Gauge invariance and Compton scattering from relativistic composite systems

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Using the Ward-Takahashi (WT) identity and the Bethe-Salpeter wave equation, we investigate the dynamical requirements imposed by electromagnetic gauge invariance on Compton scattering from relativistic composite systems. The importance of off-shell rescattering in intermediate states, which is equivalent to final state interactions in inclusive processes, is clarified in the context of current conservation. It is shown that, if the nuclear force is nonlocal, there will be both two-photon interaction currents and rescattering contributions to terms involving one-photon interaction currents. We derive the two-body WT identity for the two-photon interaction currents, and obtain explicit forms for the interaction current operators for three illustrative models of nuclear forces: (a) two-pion exchange forces with baryon resonances, (b) covariant separable forces, and (c) charged one-pion exchange.

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I. INTRODUCTION

Inclusive electron scattering from composite systems is an important subject common to nuclear physics [1], high-energy particle physics [2], and condensed matter physics [3]. Using this technique, the underlying dynamics of composite systems can be investigated in different energy regions, and we can study the scaling laws important to each region, such as Bjorken scaling [4] in particle physics, and y scaling [5, 6] in nuclear physics. The structure functions of inclusive electron scattering are related to matrix elements of the commutator of the electromagnetic current operator, and they can be expressed in terms of the imaginary part of the virtual Compton amplitude for scattering from the ground state of the system. Thus Compton scattering is important both because of its close connection to the physics of inclusive process and also because observables extracted from Compton scattering, such as the polarizability of hadrons [7, 8], are a sensitive indication of hadron structure. In this paper we discuss how relativistic calculations of Compton scattering from few-body composite systems can be done in a gauge invariant manner.

In high-energy particle theory, the current-current correlator appearing in the inclusive amplitude is assumed to be gauge invariant from the beginning,

$$\int d^4x e^{iqx} \langle p|T[J^\mu(x)J^\nu(0)]|p\rangle = [q^\mu q^\nu - g^{\mu\nu}q^2]\mathcal{O}(q)\,,$$

and the essential quantity, $\mathcal{O}(q)$, is sometimes evaluated using current algebra [9]. Alternatively, the coefficients (C_n) in the operator product expansion [10], $\mathcal{O}(q) = \sum_n C_n\; q^{-2n}$, can be calculated perturbatively [11, 12] for an asymptotically free theory [13]. In these approaches it is not explained how to obtain a gauge invariant result from the underlying few-body dynamics which describes the target system, and difficulties are encountered in calculating the electromagnetic polarizability, or gauge invariant Compton amplitudes with composite models of hadron structure [14], such as quark models. In general, it is a very difficult task to evaluate the photohadronic four-point functions because electromagnetic gauge invariance is closely related to the dynamics of the strong interaction. The solution requires, in principle, that we understand the structure of all composite intermediate states which can be excited by the photon. For example, the photoproduction of pions from a nucleon involves contributions from many excited states of the nucleon, depending upon the energy transferred to the intermediate state. In order to conserve electromagnetic current, the dynamics describing the final state interaction between the pion and nucleon should be consistent with the one describing the intermediate excited states.

In nuclear physics a dynamical approach is both feasible and necessary. For example, many quantitative estimates show the importance of final state interactions (FSI) in inclusive processes within both a nonrelativistic [15–17] and a relativistic [18, 19] framework. Issues of gauge invariance in Compton scattering have been intensively studied within a nonrelativistic framework [20, 21], but few systematic studies have been carried out in a relativistic framework [22]. Compton scattering from the deuteron has been recently studied in Ref. [23]. In this paper we investigate, within the Bethe-Salpeter (BS) formalism, the dynamical requirements imposed by gauge invariance on the amplitudes of nuclear Compton scattering and electrodisintegration.

Our analysis can be regarded as an extension of the work reported in Refs. [24–26]. In these references a general constraint on two-body electromagnetic currents (*interaction currents*) is derived and expressed directly in terms of the nuclear interaction. This so-called *two-*

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body Ward-Takahashi (WT) identity can be derived from the one-body WT identity [27] using current conservation and two-body wave equations to relate the two-body nuclear force to the divergence of the interaction current. The general result is independent of the details of the nuclear force model once the dynamical degrees of freedom are specified. Recently the interaction current operator implied by a covariant separable interaction has been analytically derived [26] by using minimal substitution [28]. The result satisfies the two-body WT identity, and confirms the existence of an interaction current. The resulting interaction current contributions to electromagnetic form factors have been studied by evaluating their matrix elements between bound state wave functions, and they are not negligible. In this paper we take a similar approach and study, for Compton amplitudes, the dynamical connections between the (a) impulse approximation, (b) off-shell rescattering processes, and (c) interaction currents. In particular, we find that a new interaction current that involves two photons (*the two-photon interaction current*) is required in order to satisfy gauge invariance. The efficiency of our rather formal approach, introduced by Gross and Riska [24], will be a help in our analysis of the dynamically complex four-point function describing relativistic nuclear Compton scattering.

In general, interaction currents will be present whenever the nuclear interaction is nonlocal, even if there are no charge exchange forces involved. Realistic nucleon-nucleon (NN) interactions are nonlocal in the mid- to short-range regions, and, as a result of this nonlocality, the NN interaction potential (or relativistic kernel) depends on three of the four coordinates of the particles involved in the interaction [see Fig. 1(a)]. This nonlocality may originate from the fact that the hadrons are spatially extended objects, leading to a short-range nuclear correlation. The nonlocal NN force has been well approximated by introducing a number of heavy mesons [29] whose Compton wavelengths are even shorter than the size of nucleon, or by a superposition of local Yukawa functions for 12 different mass parameters [30]. These local models give a fairly good description of the phase shifts. For example, the local "σ" exchange is a good approximation for the nonlocal force [31] generated by the two-pion exchange mechanism with excited baryons [Fig. 1(b)]. Another source of the nonlocality is the quark-gluon exchange process, which generates a short-range NN repulsion, and this becomes nonlocal if it is expressed as an effective NN interaction [32,33]. In spite of these interesting, clearly physical mechanisms, it is hard to distinguish local interactions from nonlocal ones within the context of elastic NN scattering.

If the electromagnetic field is present, however, we may expect a new mechanism to take place as a consequence of the nonlocality: within the *nonlocal region*, $d = |x_1' - x_1| \sim |x_2' - x_2|$ in Fig. 1(a), the photon field may interact with the charged constituents participating in the nonlocal interaction, $V(x_1', x_2'; x_1, x_2)$. For example, a photon may interact with a pion or an excited baryon in the two-pion exchange potential [Fig. 1(b)], or couple to a quark in the quark exchange process [34]. In this way, interaction currents [Figs. 2(a) and 2(b)] are induced by nonlocal nuclear forces, and this happens even if the effective nuclear force does not involve any charge exchange.

