Modified Skyrme model and the nucleon-nucleon interaction

German Kälbermann
Racah Institute of Physics, Hebrew University of Jerusalem, Jerusalem 91904, Israel
(Received 7 December 1992)

We present a Skyrme model with modified kinetic-energy terms. The baryon current of the model supports both fractional and integer baryon number solutions. Chiral symmetry breaking renders the fractional baryon number states absolutely confined. In the unbroken phase the solutions bear resemblance to the constitutent quarks of the nonrelativistic quark model. The model predicts correctly the nucleon and delta resonance masses using a value for the pion decay constant F_π very close to experiment. The model yields an attractive central potential between nucleons.

PACS number(s): 11.30.Rd, 13.75.Cs

I. INTRODUCTION

The Skyrme model [1] pictures baryons as topological solitons in a nonlinear meson field theory. It has been rather successful in describing the low-energy phenomena of strong interactions [2]. The minimal version of the model predicts baryon masses and properties quite well, by using essentially two free parameters: The pion decay constant F_π and Skyrme's parameter e. It is found that, in order to fit the baryon masses, F_π has to be unrealistically small as compared to the measured value of 186 MeV. In SU(2) the model necessitates [2] a value of around 128 MeV whereas for SU(3) it becomes as small as 54 MeV when no zero-point energy subtraction is performed [3]. This disturbing feature weakens the reliability of a calculation based on the model.

The implementation of the Skyrme approach in systems of more than one baryon has met with some difficulties too. The central nucleon-nucleon interaction predicted by the straightforward application of the model lacks any attractive components [4]. This would then imply that atomic nuclei cannot exist, with the exception of the deuteron. Some refinements to the simplest version of the Skyrme model, such as the inclusion of dynamical contributions to the two-baryon ground-state wave function by means of rotational and vibrational degrees of freedom [5], can improve the situation and yield a mildly attractive potential. However, it remains unclear whether this is indeed the mechanism responsible for nuclear binding. Numerical calculations [6] seem to indicate such a component indeed exists, when mutual distortions of the solitons are allowed. In the numerical approach, it is difficult to introduce collective degrees of freedom, identifying unambiguously the soliton locations and implement rigorously the baryon number two condition. It is then still unclear if the model does indeed predict the medium range central attraction. The modified Skyrmion we propose here will be able to use a value of F_π very close to experiment. At the same time the model will give a non-negligible attractive component in the central potential between nucleons without resorting to any kind of involved mechanisms or distortions. All this will be achieved by changing the kinetic energy of the Skyrme model. This change will also have far reaching implications concerning the spectrum of states.

We will start with a different current building block for the kinetic energy. This current when inserted in the Wess-Zumino action will generate a baryon current that supports both fractional and integer baryon number states. In order to make contact with phenomenology, we will introduce a symmetry-breaking parameter in the chiral current. In doing so we will discover that the fractional baryon number solitons cease to exist and only integer baryon number solutions survive. The fractional baryon number solutions of the unbroken phase will be identified as the constituent quarks of the model. Although QCD is expected to yield confinement even with massless quarks, the chiral anomaly of QCD will spontaneously induce chiral symmetry breaking. Therefore, even at the QCD level with massless quarks, there are chiral symmetry-breaking effects. Our manifest symmetry-breaking interactions in the kinetic energy can be viewed as a poor man's mock-up of this behavior. These terms are different from the conventional meson mass terms that are commonly included in the Skyrme model and whose importance is minor. The new kinetic energy forces the constituent quarks to be confined even when the symmetry-breaking parameter is small but nonzero. At the same time, the new Lagrangian will automatically induce different coupling constants for the vector and axial-vector interactions between mesons and baryons. This is similar to the effects appearing in the chiral dynamics method [8], in which it was necessary to introduce *ad hoc* a different coupling constant for those interactions even when meson mass terms are included, in order to recover the low-energy phenomenology of pion-nucleon processes.

In the present paper SU(2) baryon properties and the nucleon-nucleon interaction are calculated. Later we will consider a more realistic treatment of the constituent quarks and mesons of the model [9].

