Analysis of the sign of $E2/M1$ multipole mixing ratio of transitions in the even Te isotopes

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Calculations of energy levels, wave functions, and electromagnetic properties in the even Te isotopes have been made assuming a quadrupole vibrator coupled to two particles. The value of $\delta(2_2^+ \to 2_1^+)$ shows a strong dependence on the coupling parameter η. A correlation exists between the signs of $\delta(2_2^+ \to 2_1^+)$ and $\Delta E = E(4_1^+) - E(2_2^+)$.

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The study of energy levels and eletromagnetic properties in nuclei with Z or N near a magic number shows many characteristics that are in general explained by the vibrational model. However, some properties differ considerably from those of a pure vibrator. This leads to the idea of a vibrator core coupled to particles in such a way that this new particle degree of freedom can explain discrepant properties such as the crossover transitions, the splitting of the two-phonons triplet, quadrupole moments different from zero, $M1$ transitions, etc. Bohr [1] was the first one to imagine nuclei with properties related both to the shell model and to the vibrational model. The so-called unified model, with collective and particle degrees of freedom, was later developed by Choudhury [2], Raz [3], Alaga [4], and Scharff-Goldhaber and Weneser [5]. The even tellurium isotopes ($Z = 52$), with two protons outside the closed shell, are interesting nuclei for the application of this model. Having the former calculations of Lopac [6] and Degrieck and Vanden Berghe [7,8] as the starting point, we have applied this model to the systematic study of energy levels, wave functions, and eletromagnetic properties of the even tellurium isotopes with mass number A between 112 and 134, obtaining a good fitting with the experimental data [9]. The objective of this work is to determine how some calculated properties, especially the multipole mixing ratio $\delta(E2/M1)$, behave with the parameters of the model.

In the model used, a quadrupolar vibrator core couples to two protons by a quadrupole force. The system is described by the Hamiltonian

$$H = H_v + H_1 + H_2 + H_{\text{int}} + H_{12}, \tag{1}$$

where in the right-hand side of the equation there is respectively the Hamiltonian of the vibrator, the Hamiltonian of the two particles, the Hamiltonian of the interaction between the vibrator and each particle (with coupling constant η according to the definition of Covello and Sartoris [10]), and finally the Hamiltonian of the surface delta interaction (SDI) between the particles with intensity G.

The diagonal part of this Hamiltonian is

$$H_d = H_v + H_1 + H_2. \tag{2}$$

We use as the basis the eigenstates of H_d

$$|(j_1, j_2)_a J, NR\nu; IM\pi\rangle = |n; IM\pi\rangle, \tag{3}$$

referring to the eigenenergies

$$E_{n;I} = N\hbar\omega + E(j_1) + E(j_2). \tag{4}$$

In this representation the two protons with angular momenta j_1 and j_2 (where j represents the quantum numbers n, l, $s = 1/2$ and j) couple antisymmetrically to give an angular momentum J, which couples to the angular momentum R of the core with N phonons and seniority ν to give a state with total angular momentum I, projection M along the z axis, and parity π.

The eigenstates of the Hamiltonian H is

$$|I_m^\pi M\rangle = \sum C_n^{I_m} |n; IM\pi\rangle, \tag{5}$$

i.e., the mth state of angular momentum I, projection M, and parity π.

The $E2/M1$ multipole mixing ratio was extensively determined and in the particular case of transitions between the first and the second 2^+ levels there are measurements [11] including a great part of the periodic table indicating that in certain cases a dependence can occur between $\delta(2_2^+ \to 2_1^+)$ and $\Delta E = E(4_1^+) - E(2_2^+)$. This is the case of the tellurium isotopes that have the value of δ determined experimentally (Table I). In a previous result, Ku-

TABLE I. Experimental values of the absolute and the relative energy differences, $\Delta E = E(4_1^+) - E(2_2^+)$ and $\Delta E/E(2_2^+)$, and of the multipole mixing ratio $\delta(2_2^+ \to 2_1^+)$ for six even tellurium isotopes with A between 120 and 130.