Much attention has been paid recently to two-photon processes, such as the polarizability of the nucleon, including the effect of the pion cloud [35]. The dynamics of the πN system involves a nonlocality through the propagation of Δ (and/or the Chew-Low-type [36] iteration scheme) for the annihilation and creation of pions. The method developed in this paper becomes extremely useful in dealing with this topic, including photopion production from a nucleon.

This paper is organized as follows. We start in Sec. II with a study of the electrodisintegration of deuteron using the conventional impulse approximation and final state interactions. The need for interaction currents when the forces are nonlocal is demonstrated. In Sec. III we study the Compton amplitude including the impulse and off-shell intermediate scattering processes. In Sec. IV the one-photon interaction current and its rescattering processes are introduced, and we show that the result is not gauge invariant unless two-photon interaction currents are introduced. In Sec. V we derive the two-photon interaction current operators from three simple models of NN force: (a) two-pion exchange interactions with

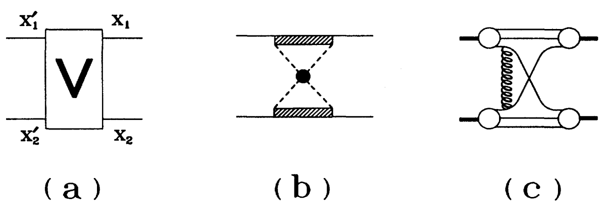

FIG. 1. (a) A general nonlocal potential in coordinate space. (b) Diagram for the nonlocal NN interaction including excited baryons. The dashed line is a pion with the correlations (the solid circle), the shaded line is an excited baryon, and the solid line is a nucleon. (c) NN interaction through the quark exchange process, where the fat solid line is a nucleon, viewed as a three-quark system, and the spiral line is a gluon.

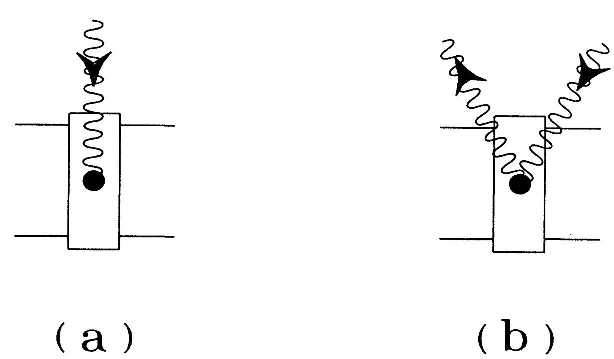

FIG. 2. Diagrams for (a) the one-photon interaction current and (b) the two-photon interaction current. The wavy line is a photon and the solid line is a nucleon.

baryon resonances, (b) covariant separable interactions, and (c) the one-pion exchange (OPE) interaction. By an extension of the results obtained by Gross and Riska [24], it can be shown that the results we obtain for the two-photon interaction currents are not unique, but this will not be discussed further in this paper. We summarize this work in Sec. VI.

Throughout this paper (except for the interaction current derived from OPE in Sec. V) we consider charge nonexchange nuclear forces only, in order to present the discussion in a simple and efficient form. We focus on the interaction currents required by the nonlocality of the nuclear dynamics. We also restrict ourselves to a nuclear two-body system, i.e., the deuteron, for simplicity. We believe that these ideas can be extended to three-body systems described by Faddeev equations, and to relativistic dynamics based on the Gross equation, but these extensions will not be presented in this paper.

II. ELECTRODISINTEGRATION

In this section we show that interaction currents are required if the forces are nonlocal, even if there is no charge exchange. As a simple exercise, we investigate the dynamical structure of electrodisintegration process, and this leads to the consideration of Compton scattering. Here, we assume that the relativistic composite system (the deuteron) is described by the Bethe-Salpeter (BS) equation, and we use the matrix representation [37] for the two-fermion system.

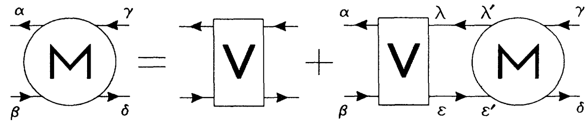

FIG. 3. Representation of the Bethe-Salpeter equation for the scattering matrix, $M(k',k;p)$, with an interaction kernel $V(k',k;p)$. The matrix representation for a two-nucleon system [35] is used.

A. Impulse amplitude and final state interaction

The BS equation for the scattering matrix (Fig. 3), $M(k',k;\,p)$, is given by

$$M_{\alpha\beta;\delta\gamma}(k',k;p) = V_{\alpha\beta;\delta\gamma}(k',k;p) + i\int \frac{d^4k''}{(2\pi)^4}\, V_{\alpha\beta;\epsilon\lambda}(k',k'';p) S_{\lambda\lambda'}\left(k''+\frac{p}{2}\right)\, M_{\lambda'\epsilon';\delta\gamma}(k'',k;p)\; S_{\epsilon'\epsilon}\left(k''-\frac{p}{2}\right), \tag{2.1}$$

where $k'=\frac{1}{2}(p_1'-p_2')$ and $k=\frac{1}{2}(p_1-p_2)$ are the relative momentum of two nucleons in the final and initial states, and $p=p_1+p_2$ is the total momentum. Here, $S_{\alpha\beta}(q)=[\not{q}-m+i\epsilon]^{-1}_{\alpha\beta}$ is the propagator for the nucleon, and greek characters are used for the Dirac indices. The solution has the form

$$M_{\alpha\beta;\delta\gamma}(k',k;p) = \frac{\Gamma_{\alpha\beta}(k';p)\overline{\Gamma}_{\delta\gamma}(k;p)}{p^2-M_B^2} + R_{\alpha\beta;\delta\gamma}(k',k;p), \tag{2.2}$$

where the first term represents a pole term due to the lowest bound state with the mass M_B, and $R(k',k;\,p)$ is regular at that energy. Inserting this expression into Eq. (2.1) and taking the residue at the ground state pole; $\lim_{p^2\to M_B^2}$ $(p^2-M_B^2)$ times Eq. (2.1), we get the wave equation (Fig. 4) for the bound state vertex, $\Gamma(k;\,p)$. Defining the BS wave function by $\Psi_{\alpha\beta}(k;p)=\left[S(k+\frac{p}{2})\Gamma(k;p)S(k-\frac{p}{2})\right]_{\alpha\beta}$, we have the wave equations for the bound state (Ψ) and the conjugate state ($\overline{\Psi}$) wave functions,

$$\Psi_{\alpha\beta}(k;\,p) = i\int \frac{d^4k'}{(2\pi)^4} S_{\alpha\gamma}\left(k+\frac{p}{2}\right) V_{\gamma\delta;\epsilon\lambda}(k,\,k';\,p) S_{\delta\beta}\left(k-\frac{p}{2}\right)\Psi_{\lambda\epsilon}(k';\,p), \tag{2.3a}$$

$$\overline{\Psi}_{\alpha\beta}(k';\,p) = i\int \frac{d^4k}{(2\pi)^4} \overline{\Psi}_{\delta\gamma}(k;\,p) S_{\alpha\epsilon}\left(k'-\frac{p}{2}\right) V_{\gamma\delta;\epsilon\lambda}(k,\,k';\,p) S_{\lambda\beta}\left(k'+\frac{p}{2}\right). \tag{2.3b}$$