II. CHIRAL SYMMETRY BREAKING IN THE SKYRME MODEL

The simplest version of the SU(2) Skyrme model starts with the Lagrangian [4]:

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$$L=-\frac{F_\pi^2}{16}\int d^3x\, \mathrm{Tr}(L_\mu L^\mu)+\frac{1}{32e^2}\int d^3x\, \mathrm{Tr}(L_\mu,L_\nu)^2\ , \tag{1}$$

where

$$L_\mu=U^\dagger\partial_\mu U \tag{2}$$

and

$$U(\mathbf{x},t)=R^\dagger(t)U_0(\theta(\mathbf{x}))R(t)\ . \tag{3}$$

U_0 is the static hedgehog ansatz, depending on the chiral angle θ, R is a collective coordinate SU(2) matrix depending on time, $F_\pi=186$ MeV experimentally is the pion decay constant, and e is the Skyrme parameter.

The action (1) is invariant under left and right transformations.

$$U\rightarrow AUB\ , \tag{4}$$

where A and B are constant SU(2) matrices. Two currents emerge from the invariances above: The vector current and the axial-vector current. The first is exactly conserved while the second is only partially conserved.

The virtue of the Skyrme model is its unified treatment of baryons and mesons by means of scalar field degrees of freedom [1,2]. Baryons are identified as the topological soliton solutions of the theory while mesons arise as fluctuations around the solitons. In the times of chiral Lagrangians [8], two separate entities were used for the same purpose. Baryons were represented as structureless Dirac particles, whereas the mesons were incorporated as separate entities by means of scalar fields coupled to the baryons. The basic premise in building field theory models of this kind was the conservation of the chiral current. The time-honored most successful model of this kind is the chiral dynamics method [8]. It is based on nonlinear realizations of chiral symmetry. The tree-level calculations based on the model correctly predict all the low-energy soft-pion process. Chiral symmetry is, however, manifestly broken to account correctly for the phenomenology of nucleon-meson interactions. Two types of breaking mechanisms appear in the model: (1) Mass terms for the mesons, which are considered to be very small and relatively unimportant (hence the soft-pion limit was quite accurate); and (2) baryon-meson couplings that break chiral symmetry by allowing a phenomenological distinction between vector and axial-vector interactions. The latter were very important for the description of meson emission and absorption from baryons and accounted for the fact that the axial-vector coupling constant is phenomenologically bigger by around 20% as compared to the vector constant. The first mechanism is easily included in the Skyrme model [10]; the second symmetry-breaking effect was never included.

Several years ago Callan *et al.* [11] developed a method to construct phenomenological Lagrangians for fields transforming according to nonlinear realizations of an internal symmetry group. Their technique can then be used to introduce a more general chiral operator instead of the one in Eq. (2) and still maintain chiral symmetry conservation. Consider the following current:

$$A_\mu=\tfrac{1}{2}(U^\dagger\partial_\mu U-U\partial_\mu U^\dagger)\ . \tag{5}$$

The normalization of the current is such that, to lowest order in the pion field, it gives the same dynamics as the current of Eq. (2) and allows us to identify the pion decay constant of Eq. (18) below to be the same as the one of Eq. (1).

Left and right invariance of Lagrangians built with Eq. (5) instead of Eq. (2) is implemented by transformations [11]

$$U\rightarrow VUB=AUV^\dagger\ , \tag{6a}$$

$$U^2\rightarrow AU^2B\ , \tag{6b}$$

where V is an SU(2) matrix depending on the field U. Explicitly, for infinitesimal left and right transformations

$$U\rightarrow(1+i\boldsymbol{\tau}\cdot\boldsymbol{\varepsilon})U\ , \tag{7a}$$

$$U\rightarrow U(1+i\boldsymbol{\tau}\cdot\boldsymbol{\delta})\ , \tag{7b}$$

and a generic SU(2) parametrization of the chiral field

$$U=\exp(i\boldsymbol{\tau}\cdot\hat{\boldsymbol{\pi}}\theta)\ , \tag{8}$$

V becomes

$$V\approx 1+\frac{1}{2}\boldsymbol{\tau}\cdot\{\boldsymbol{\delta}-\boldsymbol{\varepsilon}+\tan(\theta)[\hat{\boldsymbol{\pi}}\times(\boldsymbol{\delta}+\boldsymbol{\varepsilon})]\}\ . \tag{9}$$

If we build our Lagrangian with the current of Eq. (5) we will discover immediately that it becomes, after suitable parameter rescalings, the original Skyrme Lagrangian for twice the chiral angle. Our model, in the chiral limit, is the Skyrme model of Eq. (1) for U replaced by U^2. We will, nevertheless, keep the definition of the chiral current in Eq. (5), in terms of U and not the square root of it, because, upon introduction of symmetry breaking the above combination is the appropriate one in order to have finite-energy solutions for integer baryon number.