A	ΔE (MeV)[a]	$\Delta E/E(2_2^+)$[a] (%)	$\delta(2_2^+ \to 2_1^+)$
120	-0.0402	-3.3	-0.92 ± 0.09[b]
122	-0.0763	-6.1	-3.48 ± 0.04[c]
124	-0.0770	-5.8	-3.3 ± 0.1[d]
126	-0.0589	-4.1	$-5.6^{+0.5}_{-0.4}$[e]
128	-0.0230	-1.5	$+4.6^{+1.6}_{1.0}$[f]
130	+0.0448	+2.8	$+0.65 \pm 0.15$[g]

[a]See Ref. [13].
[b]See Ref. [14].
[c]See Ref. [15].
[d]See Ref. [16].
[e]See Ref. [11].
[f]See Ref. [17].
[g]See Ref. [18].

mar [12] had obtained a semiquantitative relation among the eletric quadrupole moment $Q(2_1^+)$ and ΔE in the W, Os, Pt region: the conclusion was that the change of sign of ΔE is a good indicator of a prolate-oblate transition. Extending the idea to the Te region, the change of sign of $\delta(2_2^+ \rightarrow 2_1^+)$ could also be related to a transition such as this. The variation of the energy of the levels 2_1^+, 4_1^+, and 2_2^+, and of $\delta(2_2^+ \rightarrow 2_1^+)$ with η using the two-particle core coupling model is shown in Figs. 1 and 2: we can see that the sign change of δ occurs concomitantly with the crossing of the levels 4_1^+ and 2_2^+. The plot of $\delta(2_2^+ \rightarrow 2_1^+)^{-1}$ in function of ΔE (Fig. 3) shows this property more clearly. The smooth curve has been drawn for convenience and corresponds to the relation

$$\delta(2_2^+ \rightarrow 2_1^+)^{-1} = -0.575 + 0.714\Delta E + 0.634 \exp(5.00\Delta E), \quad (6)$$

In the calculations taken as reference for the even tellurium and that are described below, the following standard set of the parameters of the model were used: the particle states have energies $E(g_{7/2}) = 0$, $E(d_{5/2}) = 0.75$ MeV, $E(d_{3/2}) = 1.8$ MeV, $E(s_{1/2}) = 2.0$ MeV, $E(h_{11/2}) = 2.4$ MeV; the energy of one phonon $\hbar\omega = 1.0$ MeV (states up to three phonons were used); the SDI constant $G = 0.25$ MeV and the core-particle coupling constant $\eta = 1.4$ MeV; for the calculation of the eletromagnetic properties the following constants were used: the gyromagnetic factors $g_R = 0$, $g_l = 1$, $g_s = 3.91$; the proton effective charge $e_{\text{eff}}^p = 2e$ and the vibrator effective charge $e_{\text{eff}}^v = 2.63e$ (corresponding to the value $22.5e\,\text{fm}^2$ for the constant X used by Covello and Sartoris [10]). This choice of the parameters yields energies and eletromagnetic properties that are very close to those observed for the ^{124}Te.

A deeper analysis made with different choices for the particle levels and for the maximum number of phonons of the vibrator core allowed interesting conclusions, especially in relation to the sign of $\delta(E2/M1)$:

(a) The magnetic momentum operator is given by

$$\mu(M1) = g_R\mathbf{R} + g_l\boldsymbol{l} + g_s\mathbf{s} = g_R\mathbf{I} + (g_l - g_R)\mathbf{J} + (g_s - g_l)\mathbf{s}, \quad (7)$$