We can express the amplitudes for electrodisintegration in terms of the solutions for the bound state wave function and for the scattering amplitude. Here we assign the first particle to be a proton with the charge $e_1=e_p$ and the second one to be a neutron with $e_2=e_n=0$. The impulse (IMP) amplitude [Fig. 5(a)] is given by

$$\mathcal{M}^{\mu}_{\alpha\beta}(\mathrm{IMP}) = e_p\;\gamma^{\mu}_{\alpha\delta}\Psi_{\delta\gamma}\left(k-\frac{p}{2};p\right)S^{-1}_{\gamma\beta}(k-p), \tag{2.4}$$

where the free spinor functions, $\bar{u}_\alpha(k+q)$ for the pro-

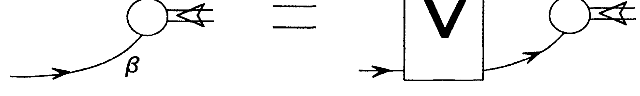

FIG. 4. The Bethe-Salpeter equation for the bound state vertex, $\Gamma(k;p)$, represented by the open circle.

ton and $v_\beta(k-p)$ for the neutron with the respective outgoing momenta $k+q$ and $p-k$, are dropped from the final state. For simplicity, we use $e_p\gamma^\mu$ for the photon-proton vertex; it is not essential in this work to use a realistic form of the off-shell one-body current, $\Lambda^\mu(q) = F_1(q)\gamma^\mu + iF_2(q)\sigma^{\mu\nu}\frac{q_\nu}{2m}$, and to include the photon-neutron coupling. [If we assume $\Lambda^\mu(q)$ satisfies the Ward-Takahashi identity, then $\gamma^\mu \to \Lambda^\mu(q)$ in Eq. (2.6) below, and all of our results also apply when $\gamma^\mu \to \Lambda^\mu(q)$. We refer to Ref. [24] for an explanation of how to define the off-shell one-body current [38] so that the results of this paper can be extended to a realistic current operator.] The amplitude with final state interaction (FSI), Fig. 5(b), is given by

$$\mathcal{M}^\mu_{\alpha\beta}(\mathrm{FSI}) = ie_p\int\frac{d^4k'}{(2\pi)^4}M_{\alpha\beta;\eta\delta}\left(k-\frac{p}{2}+\frac{q}{2},k'-\frac{p}{2}+\frac{q}{2}\ ;\ p+q\right)S_{\delta\epsilon}(k'+\ q)\gamma^\mu_{\epsilon\xi}\Psi_{\xi\eta}\left(k'-\frac{p}{2}\ ;\ p\right), \tag{2.5}$$

where $M(k',k\ ;\ p)$ is the solution of Eq. (2.1).

We test the gauge invariance of the IMP and FSI amplitudes by evaluating the divergence of the electromagnetic current for these contributions. By using the one-body Ward-Takahashi (WT) identity [27],

$$q_\mu\gamma^\mu = S^{-1}(k+q) - S^{-1}(k), \tag{2.6}$$

we get

$$\begin{aligned}q_\mu\mathcal{M}^\mu_{\alpha\beta}(\mathrm{IMP}) &= e_p\left\{S^{-1}(k+q)-S^{-1}(k)\right\}_{\alpha\delta}\Psi_{\delta\gamma}\left(k-\frac{p}{2};p\right)S^{-1}_{\gamma\beta}(k-p)\\ &= e_p\left[S^{-1}(k+q)\Psi\left(k-\frac{p}{2}\ ;\ p\right)S^{-1}(k-p)\right]_{\alpha\beta}\\ &\quad -ie_p\int\frac{d^4k'}{(2\pi)^4}V_{\alpha\beta;\epsilon\lambda}\left(k-\frac{p}{2},k'-\frac{p}{2}\ ;\ p\right)\Psi_{\lambda\epsilon}\left(k'-\frac{p}{2}\ ;\ p\right),\end{aligned} \tag{2.7}$$

for the IMP amplitude. Here, the wave equation (2.3a) is used in the second term, and the first term does not contribute when multiplied by the spinors of the external, on-shell particles. Note that the divergence of the one-body current is related to the two-body interaction, $V(k',k\ ;\ p)$. Likewise, by using Eq. (2.1) and Eq. (2.6) we can express the divergence of the FSI amplitude,

$$\begin{aligned}q_\mu\mathcal{M}^\mu_{\alpha\beta}(\mathrm{FSI}) &= ie_p\int\frac{d^4k'}{(2\pi)^4}M_{\alpha\beta;\eta\delta}\left(k-\frac{p}{2}+\frac{q}{2},k'-\frac{p}{2}+\frac{q}{2}\ ;\ p+q\right)\\ &\quad\times\left\{S_{\delta\epsilon}(k'+q)S^{-1}_{\epsilon\xi}(k'+q)\Psi_{\xi\eta}\left(k'-\frac{p}{2}\ ;\ p\right)\right.\\ &\quad\left.-\ S_{\delta\epsilon}(k'+q)\underline{S^{-1}_{\epsilon\xi}(k')\Psi_{\xi\lambda}\left(k'-\frac{p}{2}\ ;\ p\right)S^{-1}_{\lambda\rho}(k'-p)}S_{\rho\eta}(k'-p)\right\}\\ &= ie_p\int\frac{d^4k'}{(2\pi)^4}M_{\alpha\beta;\epsilon\lambda}\left(k-\frac{p}{2}+\frac{q}{2},k'-\frac{p}{2}+\frac{q}{2}\ ;\ p+q\right)\Psi_{\lambda\epsilon}\left(k'-\frac{p}{2}\ ;\ p\right)\\ &\quad+e_p\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k''}{(2\pi)^4}M_{\alpha\beta;\gamma\delta}\left(k-\ \frac{p}{2}\ +\frac{q}{2},k'-\frac{p}{2}+\frac{q}{2}\ ;\ p+q\right)\\ &\quad\times S_{\delta\rho}(k'+q)V_{\rho\eta;\epsilon\lambda}\left(k'-\frac{p}{2}\ ,\ k''-\frac{p}{2}\ ;p\right)S_{\eta\gamma}(k'-p)\Psi_{\lambda\epsilon}\left(k''-\frac{p}{2}\ ;\ p\right),\end{aligned} \tag{2.8}$$

where we have used the bound state wave equation, Eq. (2.3a), at the underlined part. The total momentum entering into the two-body interaction is p while the one entering into the scattering matrix is $p+q$. We re-express the scattering matrix in the first term of Eq. (2.8) in terms of the scattering equation, Eq. (2.1). This gives the formula

