Specification of Kowalski-Noyes f ratios for coupled channels

P. J. Dortmans and K. Amos
School of Physics, University of Melbourne, Parkville, 3052 Victoria, Australia
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We show that by a slight redefinition, the (real) Kowalski-Noyes f ratios can be defined for coupled as well as uncoupled channels, and, with this, we find a separable representation of t matrices whose complex nature is dependent only on on-shell observables.

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The Kowalski-Noyes f ratios [1] have been used often as a simple method by which diverse off-shell effects of two nucleon (NN) t or G matrices (e.g., when comparing different interactions [2] or nuclear medium effects [3]) may be displayed. These f ratios, defined by

$$f_L(p;k) = \frac{t_L(p,k)}{t_L(k,k)} \tag{1}$$

for uncoupled channels (where k is the on-shell momentum), are purely real even though the t matrices of which they are composed are complex. This is seen if one applies the Heitler equation to Eq. (1), and further the f ratios may then be expressed in terms of the real reactance matrices; viz.,

$$f_L(p;k) = \frac{K_L(p,k)}{K_L(k,k)}. \tag{2}$$

But for coupled channels, this simple relationship does not hold since the Heitler equation is coupled so that

$$f_{LL'}(p;k) = \frac{t_{LL'}(p,k)}{t_{LL'}(k,k)} \neq \frac{K_{LL'}(p,k)}{K_{LL'}(k,k)} \tag{3}$$

and f now is complex.

However, if one is to redefine the f ratio in a slightly different form (without any alteration to the uncoupled channel definition) as

$$f_{LL'}(p;k) = \sum_l t_{Ll}(p,k)\,[t(k,k)]^{-1}_{lL'}\,, \tag{4}$$

where $[\ ]^{-1}$ represents the inverse of a 2×2 matrix, it follows that

$$\sum_l t_{Ll}(p,k)\,[t(k,k)]^{-1}_{lL'} = \sum_l K_{Ll}(p,k)\,[K(k,k)]^{-1}_{lL'} \tag{5}$$

and so f is a now a 2×2 matrix with real components for coupled channels, while remaining the scalar quantity for the uncoupled channels.

As an example, we show the f_{00} ratios for the Paris interaction at 200 MeV with the coupled ($^3S_1$-$^3D_1$) channel. The solid line represents the free t-matrix ratios, the dashed line represents the Pauli blocked G-matrix ratios and the dashed line represents the Pauli blocked/auxiliary potential ratios (see Ref. [3]). The density is chosen to be 0.185 nucleons/fm^3 (k_f = 1.4 fm^{-1}). As can be seen, such representations are helpful, since they show that the effects of Pauli blocking and auxiliary potential are additive for low momenta ($< k$) but have the opposite effect at higher off-shell momenta.

Armed with this extended definition of the f ratios, we are able to obtain a W-matrix-like separable representation of t matrices [4] at rank 1 and rank 2, for uncoupled and coupled channels, respectively. This has the form

$$t_{LL'}(p,p';E) = \sum_{ll'} f_{Ll}(p;k)t_{ll'}(k,k;E)f_{L'l'}(p';k) + R_{LL'}(p,p';E), \tag{6}$$

where the remainder term, $R_{LL'}$, is a purely real quantity which is null on- and half off-shell (i.e., p or $p' = k$). Here we may use t, K, or W matrices [4] to define the f ratios since all give the same result. So the complex nature of the t matrices are due entirely to the on-shell physical observables (via the t-matrix elements on-shell).

Specification of the separable t matrices as per Eq. (6) implies a number of interesting features. As with all W-like separable representations, they are exact on- and half off-shell. The imaginary component of the t matrices is exact also fully off-shell. Also there can be a zero phase

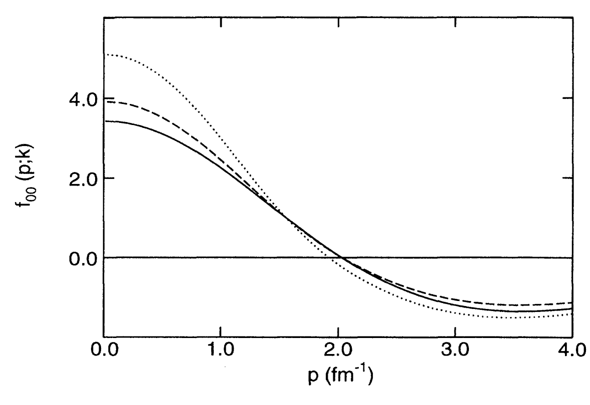

FIG. 1. The Kowalski-Noyes f ratios obtained with a Fermi momentum of 1.4 fm^{-1} for the f_{00} element of the $^3S_1$-$^3D_1$ channel. The results were obtained using an energy 200 MeV and from free, Pauli blocked, and Pauli blocked plus auxiliary potential calculations based upon the Paris interaction. These results are shown by the continuous, dashed, and dotted lines, respectively.

pathology [5, 6] (when the denominator of the f ratio, or its determinant, passes through zero) whereby the f ratios themselves become infinite. But as shown elsewhere [6], such problems can be overcome quite easily by using Sturmian expansions. By considering separately the attractive and repulsive components of a sufficiently high rank Sturmian expansion (Sturmian separation method), we are able to define rank 2 (uncoupled) or rank 4 (coupled) separable t matrices for which the on-shell sum of the matrix elements when zero, is due to the cancellation to two equal but opposite terms. This allows one to offset any such pathology since the individual denominators of the f ratios are nonzero (for a detailed account, see Ref. [6]).

We suggest that the generalization of the Kowalski-Noyes f ratios given herein is a useful scheme to delineate off-shell properties of interactions since all quantities are purely real and then one is able to obtain a separable W-like t-matrix representation whose complex nature is solely due to the on-shell t matrices.

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