Coupling between rotational and vibrational motions with the cranking Bohr-Mottelson Hamiltonian

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A cranking Bohr-Mottelson Hamiltonian is proposed and a formula for the rotational spectrum is derived, showing explicitly the coupling between rotational and vibrational motions. The formula is applied to the rotational bands of even-even nuclei in the rare-earth and actinide regions. The ground-state rotational band (below backbending) and the associated β and γ bands can be represented by a single formula. Nuclear structure information on the parameters is discussed.

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I. INTRODUCTION

Nuclear rotational spectra have been well studied both theoretically and experimentally. Spectra up to high spin values have been measured with high precision for many nuclei. Since the moments of inertia of the actinide nuclei are about twice those of the rare-earth nuclei, the two-quasiparticle S band does not compete with the ground-state band until much higher spins [1]. Therefore, the yrast levels with even spin and parity in the actinide nuclei may belong to the ground-state bands to higher spins than those in the rare-earth nuclei. Thus, the recent data provide an ideal opportunity to test the applicability of various models for rotational spectra.

Theoretically, both algebraic and geometric methods have been adopted to analyze the spectra. For the algebraic approach, various forms of interacting boson models [2,3] were applied. The models are good at the general representation of the spectra and can be extended to high spin values with suitable attention to boson numbers, but detailed numerical precision is not expected and it is rather difficult to connect the results of analysis with nuclear structure information

In the geometrical approach, most detailed studies have been done by Kumar and co-workers [4–7], with the Bohr-Mottelson (BM) Hamiltonian. The successes of his works show that the BM Hamiltonian can give an adequate description of the collective spectra of all kinds of even-even nuclei. However, his works also show that it will be rather difficult to obtain a quantitative description of the rotational spectra to higher spin values, with calculated energy surface and mass parameters.

Semiempirical approaches may be more suitable to give a quantitative description of rotational spectra and to extract nuclear structure information from the spectra. With a somewhat artificial potential energy term and other simplifications, Wu and Zeng [8,9] have derived a two-parameter formula for the rotational spectra of well-deformed nuclei. The formula is quite successful in the quantitative representation of the spectra, but the physical significances of the parameters are not self-evident. Even for intimately connected spectra, such as the ground-state band (g band), the β and γ bands of a given nucleus, the parameters must be determined separately. Empirical expansion formulas, such as expansions in powers of ω^2 [10] or $I(I+1)$ [11], can be applied to give a good quantitative account of the rotational spectrum, but little nuclear structure information can be extracted.

In this work, the interaction between rotational and vibrational motions will be studied with the cranking BM Hamiltonian proposed by us. A brief sketch of the formulation and the physical meaning of various terms will be given in the next section. Applications of the formula to rotational spectra of even-even nuclei will be presented in the third section, with special attention to the unified presentation of the g, β, and γ bands. Numerical applications will show that the formula in Sec. II gives, for most cases, a very good quantitative representation of the rotational spectra up to high spin values (below backbending). In the last section, a brief discussion in connection with nuclear structure information will be given.

II. FORMULATION

In the cranked shell model [12], the Hamiltonian in the rotational frame of reference is given by

$$H' = H(a_0, a_2) - \boldsymbol{\omega}\cdot\mathbf{J} \ . \tag{1}$$

The shell-model Hamiltonian with quadrupole deformation may be written as

$$H(a_0,a_2) = \sum_{i=1}^{N} \{ t_i + v_i(r_i) + f_i(r_i)[a_0 Y_{20}(\theta_i,\varphi_i) + a_2 Y_{22}(\theta_i,\varphi_i) + a_2 Y_{2-2}(\theta_i,\varphi_i)] \} \ . \tag{2}$$

Considering a_0 and a_2 as slowly varying parameters, and treating the problem in the adiabatic approximation, one obtains

$$\left[H(a_0,a_2)-\omega\cdot\mathbf{J}-i\hbar\left[\dot{a}_0\frac{\partial}{\partial a_0}+\dot{a}_2\frac{\partial}{\partial a_2}\right]\right]\Phi=\Lambda\Phi\ . \quad (3)$$