$$q_\mu\left[\mathcal{M}^\mu(\mathrm{IMP})+\mathcal{M}^\mu(\mathrm{FSI})\right]_{\alpha\beta} = ie_p\int\frac{d^4k'}{(2\pi)^4}\mathcal{T}_{\alpha\beta;\epsilon\lambda}(k,k'\ ;\ p,q)\ \Psi_{\lambda\epsilon}\left(k'-\frac{p}{2}\ ;\ p\right), \tag{2.9}$$

where $\mathcal{T}_{\alpha\beta;\epsilon\lambda}(k,k'\ ;\ p,q\ )$ is given by

$$\begin{aligned}\mathcal{T}_{\alpha\beta;\epsilon\lambda}(k,k'\ ;\ p,q\ ) &= \left[V\left(k-\frac{p}{2}+\frac{q}{2},k'-\frac{p}{2}+\frac{q}{2}\ ;\ p+q\right)-V\left(k-\frac{p}{2},k'-\frac{p}{2}\ ;\ p\right)\right]_{\alpha\beta;\epsilon\lambda}\\ &\quad+i\int\frac{dk''}{(2\pi)^4}M_{\alpha\beta;\gamma\delta}\left(k-\frac{p}{2}+\frac{q}{2},k''-\frac{p}{2}+\frac{q}{2}\ ;\ p+q\right)S_{\delta\rho}\left(k''+q\right)\\ &\quad\times\left[V\left(k''-\frac{p}{2}+\frac{q}{2},\ k'-\frac{p}{2}+\frac{q}{2}\ ;\ p+q\right)-V\left(k''-\frac{p}{2},k'-\frac{p}{2}\ ;\ p\right)\right]_{\rho\eta;\epsilon\lambda}S_{\eta\gamma}(k''-p).\end{aligned} \tag{2.10}$$

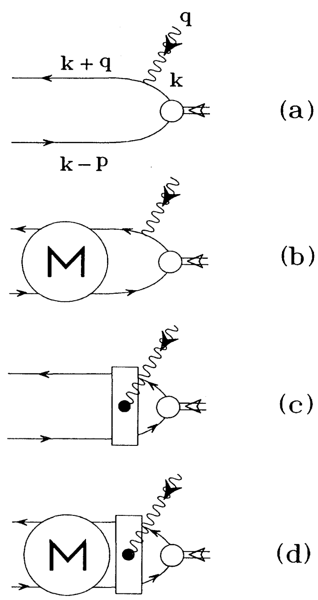

FIG. 5. Diagrams for the electrodisintegration of the deuteron, where the solid line is a nucleon, the wavy line is a photon with momentum q, and the open circle is the deuteron vertex function: (a) impulse approximation (IMP); (b) final state interaction (FSI), where M is the scattering matrix; (c) direct interaction current (DIC), where the rectangular box with a solid circle represents the interaction current; (d) rescattering of the interaction current (RIC).

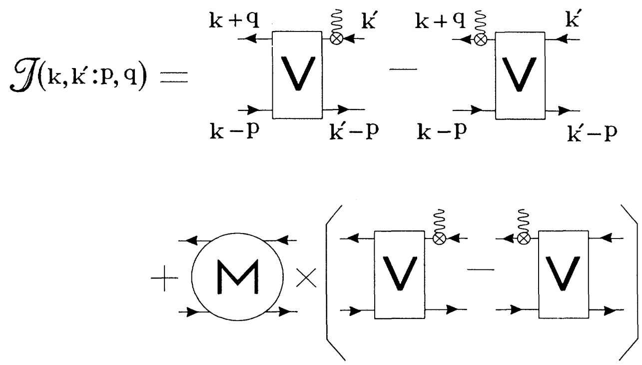

FIG. 6. Diagrammatic representation of Eq. (2.10), where the crossed circle refers to insertion of momentum q by the photon.

This equation expresses the violation of current conservation in the IMP + FSI amplitudes in terms of the nuclear potential and the off-shell scattering matrix.

We now distinguish two cases. If the interaction is local and energy independent, it depends on the momentum transfer only, and $V(k',k;p) = V(k'-k)$. In this case the contents in each square bracket in Eq. (2.10) vanish identically, and the IMP+FSI amplitudes satisfy current conservation. This is the fundamental reason why conventional approaches using local potentials work for the electrodisintegration problem. If the interaction is nonlocal, however, current conservation is not satisfied by these IMP+FSI processes. The violation of current conservation depends on the nonlocality, and is related to the difference between the nuclear potential with the photon momentum (q) inserted *before* the interaction, and *after* the interaction (see Fig. 6). These terms do not cancel for nonlocal interactions, except for the $q = 0$ case. We must find a way to recover current conservation.

B. Current conservation and interaction currents

It has been shown [24–26] that there is usually a two-body electromagnetic current associated with a two-body nuclear interaction (referred to as the *interaction current)*, and that even a charge nonexchange interaction can generate an interaction current, $J^\mu(k',k;[p,q])$, if the interaction is nonlocal [26] [see Fig. 2(a)]. This is because the electromagnetic field can interact with the charged constituents within the nonlocal interaction region. Here, k' (k) is the relative four-momentum of the two nucleons in the final (initial) state, and p is defined to be the total momentum before absorbing a photon with the momentum q. From a study of *elastic* electromagnetic form factors using the bound state BS equation and the one-body WT identity one obtains the following general expression for the divergence of the interaction current:

$$q_\mu J^\mu_{\alpha\beta;\gamma\delta}(k',\ k\ ;[p,q]) = e_P \left[V\left(k'-\frac{q}{2},\ k\ ;\ p\right)\ -\ V\left(k',k+\ \frac{q}{2}\ ;\ p+q\right)\right]_{\alpha\beta;\gamma\delta}. \tag{2.11}$$

This two-body WT identity is a necessary condition which any dynamical model of the interaction current must satisfy. Now we add the interaction current and the rescattering of the interaction current to the amplitude of electrodisintegration, so that all possible first-order processes in the electromagnetic coupling are included. The amplitude for the direct interaction current process (DIC), Fig. 5(c), is given by

$$\mathcal{M}^\mu_{\alpha\beta}(\mathrm{DIC}) = i\int\frac{d^4k'}{(2\pi)^4}J^\mu_{\alpha\beta;\epsilon\lambda}\left(k-\frac{p}{2}+\frac{q}{2},k'-\frac{p}{2}\ ;\ [p,q]\right)\Psi_{\lambda\epsilon}\left(k'-\frac{p}{2}\ ;\ p\right), \tag{2.12}$$

and the one for the rescattering of the interaction current (RIC), Fig. 5(d), is given by

$$\begin{aligned}\mathcal{M}^\mu_{\alpha\beta}(\mathrm{RIC}) = -\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k''}{(2\pi)^4}M_{\alpha\beta;\gamma\delta}\left(k-\frac{p}{2}+\frac{q}{2},k''-\frac{p}{2}+\frac{q}{2}\ ;\ p+q\right)&\\ \times S_{\delta\rho}(k''+q)J^\mu_{\rho\lambda;\epsilon\sigma}\left(k''-\frac{p}{2}+\frac{q}{2},k'-\frac{p}{2}\ ;\ [p,\ q]\right)S_{\lambda\gamma}(k''-p)\ \Psi_{\sigma\epsilon}\left(k'-\frac{p}{2}\ ;\ p\right).&\end{aligned} \tag{2.13}$$