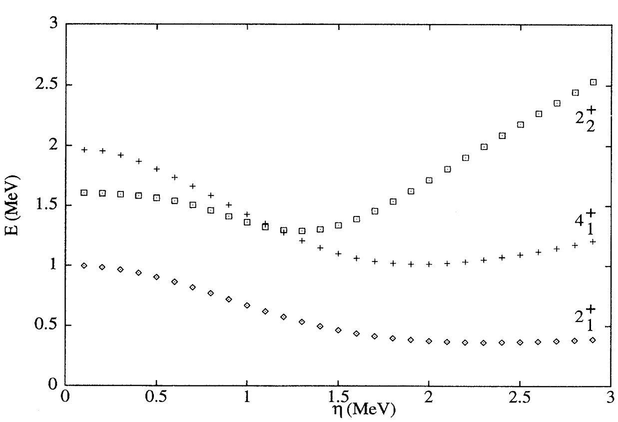

FIG. 1. Calculated energy of the three first excited levels, 2_1^+, 4_1^+, and, 2_2^+ (MeV) as a function of η (MeV).

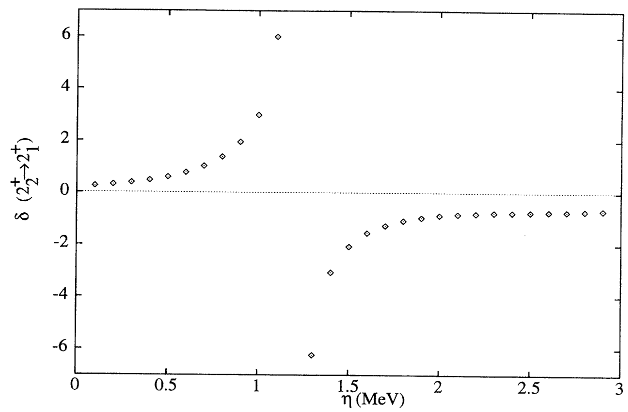

FIG. 2. Calculated multipole mixing ratio $\delta(2_2^+ \rightarrow 2_1^+)$ as a function of η (MeV).

yielding the $M1$ matrix elements

$$\langle 2_2^+ \| M(M1) \| 2_1^+ \rangle = [Cg_R + D(g_l - g_R) + E(g_s - g_l)](e\hbar/2m), \quad (8)$$

which is basically proportional only to the difference $g_l - g_R$ with proportionality constant D. The part proportional to $g_s - g_l$ is of little importance in the calculation because its proportionality constant E is small for the considered values of η. The constant g_s therefore affects slightly the result of the multipole mixing ratio. On the other hand, the part proportional only to g_R is null since it only connects each state with itself and is useful only in the determination of the magnetic dipole moments of the energy levels. Thus the constant g_R acts only as a scale factor in the calculation of δ. The behavior of δ as a function of η, using the value $g_R = Z/A \cong 0.42$, shows the same basic characteristic of the sign change for $\eta \simeq 1.2$ MeV. The constant g_R is of great importance in the calculation of the magnetic moment $\mu(2_1^+)$ and, using its experimental value, can be calibrated. The value $g_R = 0$ was obtained with the experimental value $\mu(2_1^+) =$

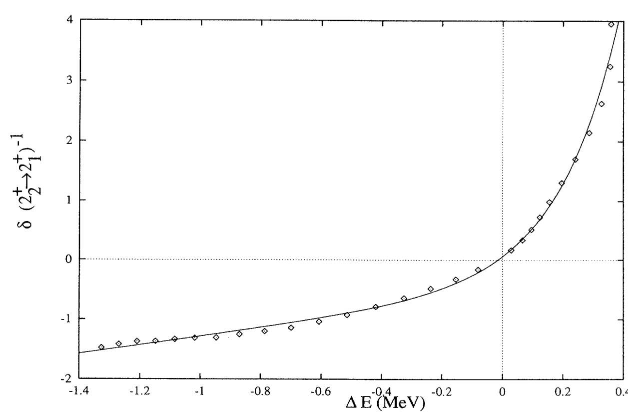

FIG. 3. Inverse of calculated multipole mixing ratio $\delta(2_2^+ \rightarrow 2_1^+)^{-1}$ as a function of the energy difference $\Delta E = E(4_1^+) - E(2_2^+)$ (MeV). The smooth curve is given by Eq. (6).