Let

$$U=-\omega\cdot\mathbf{J}-i\hbar\left[\dot{a}_0\frac{\partial}{\partial a_0}+\dot{a}_2\frac{\partial}{\partial a_2}\right] \quad (4)$$

and, treated as perturbation, we have

$$\Phi=\psi_0(a_0,a_2)-\sum_{n\neq 0}\frac{\langle\psi_n|U|\psi_0\rangle}{E_n-E_0}\psi_n(a_0,a_2) \quad (5)$$

and

$$\Lambda=E_0(a_0,a_2)+\langle\psi_0|U|\psi_0\rangle-\sum_{n\neq 0}\frac{|\langle\psi_n|U|\psi_0\rangle|^2}{E_n-E_0}\ , \quad (6)$$

where $\psi_n(a_0,a_2)$ is the solution of the unperturbed Schrödinger equation

$$H(a_0,a_2)\psi_n(a_0,a_2)=E_n(a_0,a_2)\psi_n(a_0,a_2)\ . \quad (7)$$

Hence, the energy in the rotating frame of reference is given by

$$\begin{aligned}E'&=\langle\Phi|H'|\Phi\rangle\\&=\Lambda+i\hbar\left\langle\Phi\left|\left[\dot{a}_0\frac{\partial}{\partial a_0}+\dot{a}_2\frac{\partial}{\partial a_2}\right]\right|\Phi\right\rangle\ .\end{aligned} \quad (8)$$

Making use of the symmetrical properties of $H(a_0,a_2)$ and the relation

$$\langle\psi_0|(UH-HU)|\psi_n\rangle=(E_n-E_0)\langle\psi_0|U|\psi_n\rangle\ , \quad (9)$$

formula (8) can be written in the simplified form

$$\begin{aligned}E'=&\tfrac{1}{2}B_0\dot{a}_0^2+B_2\dot{a}_2^2+B_{02}\dot{a}_0\dot{a}_2-\tfrac{1}{2}B_x(\sqrt{3}a_0+\sqrt{2}a_2)^2\omega_x^2\\&-\tfrac{1}{2}B_y(\sqrt{3}a_0-\sqrt{2}a_2)^2\omega_y^2-4B_z a_2^2\omega_z^2+E_0(a_0,a_2)\ ,\end{aligned} \quad (10)$$

where B_0, B_2, B_{02}, B_x, B_y, and B_z are the mass parameters. In the lowly excited collective states, the values of a_0 and a_2 are kept near the ground-state deformations $\bar{a}_0$ and $\bar{a}_2$. For axisymmetrical nuclei, $\bar{a}_2=0$. In the expressions of the mass parameters, we shall use approximately the axisymmetrical solution of Eq. (7) with $a_0=\bar{a}_0$ and $a_2=\bar{a}_2=0$. Hence, for the axisymmetrically deformed nuclei, we have

$$B_{02}=0\ ,\quad B_x=B_y=B_1\ ,\quad B_z=B_2\ .$$

The three mass parameters B_1, B_0, and B_2 are now given by

$$B_k=2\hbar^2\sum_{n\neq 0}\frac{|\langle\psi_0(\bar{a}_0,0)|\sum_i f_i(r_i)Y_{2k}(\theta_i,\varphi_i)|\psi_n(\bar{a}_0,0)\rangle|^2}{[E_n(\bar{a}_0,0)-E_0(\bar{a}_0,0)]^3} \quad (k=0,1,2)\ . \quad (11)$$

In the axisymmetrically deformed nuclei, we assume that ω is perpendicular to the symmetrical axis, the z axis. There is no physical significance to distinguish the x axis and y axis, hence the average values $\omega_x=\omega_y=\omega/\sqrt{2}$ are inserted in Eq. (10), which gives, after quantization of a_0 and a_2, the Hamiltonian of collective motion in the rotating frame

$$\begin{aligned}H'=&-\frac{\hbar^2}{2B_0}\frac{\partial^2}{\partial a_0^2}-\frac{\hbar^2}{4B_2}\frac{\partial^2}{\partial a_2^2}-\frac{1}{2}B_1(3a_0^2+2a_2^2)\omega^2\\&+E_0(a_0,a_2)\ .\end{aligned} \quad (12)$$

For well-deformed nuclei, it is reasonable to assume the deformation dependent part of $E_0(a_0,a_2)$ as [6]

$$V(a_0,a_2)=\tfrac{1}{2}C_0(a_0-\bar{a}_0)^2+\tfrac{1}{2}C_2a_2^2\ . \quad (13)$$