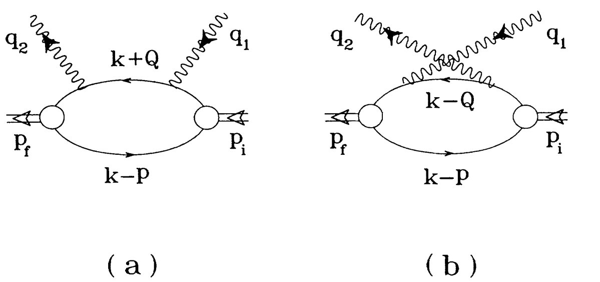

FIG. 7. Diagrams for Compton scattering from the deuteron: (a) impulse amplitude, $\mathcal{C}^{\mu\nu}$(IMP), and (b) the crossed impulse amplitude, $\mathcal{C}^{\mu\nu}(\overline{\mathrm{IMP}})$.

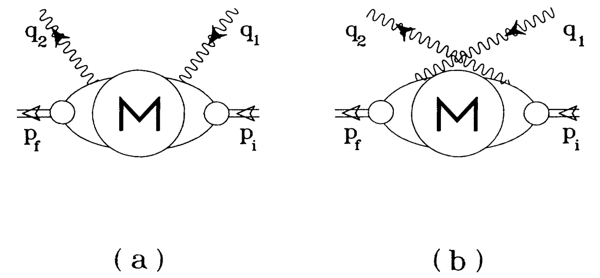

FIG. 8. Diagrams for Compton scattering involving the scattering matrix M: (a) final state interaction, $\mathcal{C}^{\mu\nu}$(FSI), and (b) the crossed FSI process, $\mathcal{C}^{\mu\nu}(\overline{\mathrm{FSI}})$.

By using Eq. (2.11) we observe that the divergence in the DIC and RIC amplitudes cancels with the one in the IMP and FSI processes,

$$q_\mu[\mathcal{M}^\mu(\mathrm{IMP}) + \mathcal{M}^\mu(\mathrm{FSI}) + \mathcal{M}^\mu(\mathrm{DIC}) + \mathcal{M}^\mu(\mathrm{RIC})] = 0. \tag{2.14}$$

The total amplitude, $\mathcal{M}^\mu$(IMP)+$\mathcal{M}^\mu$(FSI)+$\mathcal{M}^\mu$(DIC) +$\mathcal{M}^\mu$(RIC), is gauge invariant.

The two-body WT identity is a general constraint which must hold for any model of the nuclear force. It is a *necessary* condition which the interaction current must satisfy. It shows that interaction currents exist but does not tell us how to find them. In Sec. V we will complete the discussion by deriving explicit forms for the interaction current operator in three different models, and show that the operators we obtain satisfy Eq. (2.11).

III. COMPTON SCATTERING

In this section, we apply the procedures developed in Sec. II to Compton scattering. First we consider the impulse (IMP) amplitude and the intermediate off-shell rescattering of nucleons after the absorption or emission of a photon. We refer these latter processes as "final state interaction" (FSI) processes, since they are similar to ones in electrodisintegration. The Compton amplitudes $\mathcal{C}^{\mu\nu}$(IMP) and $\mathcal{C}^{\mu\nu}(\overline{\mathrm{IMP}})$ for the impulse [Fig. 7(a)] and crossed impulse [Fig. 7(b)] processes are given by

$$\mathcal{C}^{\mu\nu}(\mathrm{IMP}) = ie_p^2 \int \frac{d^4k}{(2\pi)^4} \overline{\Psi}_{\alpha\beta}(K_f;p_f)\,[\gamma^\mu S(k+Q)\gamma^\nu]_{\beta\gamma}\,\Psi_{\gamma\delta}\,(K_i\,;\,p_i) S^{-1}_{\delta\alpha}(k-p), \tag{3.1a}$$

$$\mathcal{C}^{\mu\nu}(\overline{\mathrm{IMP}}) = ie_p^2 \int \frac{d^4k}{(2\pi)^4} \overline{\Psi}_{\alpha\beta}(K_f;p_f)\,[\gamma^\nu S(k-Q)\gamma^\mu]_{\beta\gamma}\,\Psi_{\gamma\delta}\,(K_i\,;\,p_i) S^{-1}_{\delta\alpha}(k-p), \tag{3.1b}$$

where $K_f = k - \frac{p}{2} + \frac{q}{4}$, $K_i = k - \frac{p}{2} - \frac{q}{4}$, $p = \frac{1}{2}(p_f + p_i)$, $Q = \frac{1}{2}(q_1 + q_2)$, and $q = q_1 - q_2$. Here p_i (p_f) and q_1 (q_2) are the four momenta of deuteron and photon in the initial (final) state, respectively. The amplitudes for the FSI [Fig. 8(a)] and $\overline{\mathrm{FSI}}$ [the crossed FSI process, Fig. 8(b)] are given by

$$\mathcal{C}^{\mu\nu}(\mathrm{FSI}) = -e_p^2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} \left[\overline{\Psi}(K_f;p_f)\gamma^\mu S(k+Q)\right]_{\alpha\beta} M_{\beta\alpha;\delta\gamma}(t,t';p+Q)\,[S(k'+Q)\gamma^\nu \Psi(K'_i;p_i)]_{\gamma\delta}\,, \tag{3.2a}$$

$$\mathcal{C}^{\mu\nu}(\overline{\mathrm{FSI}}) = -e_p^2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} \left[\overline{\Psi}(K_f;p_f)\gamma^\nu S(k-Q)\right]_{\alpha\beta} M_{\beta\alpha;\delta\gamma}(\bar{t},\bar{t}';p-Q)\,[S(k'-Q)\gamma^\mu \Psi(K'_i;p_i)]_{\gamma\delta}\,, \tag{3.2b}$$

where $t = k - \frac{p}{2} + \frac{Q}{2}$, $t' = k' - \frac{p}{2} + \frac{Q}{2}$, $\bar{t} = k - \frac{p}{2} - \frac{Q}{2}$, $\bar{t}' = k' - \frac{p}{2} - \frac{Q}{2}$, and $K'_i = k' - \frac{p}{2} - \frac{q}{4}$. Using the WT identity $q_{1\nu}\gamma^\nu = \{S^{-1}(k+Q) - S^{-1}(k-\frac{q}{2})\} = \{S^{-1}(k+\frac{q}{2}) - S^{-1}(k-Q)\}$ in the IMP and $\overline{\mathrm{IMP}}$ amplitudes, we can express the divergences in the following form:

