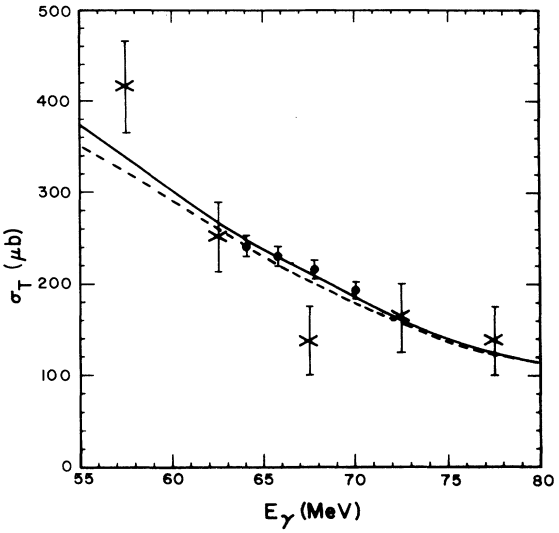

FIG. 3. Born results obtained by means of the IDEA (——) and the integral equation approach (— — —). The data are from Refs. [14] (×××) and [15] (•••).

RGM approach of Refs. [9] and [33], thus, displays a high ambiguity, which has been exploited in these papers to fit first the old and now the new data.

Let us emphasize that an ambiguity of this type did occur neither in our previous integral equation treatment [10] nor in the present integrodifferential equation approach. Instead of the rather unrealistic Gaussian potentials, to which the RGM calculations of Refs. [9] and [33] were restricted, the semirealistic MT I+III potential has been employed in these investigations, which, as discussed in Sec. III, provides binding energies close to the experimental values (note that this well-known fact contradicts the statements in Sec. 4.1 of Ref. [33]). The photon energy E_γ^m consistent with these theoretical values, therefore, differs only marginally from $E_\gamma^{\rm ex}$. It was just this important feature which suggested the choice of the MT I+III potential rather than, in other respects more realistic, interactions. Proceeding in this way we, therefore, had no freedom to adjust the theoretical results to the data. The integral equation approach and also the present model, on the contrary, predicted unambiguously the flat nonresonant behavior found in almost all recent measurements.

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