The eigenequation of collective motion is

$$H'|n_\beta,n_\gamma\rangle=E'(n_\beta,n_\gamma)|n_\beta,n_\gamma\rangle\ , \quad (14)$$

with the eigenvalue given by

$$\begin{aligned}E'(n_\beta,n_\gamma)=&\left[n_\beta+\frac{1}{2}\right]\hbar\omega_\beta\left[1-3\frac{B_1}{B_0}\frac{\omega^2}{\omega_\beta^2}\right]^{1/2}\\&+\left[n_\gamma+\frac{1}{2}\right]\hbar\omega_\gamma\left[1-\frac{B_1}{B_2}\frac{\omega^2}{\omega\gamma^2}\right]^{1/2}\\&-\frac{3}{2}B_1\bar{a}_0^2\omega^2\Big/\left[1-3\frac{B_1}{B_0}\frac{\omega^2}{\omega_\beta^2}\right]\ ,\end{aligned} \quad (15)$$

where $\omega_\beta=\sqrt{C_0/B_0}$ and $\omega_\gamma=\sqrt{C_2/2B_2}$ are the frequencies of β and γ vibrations, and n_β and n_γ are the corresponding quantum numbers. The eigenvalue wave function can be written as

$$|n_\beta,n_\gamma\rangle=f_{n_\beta}(\alpha_0(a_0-\bar{a}_0'))f_{n_\gamma}(\alpha_2a_2)\ , \quad (16)$$

where

$$\bar{a}_0'=\bar{a}_0\Big/\left[1-3\frac{B_1\omega^2}{B_0\omega_\beta^2}\right]\ ,$$

$$\alpha_0^2=\frac{B_0\omega_\beta}{\hbar}\left[1-3\frac{B_1\omega^2}{B_0\omega_\beta^2}\right]^{1/2}\ ,$$

$$\alpha_2^2=\frac{2B_2\omega_\gamma}{\hbar}\left[1-\frac{B_1\omega^2}{B_2\omega_\gamma^2}\right]^{1/2}\ ,$$

and $f_n(\alpha x)$ is the normalized eigenvalue wave function of the one-dimensional harmonic oscillator.

According to the cranked shell model [12], the energy in the laboratory frame of reference is give by

$$\begin{aligned}E&=\langle\Phi|H(a_0,a_2)|\Phi\rangle\\&=\langle\Phi|H'+\omega\cdot\mathbf{J}|\Phi\rangle\ .\end{aligned} \quad (17)$$

Using formulas (5), (15), and (16), one obtains

$$E=E'(n_\beta,n_\gamma)+B_1\omega^2\langle n_\beta,n_\gamma|3a_0^2+2a_2^2|n_\beta,n_\gamma\rangle$$
$$=\left[n_\beta+\frac{1}{2}\right]\hbar\omega_\beta\left[1-3\frac{B_1}{B_0}\frac{\omega^2}{\omega_\beta^2}\right]^{-1/2}+\left[n_\gamma+\frac{1}{2}\right]\hbar\omega_\gamma\left[1-\frac{B_1}{B_2}\frac{\omega^2}{\omega_\gamma^2}\right]^{-1/2}$$
$$+\frac{3}{2}B_1\bar{a}_0^2\omega^2\left[1+3\frac{B_1}{B_0}\frac{\omega^2}{\omega_\beta^2}\right]\left[1-3\frac{B_1}{B_0}\frac{\omega^2}{\omega_\beta^2}\right]^{-2}. \tag{18}$$

The physical meaning of the above formula is quite transparent, the first two terms give the β and γ vibrational energies, modified by the rotational motion, and the third term is the rotational energy, with moment of inertia

$$\mathcal{J}=3B_1\bar{a}_0^2\left[1+3\frac{B_1}{B_0}\frac{\omega^2}{\omega_\beta^2}\right]\left[1-3\frac{B_1}{B_0}\frac{\omega^2}{\omega_\beta^2}\right]^{-2}, \tag{19}$$

which changes with ω due to the coupling with β vibration. In the cranked shell model [12], the average angular momentum in the direction of ω, $\langle J_\omega\rangle$, is given by