$$\begin{aligned} q_{1\nu}\mathcal{C}^{\mu\nu}(\mathrm{IMP}) &= ie_p^2 \int \frac{d^4k}{(2\pi)^4} \overline{\Psi}_{\alpha\beta}(K_f;p_f) \left\{ \gamma^\mu S(k+Q) \left[S^{-1}(k+Q) - S^{-1}\left(k - \frac{q}{2}\right)\right]\right\}_{\beta\gamma} \Psi_{\gamma\delta}(K_i\,;p_i) S^{-1}_{\delta\alpha}(k-p) \\ &= ie_p^2 \int \frac{d^4k}{(2\pi)^4} \Psi_{\alpha\beta}(K_f;p_f) \left[\gamma^\mu - \gamma^\mu S(k+Q) S^{-1}\left(k - \frac{q}{2}\right)\right]_{\beta\gamma} \Psi_{\gamma\delta}(K_i\,;\,p_i) S^{-1}_{\delta\alpha}(k-p), \end{aligned} \tag{3.3a}$$

and

$$q_{1\nu}\mathcal{C}^{\mu\nu}(\overline{\mathrm{IMP}}) = ie_p^2 \int \frac{d^4k}{(2\pi)^4} \overline{\Psi}_{\alpha\beta}(K_f;p_f) \left[S^{-1}\left(k + \frac{q}{2}\right) S(k-Q)\gamma^\mu - \gamma^\mu\right]_{\beta\gamma} \Psi_{\gamma\delta}(K_i\,;p_i) S^{-1}_{\delta\alpha}(k-p). \tag{3.3b}$$

Note that the γ^μ terms in the square brackets of Eq. (3.3a) and Eq. (3.3b) cancel each other. Likewise, we express the divergence in the FSI and $\overline{\text{FSI}}$ amplitudes,

$$q_{1\nu}\mathcal{C}^{\mu\nu}(\text{FSI}) = -e_p^2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} \left[\bar{\Psi}(K_f;p_f)\gamma^\mu S(k+Q)\right]_{\alpha\beta} M_{\beta\alpha;\delta\gamma}(t,t';p+Q) \times \left[\left\{1 - S(k'+Q)S^{-1}\left(k'-\frac{q}{2}\right)\right\}\Psi(K'_i;p_i)\right]_{\gamma\delta} \tag{3.4a}$$

and

$$q_{1\nu}\mathcal{C}^{\mu\nu}(\overline{\text{FSI}}) = -e_p^2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} \left[\bar{\Psi}(K_f;p_f)\left\{S^{-1}(k'+\frac{q}{2})S(k'-Q) - 1\right\}\right]_{\alpha\beta} \times M_{\beta\alpha;\delta\gamma}(\bar{t},\bar{t}';p-Q)\left[S(k'-Q)\gamma^\mu\Psi(K'_i;p_i)\right]_{\gamma\delta}. \tag{3.4b}$$

Omitting the γ^μ term in the IMP amplitude, which is canceled by a similar term in the $\overline{\text{IMP}}$ amplitude, the sum of the divergence of the IMP and FSI amplitudes is

$$\begin{aligned}
q_{1\nu}\left[\mathcal{C}^{\mu\nu}(\text{IMP}) + \mathcal{C}^{\mu\nu}(\text{FSI})\right] = {}& ie_p^2 \int \frac{d^4k}{(2\pi)^4}\left[\bar{\Psi}(K_f;p_f)\gamma^\mu S(k+Q)\right]_{\alpha\beta} \\
&\times\Bigg\{-\left[\underline{S^{-1}\left(k-\frac{q}{2}\right)\Psi\left(k-\frac{p}{2}-\frac{q}{4};p_i\right)S^{-1}(k-p)}\right]_{\beta\alpha} \\
&+i\int\frac{d^4k'}{(2\pi)^4}M_{\beta\alpha;\delta\gamma}(t,t';p+Q)\,\Psi_{\gamma\delta}\left(k'-\frac{p}{2}-\frac{q}{4};p_i\right) \\
&-i\int\frac{d^4k'}{(2\pi)^4}M_{\beta\alpha;\delta\gamma}(t,t';p+Q) \\
&\times\left[S(k'+Q)\underline{S^{-1}(k'-\frac{q}{2})\Psi(K'_i;p_i)S^{-1}(k'-p)}S(k'-p)\right]_{\gamma\delta}\Bigg\} \\
= {}& ie_p^2 \int \frac{d^4k}{(2\pi)^4}\left[\bar{\Psi}(K_f;p_f)\gamma^\mu S(k+Q)\right]_{\alpha\beta} \\
&\times\Bigg\{-i\int\frac{d^4k'}{(2\pi)^4}V_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}-\frac{q}{4},k'-\frac{p}{2}-\frac{q}{4};p_i\right)\Psi_{\gamma\delta}\left(k'-\frac{p}{2}-\frac{q}{4};p_i\right) \\
&+i\int\frac{d^4k'}{(2\pi)^4}M_{\beta\alpha;\delta\gamma}(t,t';p+Q)\Psi_{\gamma\delta}\left(k'-\frac{p}{2}-\frac{q}{4};p_i\right) \\
&+\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k''}{(2\pi)^4}M_{\beta\alpha;\rho\lambda}(t,t'';p+Q)S_{\lambda\sigma}(k''+Q) \\
&\times V_{\sigma\tau;\delta\gamma}\left(k''-\frac{p}{2}-\frac{q}{4},k'-\frac{p}{2}-\frac{q}{4};p_i\right)S_{\tau\rho}(k''-p)\Psi_{\gamma\delta}\left(k'-\frac{p}{2}-\frac{q}{4};p_i\right)\Bigg\} \\
= {}& e_p^2\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}\left[\bar{\Psi}(K_f;p_f)\gamma^\mu S(k+Q)\right]_{\alpha\beta}\{\odot\}_{\beta\alpha;\delta\gamma}\Psi_{\gamma\delta}(K'_i;p_i),
\end{aligned} \tag{3.5a}$$

where we have used the bound state wave equation (2.3a) at the underlined parts, and

$$\begin{aligned}
\{\odot\}_{\beta\alpha;\delta\gamma} \equiv {}& V_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}-\frac{q}{4},k'-\frac{p}{2}-\frac{q}{4};p_i\right) - M_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}+\frac{Q}{2};p+Q\right) \\
&+i\int\frac{d^4k''}{(2\pi)^4}M_{\beta\alpha;\rho\lambda}(t,t'';p+Q)S_{\lambda\lambda'}(k''+Q)V_{\lambda'\rho';\delta\gamma}\left(k''-\frac{p}{2}-\frac{q}{4},k'-\frac{p}{2}-\frac{q}{4};p_i\right)S_{\rho'\rho}(k''-p),
\end{aligned} \tag{3.5b}$$

where $t'' = k'' - \frac{p}{2} + \frac{Q}{2}$. Using the scattering equation (2.1) to replace the second term in Eq. (3.5b), we get the following compact expression;