Let us consider the baryon current of the model. Following Witten [7] we can write the Wess-Zumino action

$$n\Gamma=\frac{-in\,\varepsilon^{\mu\nu\alpha\beta\gamma}}{240\pi^2}\int \mathrm{Tr}(A_\mu A_\nu A_\alpha A_\beta A_\gamma)d^5x\ , \tag{10}$$

where n is an integer and A_μ is the current of Eq. (5). We still use the original normalization of the Wess-Zumino current because the relevant chiral field here is U^2 and not U. If one calculates the Jacobian of the transformation between the field manifold and the five-dimensional target space we will find the normalization of Eq. (10) provided that the current is defined as in Eq. (5). This can be seen by expanding the Wess-Zumino action to lowest order in the pion field.

Applying the U(1) gauge transformation

$$U\rightarrow i\varepsilon(x)[Q,U] \tag{11}$$

where ε is an infinitesimal parameter and Q the quark charge matrix, and integrating over the fifth coordinate, we find the electromagnetic current of the Wess-Zumino action:

$$J^{\mu}=\frac{n\,\varepsilon^{\mu\nu\alpha\beta}}{192\pi^2}\int[\mathrm{Tr}(QR_\nu R_\alpha R_\beta)+\mathrm{Tr}(QL_\nu L_\alpha L_\beta)]d^4x\ , \tag{12}$$

where

$$L_\mu=\tfrac{1}{2}U^{\dagger 2}\partial_\mu U^2 \tag{13a}$$

and

$$R_\mu=\tfrac{1}{2}\partial_\mu U^2 U^{\dagger 2}\ . \tag{13b}$$

Equation (12) is 4 times smaller than the one obtained in the standard Skyrme model due to the normalization of the currents in Eq. (5). Following Ref. [7] we calculate the amplitude for the process $\pi_0\to\gamma\gamma$. Expanding U to lowest order in the pion field and replacing the derivatives of Eq. (14) by covariant derivatives,

$$\partial_\mu\to\partial_\mu+ieA_\mu\ , \tag{14}$$

we will find [see Eq. (21) of Ref. [7]]

$$A=\frac{ne^2}{768\pi^2F_\pi}\pi^0\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\ . \tag{15}$$

The amplitude of Eq. (15) is 16 times smaller than the expression of Ref. [7] due to the normalization factors of the currents of Eq. (5). Although we have five currents in the expression of the Wess-Zumino action above producing a denominator of 2^5 smaller than the one we obtain with the current of Eq. (2), a factor of 2 still appears upon expansion of U^2 in terms of the pionic fields. The electromagnetic coupling is identical in both cases and does not introduce any extra factors. The denominator then becomes 16 times smaller. In order to reproduce the measured value of the pion decay rate, the factor n in Eq. (15) has to be equal to 3 times 2^4.

The neutral pion decay amplitude is inversely proportional to f_π. The decay width of the neutral pion of 7.63 eV needs a value for f_π of 93 MeV. In the Skyrme model f_π is varied. A fit to nucleon and delta masses needs a much lower value, implying a decay rate as much as a factor of 4 bigger than the experiment. In the present approach we will see that we can use a realistic value of f_π and therefore our prediction of the neutral pion decay rate is more appropriate. Due to the normalization factor n of Eq. (15), the $VAAA$ vertex is here a factor of 4 bigger than the prediction of the anomaly [7]. The amplitude for the $VAAA$ vertex process is, nevertheless, inversely proportional to f_π^3 and, therefore, the prediction of the present model will be numerically almost the same—and even better if one uses the value of f_π needed in the SU(3) case [3]—as the conventional Skyrme model prediction. (A measurement of this amplitude was recently performed by Antipov *et al.* [15] and found to be in agreement with the QCD prediction.) Moreover, the present model provides us with an alternative picture of mesons as quark-antiquark composites. The calculation of the various photon (and related) vertices should therefore proceed along a different avenue as the one defined by the Wess-Zumino Lagrangian. It is then reasonable to assume that the higher-order vertices predicted by the Wess-Zumino tree-level amplitude are not to be taken as a strong constraint on the value of the constant n.

We, therefore, chose to fix n by comparing to the decay of the neutral pion only. (In the future work we will address the quark-antiquark picture of mesons [9].)