$$\langle J_\omega\rangle=-\frac{1}{\hbar}\frac{\partial E'(n_\beta,n_\gamma)}{\partial\omega}$$
$$=\left[n_\beta+\frac{1}{2}\right]3\frac{B_1}{B_0}\frac{\omega}{\omega_\beta}\left[1-3\frac{B_1}{B_0}\frac{\omega^2}{\omega_\beta^2}\right]^{-1/2}+\left[n_\gamma+\frac{1}{2}\right]\frac{B_1}{B_2}\frac{\omega}{\omega_\gamma}\left[1-\frac{B_1}{B_2}\frac{\omega^2}{\omega_\gamma^2}\right]^{-1/2}+\frac{1}{\hbar}3B_1\bar{a}_0^2\omega\left[1-3\frac{B_1}{B_0}\frac{\omega^2}{\omega_\beta^2}\right]^{-2}, \tag{20}$$

where $\langle J_\omega\rangle$ is connected to the quantum numbers I and K by

$$\langle J_\omega\rangle=\sqrt{I(I+1)-K^2}. \tag{21}$$

The above derivation remains unchanged if pairing interaction is included in the shell-model Hamiltonian.

III. APPLICATION TO THE ROTATIONAL BANDS OF WELL-DEFORMED EVEN-EVEN NUCLEI

Formulas (18) and (20) can be used jointly to determine the energy spectra of rotational bands. They are most suitable for application to g, β, and γ bands. In this case, $\hbar\omega_\beta$ can be identified as the bandhead energy of the β band. For the γ band, the bandhead energy can be calculated from Eqs. (18) and (20), and the value of $\hbar\omega_\gamma$ can be determined by identifying the empirical γ bandhead energy with the calculated value. In this way, one obtains, to a good approximation, the value of $\hbar\omega_\gamma$ equal to the empirical γ bandhead energy minus the expression $\hbar^2/(3B_1\bar{a}_0^2)$ which amounts to a few percent of the γ bandhead energy. We are left with three parameters B_0/B_1, B_2/B_1, and $B_1\bar{a}_0^2$ to be determined by fitting rotational spectra. It is found that the quality of the fitting is insensitive to the value of B_2. There are reasons to expect that, for most of the deformed nuclei, the three B values are not very different from each other. So, for simplicity, we take $B_2=B_1$, and the two remaining parameters $B_1\bar{a}_0^2$ and B_0/B_1 are determined by fitting the first three levels of the ground-state band. The spectra of the whole g band (below backbending) and the associated β and γ bands are calculated with the parameter values thus determined.

All well-deformed even-even nuclei in the rare-earth and actinide regions, with sufficient empirical data, are investigated in this way. To demonstrate the quality of fitting, we may define the root mean square (rms) deviation between the calculated and the experimental spectrum as

$$D=\left[\frac{1}{N}\sum_I\{[E_{\rm cal}(I)-E_{\rm exp}(I)]/E_{\rm exp}(I)\}^2\right]^{1/2}, \tag{22}$$

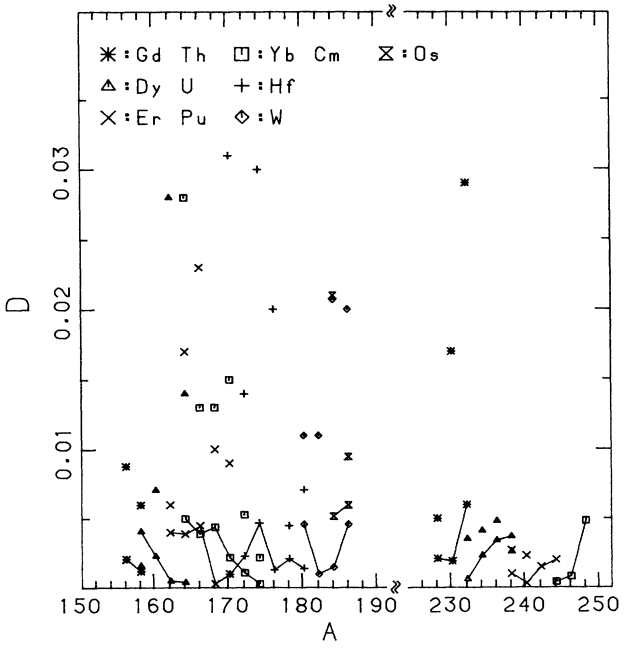

FIG. 1. The rms deviations between the calculated spectra and the experimental data for the g and γ bands of even-even deformed nuclei. The deviations of the g bands are denoted by linking isotopes. The deviations of the γ bands are not linked.