$$q_{1\nu}\left[\mathcal{C}^{\mu\nu}(\text{IMP}) + \mathcal{C}^{\mu\nu}(\text{FSI})\right] = -e_p^2\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}\left[\bar{\Psi}(K_f;p_f)\gamma^\mu S(k+Q)\right]_{\alpha\beta} \times\mathcal{T}_{\beta\alpha;\delta\epsilon}\left(k-\frac{q}{2},k'-\frac{q}{2};p-\frac{q}{2},Q+\frac{q}{2}\right)\Psi_{\epsilon\delta}(K'_i;p_i), \tag{3.6a}$$

where $\mathcal{T}(k,k';p,q)$ was defined in Eq. (2.10). Likewise, the divergence of the crossed term is

$$q_{1\nu}[\mathcal{C}^{\mu\nu}(\overline{\rm IMP})+\mathcal{C}^{\mu\nu}(\overline{\rm FSI})]=e_p^2\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}\overline{\Psi}_{\alpha\beta}(K_f;p_f)\mathcal{T}_{\beta\alpha;\delta\gamma}\left(k+\frac{q}{2},k'+\frac{q}{2};p+\frac{q}{2},-Q-\frac{q}{2}\right)$$
$$\times\,[S(k'-Q)\gamma^\mu\Psi(K'_i;p_i)]_{\gamma\delta}\,. \qquad (3.6b)$$

The violation of current conservation is again expressed in terms of $\mathcal{T}(k',k;p,q)$ defined by Eq. (2.10). If the interaction is local and independent of energy, the Compton amplitudes $\mathcal{C}^{\mu\nu}(\text{IMP})+\mathcal{C}^{\mu\nu}(\text{FSI})+\mathcal{C}^{\mu\nu}(\overline{\text{IMP}})+\mathcal{C}^{\mu\nu}(\overline{\text{FSI}})$ are gauge invariant because $\mathcal{T}$ vanishes identically. We emphasize that $\mathcal{C}^{\mu\nu}(\text{IMP})$ and $\mathcal{C}^{\mu\nu}(\overline{\text{IMP}})$ are correlated through the γ^μ terms, which cancel only when both terms are present, and hence all of the diagrams in Figs. 7 and 8 are needed. If the interaction is nonlocal, the sum of all of these processes is not gauge invariant, except for a special case where the momentum of the photons is zero ($\mathcal{T}=0$ for $Q=q=0$). This is the principal conclusion of this section. In the next section, we introduce the additional currents, associated with the nonlocal dynamics, which are needed to conserve current.

IV. TWO-PHOTON WARD-TAKAHASHI IDENTITY

In Sec. II, we discussed the role that the interaction current, with rescattering, plays in the electrodisintegration process. If the interaction is nonlocal these additional terms are necessary to give a gauge invariant amplitude. In this section we include the same terms in the Compton scattering case.

A. Impulse and interaction currents

We now include all possible processes to order e_p^2 which arise from the lowest-order impulse ($\sim e_p$) and interaction ($\sim e_p$) currents. Many diagrams are generated through different combinations of these basic elements, with and without rescattering by M. It is very efficient to use a unified current obtained by adding the impulse current, $ie_p\gamma^\mu$, and the interaction current $J_3^\mu\equiv J^\mu(k',k;[p,q])$:

$$\mathcal{J}^\mu_{\beta\alpha;\delta\rho}(k',k;[p,q])=J_1^\mu+J_3^\mu, \qquad (4.1)$$

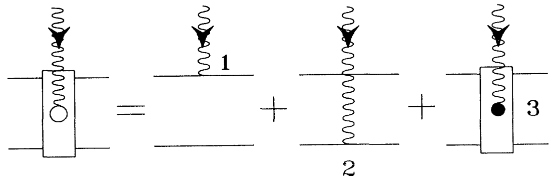

FIG. 9. The unified current $\mathcal{J}^\mu$, Eq. (4.1), is represented by the rectangular box with an open circle. The other rectangular box with a solid circle is the interaction current, J_3. (In this work the neutron current, J_2, is zero.)

where

$$J_1^\mu=ie_p\gamma^\mu_{\beta\rho}S^{-1}_{\alpha\delta}\left(k-\frac{p}{2}\right)(2\pi)^4\delta^4\left(k'-k-\frac{p}{2}\right).$$

This current is illustrated in Fig. 9. The extra factors in J_1^μ convert this one-body operator into a two-body form, so that both currents in Eq. (4.1) have a standard two-body structure. The neutron current, J_2^μ, is zero in the examples discussed in this paper. We can systematically generate all the processes of order e_p^2 by evaluating the four types of diagrams shown in Fig. 10:

$$\mathcal{C}^{\mu\nu}(WXYZ)=\mathcal{C}^{\mu\nu}(W)+\mathcal{C}^{\mu\nu}(X)+\mathcal{C}^{\mu\nu}(Y)+\mathcal{C}^{\mu\nu}(Z). \qquad (4.2)$$

These diagrams are obtained by replacing the one-body current operators with the unified current, $\mathcal{J}^\mu$, in the IMP, FSI, and their crossed diagrams, Eqs. (3.1a) and (3.1b) and Eqs. (3.2a) and (3.2b). Note that each of these diagrams generates four separate diagrams through the choice of J_1^μ or J_3^μ for the coupling with the first and second photons. We distinguish them by the subscripts "1" (for J_1^μ) and "3" (for J_3^μ). For example, the diagram in Fig. 7(b) is denoted as $\mathcal{C}^{\mu\nu}(X_{11})$ and the one of Fig. 8(a) as $\mathcal{C}^{\mu\nu}(Y_{11})$.

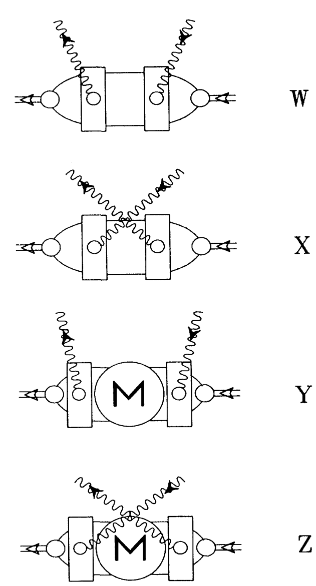

FIG. 10. The four types of amplitudes $\mathcal{C}^{\mu\nu}(W)$, $\mathcal{C}^{\mu\nu}(X)$, $\mathcal{C}^{\mu\nu}(Y)$, and $\mathcal{C}^{\mu\nu}(Z)$ in Eq. (4.2). The rectangular box is the unified current, $\mathcal{J}^\mu$.