$(0.50 \pm 0.06)\mu_N$ of ^{124}Te. Calculations were made for many values of the coupling constant η showing that the constant D is really responsible for the sign change of the $M1$ operator matrix element and also for the sign change of the multipole mixing ratio $\delta(2_2^+ \to 2_1^+)$, because the $E2$ matrix element sign does not change in this range of η.

(b) With the five particle states and the ten collective states up to three phonons used, we obtained a basis with 205 vectors for states with angular momentum 2 and positive parity; these vectors are generally written as $|(j_1, j_2)_a J, NR\nu; 2M+\rangle = |n; 2M+\rangle$ and are numbered in order of increasing values for N, R, J and $E(j_1) + E(j_2)$. The constant D is therefore defined as being

$$D = (3/4\pi)^{1/2} \sum_n \sum_{n'} (-1)^{J+J'+R'+1} C_n^{I_i} C_{n'}^{I_f} [(2I_f+1)(2I_i+1)(2J+1)(J+1)J]^{1/2} \times \delta_{NN'} \delta_{RR'} \delta_{\nu\nu'} \delta_{j_1 j_1'} \delta_{j_2 j_2'} \begin{pmatrix} J' & J & 1 \\ I_i & I_f & R' \end{pmatrix}, \quad (9)$$

where C_n^I are the coefficients of the expansion of the eigenstate I in the basis of states $|n; IM+\rangle$. Here $I_i = 2_2^+$ and $I_f = 2_1^+$. The plot of the partial results for D near the region where the sign change of δ occurs ($\eta = 1.0$ MeV, 1.2 MeV, 1.4 MeV), as a function of the variation of n' in the second sum, is in Fig. 4; it is evident that this change does not occur due to some special component of the levels of the transition, but due to an effect of the addition of multiple components. In these calculations the levels 2_1^+ and 2_2^+ have some complementary properties, e.g., when the magnetic dipole moment of the first increases with η, the inverse happens to the second.

(c) The use of five particle states is necessary, since the suppression of some of them leads to results much different from those calculated using energies near 2 MeV for the states $d_{3/2}$, $s_{1/2}$, and $h_{11/2}$. We realized that even energies of 10 MeV for these states still resulted in numbers reasonably different from those obtained using the basis with only the two lower particle states. Intermediate calculations were made with the energy value of the three higher particle states equal to 4 MeV and 10 MeV. Both the electric quadrupole moment of the level 2_1^+ and the multipole mixing ratio $\delta(2_2^+ \to 2_1^+)$ have their signs changed if the number of particle states is reduced to only two, or equivalently, if the three higher particle states have their energies substantially increased. With the reduction of the basis of vectors, the multipole mixing ratio $\delta(2_2^+ \to 2_1^+)$ stops changing sign when the value of η is varied. The behavior of the energy levels, especially the level 6_1^+, also is significantly changed with the reduction of the basis of particle states. The importance of the higher particle states can be due to a greater distribution of the values of their angular momenta (3/2, 1/2, and 11/2) in comparison with the angular momenta of the lower levels (7/2 and 5/2). To determine which of these three particle states causes the sign change of δ, calculations were made in which the particle basis was restricted to the two lower states plus one of the three higher states: the sign change only occurred in the case in which the state $s_{1/2}$ was mantained, indicating that this is the state responsible for this property. This can be due to the fact that the state $s_{1/2}$ has an angular momentum very different from $g_{7/2}$ and $d_{5/2}$ and the same parity of these states. For a deeper understanding of the role of the particle states, calculations were made changing the position of the levels $g_{7/2}$ and $d_{5/2}$, i.e., using the second as a ground state and the first with an energy of 0.75 MeV. Even with this change in energies, $\delta(2_2^+ \to 2_1^+)$ continued changing its sign for a critical value of η, due to the change of the sign of the $M1$ operator matrix element. Therefore the use of all the five particle states between the magic numbers 50 and 82 has effectively even greater importance than the value assumed by their energies.