TABLE I. Experimental and calculated rotational spectra (in keV) of some even-even deformed nuclei. For comparison, the results obtained by the two-parameter Harris expansion for the g bands with same fitting method are also listed.

		Spin	2(3)[a]	4(4)	6(5)	8(6)	10(7)	12(8)	14(9)	16(10)	18(11)	20(12)	22(13)	24(14)	26(15)	28(16)
^{234}U	g	Exp.[b]	43.48	143.34	296.04	497.02	741.2	1024.0	1340.8	1688.0	2063.0	2664.2	2889.7	3339	3808	4297
		Cal.	43.48	143.32	296.05	497.12	741.7	1025.0	1342.8	1691.2	2067.1	2467.4	2889.8	3332.1	3792.6	4269.6
		Har.	43.49	143.31	296.06	497.52	743.4	1029.7	1353.0	1710.1	2098.5	2515.9	2960.4	3430.2	3923.8	4440.0
	β	Exp.[b]	851.72	947.85	1095.8	1292.6										
		Cal.	851.40	946.88	1093.4	1286.9	1523.1	1797.6	2106.6							
	γ	Exp.[b]	969.1	1023.6	1091.6	1172.3	1262.2									
		Cal.	969.4	1025.7	1095.2	1177.2	1271.3	1376.8	1493.1	1619.7						
^{232}Th	g	Exp.[c]	49.37	162.12	333.2	556.9	827.0	1137.1	1482.8	1858.6	2262.9	2691.5	3144.2	3619.6	4116.2	4631.8
		Cal.	49.37	162.12	333.2	556.3	825.2	1133.8	1477.3	1851.3	2252.1	2676.7	3122.6	3587.5	4069.7	4567.6
		Har.	49.38	162.09	333.2	557.0	828.0	1141.4	1493.2	1879.8	2298.5	2746.8	3222.6	3724.2	4250.0	4798.6
	β	Exp.[c]	774.1	873.0	1023.1	1221.1	1469.3	1755.0	2080.3	2440.7	2831.6	3249				
		Cal.	777.3	884.9	1048.7	1263.3	1523.0	1822.4	2156.6	2521.5	2913.7	3330.0	3768.0	4225.4	4700.4	
	γ	Exp.[c]	829.6	890.1	960.4	1049.9		1258.7	1370	1511.9	1640	1801.2		2117.3		2445.7
		Cal.	833.5	896.8	974.6	1066.1	1170.5	1287.1	1415.1	1553.7	1702.2	1860.1	2026.6	2201.2	2383.5	2572.9
^{176}Hf	g	Exp.[d]	88.35	290.18	597.0	998.0	1481.3	2034.9	2646.7							
		Cal.	88.34	290.25	597.0	997.5	1480.8	2036.4	2655.3	3329.8	4053.3	4820.3				
		Har.	88.35	290.19	597.0	998.7	1485.6	2049.3	2682.4	3378.8	4133.3	4941.6				
	β	Exp.[d]	1226.6	1390.2	1628.6	1932.7	2295.0									
		Cal.	1234.6	1428.6	1724.2	2111.7	2580.9	3122.1	3726.7	4387.2						
	γ	Exp.[d]	1445.8	1540.2	1727.7	1861.9	2106.6	2285.0								
		Cal.	1427.4	1540.7	1679.9	1843.8	2031.1	2240.2	2470.0	2719.0	2986.0					
^{164}Er	g	Exp.[e]	91.39	299.46	614.4	1024.57	1518.00	2082.73	2702.5	3411.2	4121.2	4868.4	5651.5			
		Cal.	91.37	299.56	614.32	1023.24	1514.03	2075.65	2698.6	3375.1	4098.5	4863.3	5665.0	6499.7	7364.2	
		Har.	91.38	299.49	614.39	1024.86	1520.44	2092.10	2732.3	3434.8	4194.4	5006.6	5867.8	6774.8	7724.7	
	β	Exp.[e]	1314.54	1469.6	1706.6	2068.9										
		Cal.	1332.69	1531.0	1832.2	2225.4	2699.5	3244.2	3850.6							
	γ	Exp.[e]	946.31	1058.25	1197.49	1358.8	1545.11	1744.59	1977.12	2184.1	2479.45	2733.0				
		Cal.	948.19	1063.60	1205.21	1371.59	1561.22	1772.61	2004.31	2254.9	2523.22	2807.9	3108.0	3422.4	3750.2	4090.6

[a]The numbers in the parentheses mark the spin values for the levels of γ bands.
[b]Reference [13].
[c]Reference [14].
[d]Reference [15].
[e]Reference [16].