It is straightforward to identify the anomalous baryon current

$$B^{\mu}=\frac{1}{6\pi^2}\varepsilon^{\mu\nu\alpha\beta}\mathrm{Tr}(L_\nu L_\alpha L_\beta)\ , \tag{16}$$

where L_μ is the current of Eq. (13a). The new baryon current is four times bigger than the usual one expressed in terms of the currents of Eq. (2). Below we will find that this definition will allow fractional and integer baryon number solutions. Our world consists of integer baryon number particles while the fractional baryon number particles are absolutely confined. We will find that this phenomenology can be recovered, if we introduce manifest chiral symmetry breaking in the kinetic terms built from the currents of Eq. (5).

We build the following broken symmetry current,

$$M_\mu=\tfrac{1}{2}[(1+g)U^\dagger\partial_\mu U-(1-g)U\partial_\mu U^\dagger]\ , \tag{17}$$

where g is a parameter that reflects the degree of chiral symmetry breaking. It has to be equal to zero in the chiral limit, and close to zero in the broken phase. A suitable upper limit inspired in chiral dynamics could be taken to be of the order of 0.23.

The modified Skyrmion Lagrangian now reads

$$L=-\frac{F_\pi^2}{16}\int d^3x\ \mathrm{Tr}(M_\mu M^\mu)+\frac{1}{32e^2}\int d^3x\ \mathrm{Tr}(N_\mu,N_\nu)^2\ . \tag{18}$$

M_μ is constructed such that it gives the correct pion kinetic-energy asymptotically. N_μ differs from M_μ by the numerical value of the symmetry-breaking parameter g. [There is no reason *a priori* to demand the same value for g in both terms of Eq. (18), thus we use two different parameters g, in actual applications we fix both of them to be equal, see below.]

Expanding Eq. (18) to lowest order in the chiral field and its fluctuations, we will discover pion-nucleon couplings with a different strength for vector and pseudovector interactions, only when g is nonvanishing.

We now proceed to investigate the classical solutions of the model. Introducing the hedgehog ansatz [1,2] in the modified Lagrangian and performing the adiabatic rotation of U with $R(t)$ of Eq. (3) we find the static mass of the Skyrmion M and the moment of inertia Λ to be [see Eqs. (2) and (4) of Ref. [2] for notation]

$$M=\frac{\pi F_\pi^2}{2}\int r^2dr\left[\theta'^2+\frac{\sin^2(2\theta)}{2r^2}+\frac{2g_1^2\sin^4(\theta)}{r^2}\right]$$
$$+\frac{\pi}{e^2}\int r^2dr\left[\left[\theta'^2+\frac{\sin^2(2\theta)}{8r^2}\right]\frac{\sin^2(2\theta)}{r^2}+\left[4\theta'^2+\frac{\sin^2(2\theta)}{r^2}\right]\frac{g_2^2\sin^4(\theta)}{r^2}+\frac{2g_2^4\sin^8(\theta)}{r^4}\right], \tag{19a}$$

$$\frac{3}{2\pi}\Lambda=\int r^2dr\,F_\pi^2\left[\frac{\sin^2(2\theta)}{4}+g_1^2\sin^4(\theta)\right]$$
$$+\int r^2dr\frac{1}{e^2}\left[\left[\theta'^2+\frac{\sin^2(2\theta)}{4r^2}\right]\sin^2(2\theta)+4g_2^2\sin^4(\theta)\left[\theta'^2+\frac{\sin^2(2\theta)}{2r^2}\right]+\frac{4g_2^4\sin^8(\theta)}{r^2}\right], \tag{19b}$$

where g_1 (g_2) are the symmetry-breaking parameters corresponding to the first (second) term in Eq. (18), and a prime denotes radial derivative.

From Eq. (19) we see that finite-energy solutions demand the boundary condition for the chiral angle at the origin to be $\theta=n\pi$ with g_1 and g_2 nonzero, while for $g_1=g_2=0$ the boundary condition is $\theta=n\pi/2$.

Inserting the hedgehog ansatz into the baryon current of Eq. (17) we find the baryon density

$$B^0(r)=-\theta'(r)\frac{\sin^2[2\theta(r)]}{2\pi^2r^2}\ . \tag{20}$$

The corresponding baryon charge becomes (taking $\theta=0$ at infinity)

$$B=\frac{1}{\pi}\left[\theta(0)-\frac{\sin[2\theta(0)]}{4}\right]. \tag{21}$$

From Eq. (21) we see that for g nonzero and $\theta(0)=n\pi$ the baryon number is integer, whereas for $g=0$ and $\theta(0)=n\pi/2$, the baryon number is a multiple of $\frac{1}{2}$. [In a later work we treat the quark and meson sectors in detail. We will find there that it is easy to modify the current of Eq. (18) in order to have constituent quarks with $\frac{1}{3}$ baryon number and spin $=\frac{1}{2}$ [9].]