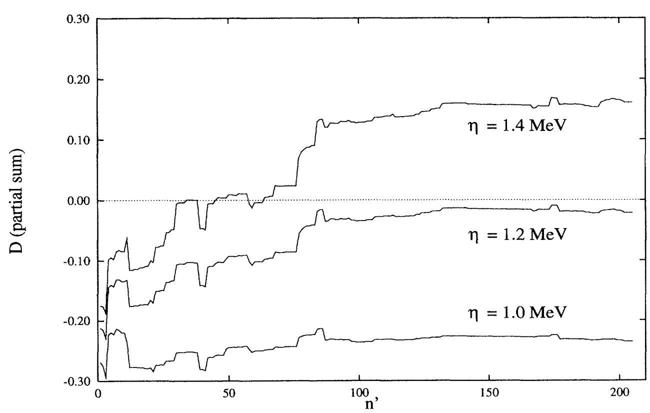

FIG. 4. Partial results for D as a function of the variable n' of the second sum of Eq. (9), with $\eta = 1.0$ MeV, 1.2 MeV, 1.4 MeV.

(d) The introduction of four-phonons collective levels with energies of 4 MeV leads to effective changes in the results, in spite of perhaps not being so important as in the case of the change of the particle states basis. A good comparison was made with the result obtained using the same 4 MeV for the energies of the three higher particle states. Generally the calculated levels have their energies reduced when we introduce four-phonons states. While the change in the quadrupole moment $Q(2_1^+)$ is not significant, the energy of 6_1^+ decreases strongly. Again the multipole mixing ratio $\delta(2_2^+ \to 2_1^+)$ continues having a divergence for a critical value of η. In general the introduction of collective states affects more the energies (including the energy of 2_1^+ which decreases from 0.496 MeV to 0.347 MeV, i.e., 30%) than the eletromagnetic properties, while the introduction of particle states affects deeply properties like $Q(2_1^+)$ and $\delta(2_2^+ \to 2_1^+)$.

(e) The multipole mixing ratio of the transition $4_2^+ \to 4_1^+$ also changes the sign in the region in which η is approximately equal to 1 MeV due to the sign change of the $E2$ matrix element and no more of the $M1$ matrix element. The multipole mixing ratio $\delta(2_3^+ \to 2_1^+)$ has negative sign for η between 0.6 MeV and 2.2 MeV.

(f) Calculations were made for the ^{142}Ce,with two neutrons outside the closed shell $N = 82$, using the particle states $f_{7/2}$ and $g_{9/2}$ (with energy of 1.35 MeV), energy of 1 phonon $\hbar\omega = 1.6$ MeV, and coupling constant between the core and the neutrons $\eta = 1.2$ MeV. For certain critical values of η, $E_{9/2}$, and $\hbar\omega$, the two multipole mixing ratios, $\delta(2_2^+ \to 2_1^+)$ and $\delta(2_3^+ \to 2_1^+)$, change their signs almost simultaneously due to the sign change of the matrix element of the operators $E2$ and $M1$ respectively. This can be related to the crossing of the levels 2_2^+ and 2_3^+, i.e., to the change of the mutual characteristics between these two levels.

The two-particle core coupling model indicates that in general eletromagnetic properties are sensitive to the coupling constant η and to the configuration of vectors used. An important result is the sign change of $\delta(2_2^+ \to 2_1^+)$ with the coupling constant η and following the sign of $\Delta E = E(4_1^+) - E(2_2^+)$. Although the model is not complete, it shows us clearly the origins of certain results, in order to allow better fittings.

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