TABLE II. The fitted values of the mass parameters B_1 and B_0 for the nuclei in Table I and the rms deviations for the g bands of the nuclei.

	^{234}U	^{232}Th	^{176}Hf	^{164}Er
$\bar{a}_0$[a]	0.2719	0.2608	0.2953	0.3333
$B_1/\hbar^2$ (keV^{-1})	0.2996	0.2850	0.1247	0.0934
$B_0/\hbar^2$ (keV^{-1})	0.3352	0.3733	0.2201	0.1283
rms (our cal.)	0.0023	0.0062	0.0013	0.0039
rms (Harris)	0.0177	0.0183	0.0060	0.0159

[a]Reference [17].

where N is the number of the calculated levels. The values of D for g bands (below backbending) and γ bands ($N \geq 3$) are plotted in Fig. 1. It can be seen that the rms deviations for the g bands are less than 0.6%. For clearness of illustration, the rms deviations for β bands are not marked in the figure. They are usually several times larger than the corresponding deviations of g or γ bands.

Some detailed examples are given in Table I, in which the calculated energies are compared with the experimental data. For comparison, a similar fitting for the g bands with the usual two-parameter Harris parametrization is also included in the same table. It is obvious that the fitting by our proposed formula is superior to the fitting by the two-parameter Harris formula, especially for the levels with higher spin values. Table II lists the values of parameters determined in our fitting for the nuclei in Table I, and the rms deviations of the g bands.

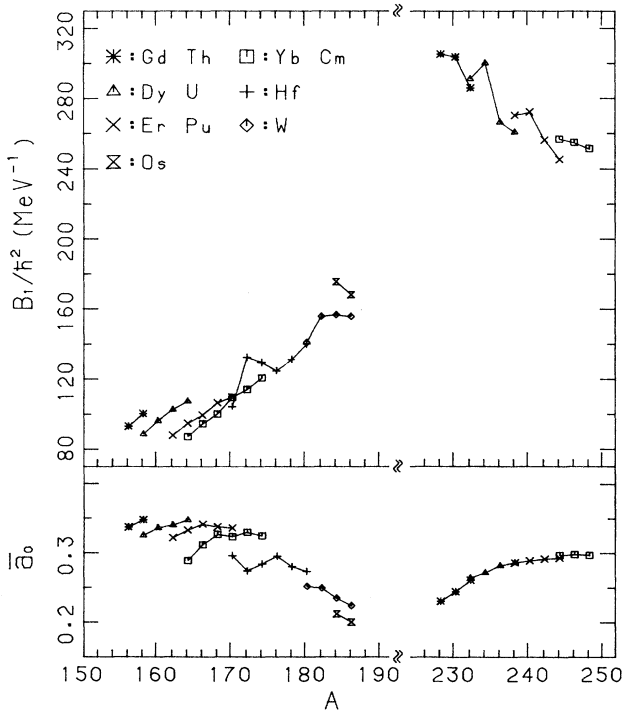

FIG. 2. The fitted B_1 values of the same nuclei as in Fig.1. The quadrupole deformations $\bar{a}_0$ [17] of the corresponding nuclei are plotted in the bottom part of the figure.