In the conventional Skyrme model, baryons are sought to arise as topological solitons in the large N_c limit of QCD. The chiral condensate of pions generates the baryons through topology. If manifest symmetry breaking is ignored, the strong interaction world described by it, is one of massless mesons and massive topological solitons recognized as baryons. Confined quarks do not appear at all in the picture.

The present approach allows for a different interpretation of the topological soliton model. In the chiral limit the parameter of manifest breaking $g=0$. Fractional baryon number solutions are possible in that limit. Those solutions can be identified as the constituent quarks of the model. These solutions cease to exist when symmetry breaking is introduced. The quarks cannot be free, because their energy would be infinite (boundary condition at the location of the source would lead to the divergence). Chiral symmetry breaking and confinement are more intimately related here. As in the nonrelativistic quark model, pions will arise, not as Goldstone bosons, but as quark-antiquark states, obeying topological constraints and partially fulfilling chiral symmetry. (For this reason we omit explicit meson mass terms from the Lagrangian.) As in the case of baryons, pions, and the mesons in general, cannot decay into isolated quarks, because quarks cannot exist as free objects in the chirally broken phase. The quarks arise as topological solitons in the pion condensate as in the qualiton model [12]. There, however, quarks arise as solitons in an SU(3) condensate of *color* and confinement is expected to originate from the coupling to gluons. In the present approach, chiral symmetry is manifestly broken in the kinetic-energy terms. No mater how small the symmetry-breaking parameter is it will have a leading role in determining the spectrum of states of the model.

The Euler-Lagrange equation of motion for the static hedgehog soliton is

$$\theta''\left[1+\frac{\sin^2(2\theta)}{2x^2}+\frac{2g_2^2\sin^4(\theta)}{x^2}\right]+\frac{2\theta'}{x}+\theta'^2\left[\frac{\sin(4\theta)}{2x^2}+\frac{2g_2^2\sin(2\theta)\sin^2(\theta)}{x^2}\right]-\frac{\sin(4\theta)}{2x^2}$$
$$-\frac{2g_1^2\sin(2\theta)\sin^2(\theta)}{x^2}-\frac{\sin(4\theta)\sin^2(2\theta)}{8x^4}-\frac{g_2^2\sin^3(2\theta)\sin^2(\theta)}{2x^4}-\frac{g_2^2\sin(4\theta)\sin^2(\theta)}{2x^4}-\frac{2g_2^4\sin(2\theta)\sin^6(\theta)}{2x^4}=0\ , \tag{22}$$

where $x=eF_\pi r$ and primes denote derivatives with respect to x. Two easy checks of Eqs. (19) and (22) are at hand. In the limit of $g_1=g_2=1$ we have to recover the expressions of Ref. [2] and indeed we do. The second limit, when both parameters are set to zero, has to yield the same expressions as those of Ref. [2], but for double the chiral phase and suitable factors of $\frac{1}{4}$ and $\frac{1}{16}$ for each the first and second terms of Eq. (18), again the formulas prove to be correct.

Static observables for the modified Skyrmion can be found in the same way as for the standard Skyrmion [2]. In order to do so we generated the left and right unbroken currents with the transformations of Eq. (6). The moment of inertia of Eq. (19b) enters in the expression of some observables whereas for the calculation of the axial-vector coupling constant g_A we need the parameter D' [see Eq. (16) of Ref. [2]]:

$$D'=F_\pi^2\left[\theta'+\sin(2\theta)\frac{\cos^2(\theta)+g_1^2\sin^2(\theta)}{r}\right]+\frac{4[\cos^2(\theta)+g_2^2\sin^2(\theta)]}{e^2}\left[\frac{\sin(2\theta)\theta'^2}{r}+\frac{2\sin^2(\theta)\theta'}{r^2}+\sin(2\theta)\sin^2(\theta)\frac{\cos^2(\theta)+g_2^2\sin^2(\theta)}{r^3}\right]. \tag{23}$$

Taking values for g_1 and g_2 between 0 and 0.22 we find that we are able to fit the nucleon and delta resonance masses with a value of F_π of approximately 162 MeV, almost independently of g_1 and g_2 and e around 3. For $g_1=g_2=0$ and the boundary condition at the origin $\theta=\pi/2$ appropriate for the constituent quarks, we obtain soliton solutions that have a static mass of the order of 160 MeV with the same parameters.