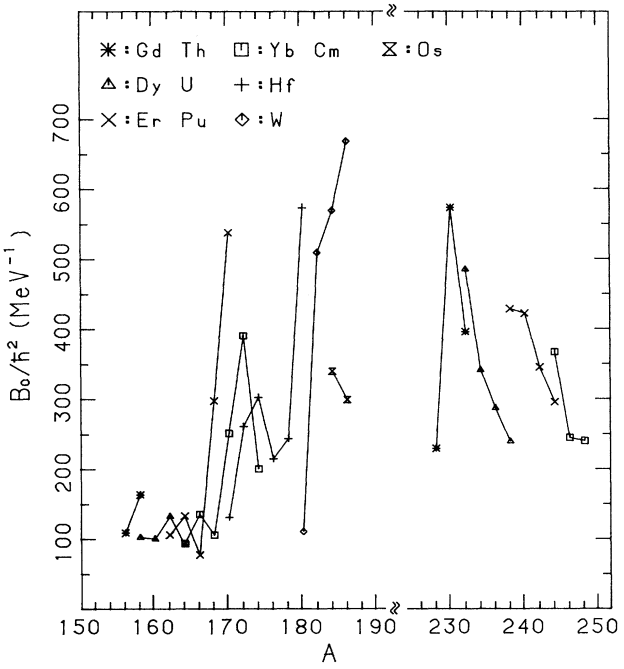

FIG. 3. The fitted B_0 values of the same nuclei as in Fig. 2.

For most of the studied nuclei, the ground-state quadrupole deformations $\bar{a}_0$ are known [17], hence the mass parameters B_1 and B_0 can be calculated from the parameters $B_1\bar{a}_0^2$ and B_0/B_1. Figures 2 and 3 show the values of B_1 and B_0 thus determined. From $\omega_\beta=\sqrt{C_0/B_0}$, C_0 (describing rigidity of nucleus) can be obtained. Figure 4

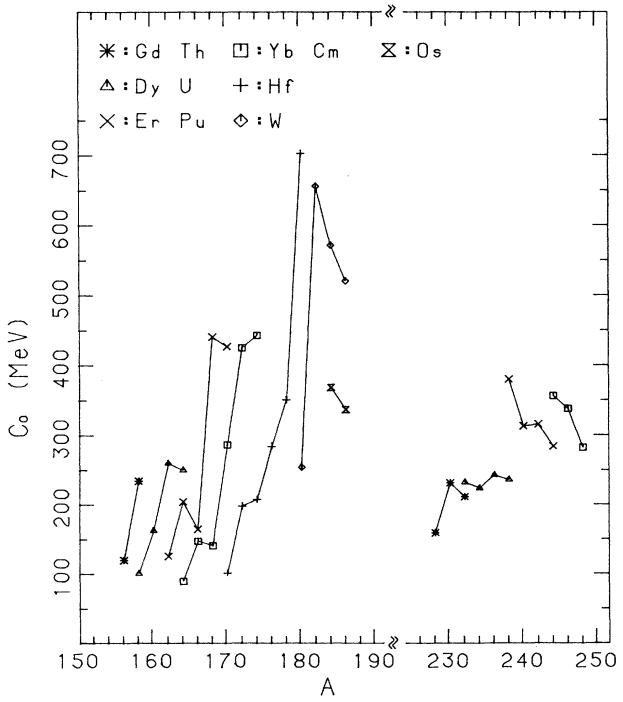

FIG. 4. The calculated C_0 values of the same nuclei as in Figs. 1–3.

TABLE III. Experimental and calculated energies of the $K=3^+$ and $K=2^-$ bands of ^{176}Hf. The values of the corresponding parameters for the g band are $B_1\bar{a}_0^2/\hbar^2=0.01088$ keV^{-1}, $\hbar\sqrt{B_0/B_1}\omega_\beta=1528$ keV.

	Energies (keV)								$B_1\bar{a}_0^2/\hbar^2$ (keV^{-1})	$\hbar\sqrt{B_0/B_1}\omega_\beta$ (keV)
I^π	3^+	4^+	5^+	6^+	(7^+)	(8^+)				
Exp.[a]	1577.5	1675.8	1798.4	1944.3	2116.9	2304.9				
Cal.	1577.5	1676.2	1798.9	1945.1	2114.4	2306.2			0.01345	2381
I^π	2^-	3^-	4^-	5^-	6^-	7^-	(8^-)	(9^-)		
Exp.[a]	1247.7	1313.3	1404.5	1508.6	1652.9	1785.0	1992.5	2137.3		
Cal.	1247.7	1315.5	1405.0	1515.5	1646.4	1796.7	1965.5	2152.1	0.01465	1477

[a]Reference [15].

gives the variation of C_0 with nuclei.