The moment of inertia for these solutions is very small, yielding a very large rotational energy, as found for qualitons [12]. A better quantization procedure is needed to handle the spin-isospin degrees of freedom of the quark. Verschelde [13] found that a careful treatment of the coupling to quantum rotations and vibrations by means of Dirac constraints yields a net reduction of the static energy equal to the rotational excitation of the nucleon of the order of 80 MeV.

If we introduce such a quantum correction, we will be able to fit the nucleon and delta resonance masses with an even better value of F_π of approximately 170 MeV. This correction is therefore only minor and operates in the right direction. On the other hand, a calculation of the Casimir energy in the Skyrme model [14] indicates a much larger subtraction of the order of 1 GeV for the nucleon. Such a subtraction would leave us with nucleon mass very close to zero. However, this calculation has several limitations, such as the neglect of the quartic, or Skyrme term, of Eq. (1): the need to introduce regularization schemes with input parameters coming from other models (such as chiral perturbation theory), and the very important omission of coupling to zero modes that prove to be crucial in order to identify the proper degrees of freedom. We therefore prefer to adhere to the exact determination of Verschelde [13] in considering quantum fluctuations a small correction.

Almost independently of g_1 and g_2, the values of the static observables for the integer baryon case are approximately as follows: nucleon isoscalar root-mean-square radius $\langle r^2\rangle_{I=0}^{1/2}=0.6$ fm, proton magnetic factor $\mu_{\text{proton}}=1.9$, neutron magnetic factor $\mu_{\text{neutron}}=-1.4$, pion-nucleon coupling constant $g_{\pi NN}=8$, and $g_A=0.65$. [The value of g_A is still small, but it is known to improve dramatically in SU(3).] These results are essentially the same as those of the original Skyrme model [2], but now with a much more realistic value of F_π.

III. THE *NN* INTERACTION

We have managed to alleviate one of the problems of the Skyrme model and fitted the masses of the nucleon and the delta with a more reasonable value of F_π. Can we improve the predictions of the Skyrme model for the nucleon-nucleon interaction with the modified Lagrangian? The answer here is affirmative too.

As mentioned in the Introduction, the intermediate-range central attraction between nucleons is notoriously absent in all the straightforward uses of the Skyrme model. In order to investigate the central interaction between nucleons we follow the procedure of previous works [4,5]. The two-body ansatz we use is the so-called product ansatz for two baryons located at positions $\mathbf{r}_1$ and $\mathbf{r}_2$. It has the virtue of carrying baryon number two and describing to separate baryons at large interbaryon separation $\mathbf{r}_1-\mathbf{r}_2$. It also allows for a rather easy quantization of the spin-isospin degrees of freedom by means of each baryon collective coordinates A and B, explicitly

$$U(\mathbf{r}_1,\mathbf{r}_2)=BU_0(\mathbf{r}_1)B^\dagger AU_0(\mathbf{r}_2)A^\dagger . \tag{24}$$

Inserting Eq. (24) into Eq. (18) and subtracting the one-baryon contributions we obtain the two-body static Lagrangian. We evaluated matrix elements of the two-body Hamiltonian between suitable two-body collective-coordinate wave functions using a Monte Carlo method with three million mesh points, obtaining an accuracy of at least 10%. Comparing to standard parametrizations

TABLE I. Nucleon-nucleon central potential V_c in MeV as a function of internucleon distance R in Fermi for $g_1=g_2=0.22$, and 0.14, 0, respectively.

R		V_c	
0	894	996	1105
0.4	371	406	437
0.8	125	145	164
1.2	29	37	44
1.6	1	4	7
2.0	−4	−3	−2
2.4	−2	−1	0
2.8	−1	0	0
3.2	0	0	0

of the nucleon-nucleon interaction, we find the central potentials shown in Table I. We have chosen for the calculation g_1 and g_2 ranging from 0 to 0.22. For each case we used the appropriate soliton profile. It is clear that, increasing the value of the symmetry-breaking parameter, one gets an intermediate-range central potential of depth comparable to that obtained in phenomenological fits such as the Paris potential, and at the right internucleon separation. As can be seen from Table I, the smaller the symmetry-breaking parameters, the shallower the depth of the attractive component; until when the parameters are set to zero it almost disappears completely and the central potential is repulsive throughout. This is a most remarkable effect.

I thank Professor Judah Eisenberg and Professor J. R. Donoghue for most valuable remarks and suggestions.

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