Formulas (18) and (20) can also be applied to other rotational bands. In such cases, the state of associated vibrational motion is unknown. We can, however, use only the rotational energy part of formulas (18) and (20) with $B_1\bar{a}_0^2$ and $\sqrt{B_0/B_1}\omega_\beta$ as fitting parameters. Most of the bands can be fitted successfully in this way. Two examples are listed in Table III. The fitting is quite satisfactory, with the parameter values comparable to those determined from the g band of the same nucleus.

IV. DISCUSSION

From the second section, it can be seen that the collective motion can be easily treated with the cranking BM Hamiltonian. In the rotating frame of reference, the effect of the Coriolis and centrifugal forces of rotational motion is clearly demonstrated and easily treated. However, in the cranked shell model, ω is not a dynamic quantity and cannot be quantized. This is the main difference between the usual BM Hamiltonian and the cranking BM Hamiltonian proposed in this paper. It can be seen, from our results, that the error involved in the semiclassical treatment of angular momentum is not serious and that the proposed model gives a good representation of rotation-vibration coupling for the deformed nuclei, especially for the g, β, and γ bands (below backbending). Results of applications to other bands are also satisfactory. It seems, therefore, that reliable nuclear information can be derived from this model. Primarily, the information is embodied in the mass parameters and potential parameter given in Sec. III. As has been remarked in the above section, for most nuclei the fitting of the β band is less satisfactory (this can also be seen from the examples given in Table I). It may be caused by some deviation of the β vibration from the harmonic form. Study of such deviation may lead to more detailed knowledge about the potential energy surface.

From Figs. 2–4, it can be seen that the variation of B_1 (rotational parameter) is rather smooth, while the variations of B_0 (vibrational parameter) and C_0 (rigidity) are pronounced near $A=170$ and 180 (the neutron number $N=102$ and 108) in the rare-earth region and $A=238$ ($N=144$) in the actinide region. It may reveal some kind of shell effect in the deformed nuclei.

As a semiclassical model based on axisymmetrical deformations with a simple harmonic potential, it is out of the scope of our present model to deal with the staggering phenomena [12] of certain γ bands. However, since the general trend of the γ band is adequately represented, our fitting can demonstrate the staggering as is shown in Table I for the nuclei ^{176}Hf and to less extent ^{164}Er.

The rotational bands of odd A nuclei are more complicated because of the interaction between single-particle motion and collective motion. In some cases, the interaction can be neglected, then the above simple model can be applied. The rotational bands of odd A nuclei, the transition probabilities, and other related topics will be discussed in a future paper.

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[1] L. K. Peker, S. Pearlstein, and J. H. Hamilton, Phys. Lett. **100B**, 281 (1981).
[2] A. Arima and F. Iachello, Ann. Phys. (N.Y.) **115**, 325 (1978).
[3] A. E. L. Dieperik, Prog. Part. Nucl. Phys. **9**, 121 (1983).
[4] K. Kumar and M. Baranger, Nucl. Phys. **A110**, 529 (1968).
[5] K. Kumar, Nucl. Phys. **A231**, 189 (1974).
[6] J. B. Gupta, K. Kumar, and J. H. Hamilton, Phys. Rev. C **16**, 427 (1977).
[7] K. Kumar and J. B. Gupta, Nucl. Phys. **A304**, 295 (1978).
[8] C. S. Wu and J. Y. Zeng, Commun. Theor. Phys. **8**, 51 (1987).
[9] H. X. Huang, C. S. Wu, and J. Y. Zeng, Phys. Rev. C **39**, 1617 (1989).
[10] S. M. Harris, Phys. Rev. Lett. **13**, 663 (1964).
[11] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1965), Vol. II, pp. 22–36.

[12] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer-Verlag, Berlin, 1980), pp. 271–279.
[13] Y. A. Ellis-Akovali, Nucl. Data Sheets **40**, 523 (1983).
[14] M. R. Schmorak, Nucl. Data Sheets **63**, 139 (1991).
[15] D. J. Horen and B. Harmatz, Nucl. Data Sheets **19**, 383 (1976).
[16] E. N. Shurshikov, Nucl. Data Sheets **47**, 433 (1986).
[17] S. Raman, C. H. Malarkey, W. T. Milner, C. W. Nestor, Jr., and P. H. Stelson, At. Data Nucl. Data Tables **36**, 1 (1987).