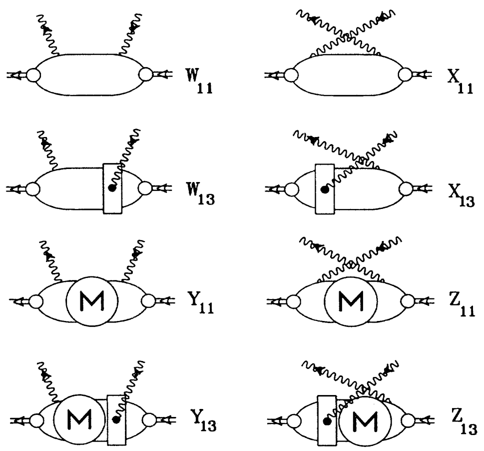

FIG. 11. The eight amplitudes in Eq. (4.3), where the incoming (outgoing) photon has momentum $q_1(q_2)$ and polarization $\epsilon_\nu(\epsilon_\mu)$.

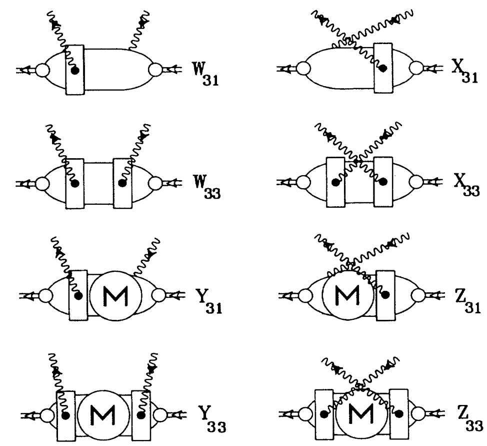

FIG. 12. The eight amplitudes in Eq. (4.4), Eq. (4.8) and Eq. (4.9). The two photons are labeled as in Fig. 11.

B. Current conservation

We will now show that the Compton amplitude given by Eq. (4.2) is not gauge invariant, even though it includes, to order e_p^2, all possible combinations of (1) impulse, (2) FSI, (3) interaction currents, and (4) rescattering of the interaction currents.

First, we prove that

$$q_{1\nu}\left\{\mathcal{C}^{\mu\nu}(W_{11})+\mathcal{C}^{\mu\nu}(W_{13})+\mathcal{C}^{\mu\nu}(Y_{11})+\mathcal{C}^{\mu\nu}(Y_{13})+\mathcal{C}^{\mu\nu}(X_{11})+\mathcal{C}^{\mu\nu}(X_{13})+\mathcal{C}^{\mu\nu}(Z_{11})+\mathcal{C}^{\mu\nu}(Z_{13})\right\}$$

$$\begin{aligned}&= ie_p^2\int\frac{d^4k}{(2\pi)^4}\left[\overline{\Psi}(K_f;p_f)\gamma^\mu S(k+Q)\right]_{\alpha\beta}\left[S^{-1}(k+Q)\Psi(K_i\,;p_i)S^{-1}(k-p)\right]_{\beta\alpha}\\ &\quad -ie_p^2\int\frac{d^4k}{(2\pi)^4}\left[S^{-1}(k-p)\overline{\Psi}(K_f\,;p_f)S^{-1}(k-Q)\right]_{\alpha\beta}\left[S(k-Q)\gamma^\mu\Psi(K_i;p_i)\right]_{\beta\alpha}\\ &=0\end{aligned}\tag{4.3}$$

so that these eight amplitudes, shown in Fig. 11, are separately gauge invariant (at least with regard to the index ν). The proof is somewhat similar to the one leading up to Eq. (2.14), and is presented in Appendix A. Briefly, the first four terms in Eq. (4.3), $\mathcal{C}^{\mu\nu}(W_{11})+\mathcal{C}^{\mu\nu}(W_{13})+\mathcal{C}^{\mu\nu}(Y_{11})+\mathcal{C}^{\mu\nu}(Y_{13})$, have the same structure as the ones in electrodisintegration, $\mathcal{M}^\mu(\text{IMP})+\mathcal{M}^\mu(\text{FSI})+\mathcal{M}^\mu(\text{DIC})+\mathcal{M}^\mu(\text{RIC})$, given in Sec. II. The difference is that the free spinor functions in the final state are replaced by the bound state wave function along with the emission of a photon (q_2), so that the only reason the divergence is not zero is because the final nucleons are off shell. However, the divergence of these four terms is precisely canceled by the four crossed ($q_1\leftrightarrow q_2$) amplitudes, $\mathcal{C}^{\mu\nu}(X_{11})+\mathcal{C}^{\mu\nu}(X_{13})+\mathcal{C}^{\mu\nu}(Z_{11})+\mathcal{C}^{\mu\nu}(Z_{13})$.

Among the remaining eight amplitudes (shown in Fig. 12), we first investigate $\mathcal{C}^{\mu\nu}(W_{31})$, $\mathcal{C}^{\mu\nu}(W_{33})$, $\mathcal{C}^{\mu\nu}(Y_{31})$ and $\mathcal{C}^{\mu\nu}(Y_{33})$. These four amplitudes are given by

$$\mathcal{C}^{\mu\nu}(W_{31})=ie_p\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}\overline{\Psi}_{\alpha\beta}(K_f;p_f)\,iJ^\mu_{\beta\alpha;\delta\gamma}\left(K_f,k'-\frac{p}{2}+\frac{Q}{2};[p+Q,-q_2]\right)\left[S(k'+Q)\gamma^\nu\Psi(K'_i;p_i)\right]_{\gamma\delta},\tag{4.4a}$$

$$\begin{aligned}\mathcal{C}^{\mu\nu}(W_{33})=i\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k''}{(2\pi)^4}\overline{\Psi}_{\alpha\beta}(K_f;p_f)iJ^\mu_{\beta\alpha;\delta\gamma}\left(K_f,k''-\frac{p}{2}+\frac{Q}{2};[p+Q,-q_2]\right)S_{\gamma\epsilon}(k''+Q)\\ \times iJ^\nu{}_{\epsilon\rho;\lambda\phi}\left(k''-\frac{p}{2}+\frac{Q}{2},K'_i;\left[p-\frac{q}{2},q_1\right]\right)S_{\rho\delta}(k''-p)\Psi_{\phi\lambda}(K'_i;p_i),\end{aligned}\tag{4.4b}$$

$$\mathcal{C}^{\mu\nu}(Y_{31}) = ie_p \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k''}{(2\pi)^4} \overline{\Psi}\alpha\beta(K_f;p_f) iJ^{\mu}_{\beta\alpha;\delta\gamma}\left(K_f, k'' - \frac{p}{2} + \frac{Q}{2}; [p+Q, -q_2]\right) S_{\gamma\epsilon}(k''+Q)$$
$$\times iM_{\epsilon\rho;\lambda\phi}\left(k'' - \frac{p}{2} + \frac{Q}{2}, k' - \frac{p}{2} + \frac{Q}{2}; p+Q\right) S_{\rho\delta}(k''-p)[S(k'+Q)\gamma^{\nu}\Psi(K_i';p_i)]_{\phi\lambda}, \tag{4.4c}$$

and

$$\mathcal{C}^{\mu\nu}(Y_{33}) = i \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k''}{(2\pi)^4} \int \frac{d^4\widetilde{k}}{(2\pi)^4} \overline{\Psi}_{\alpha\beta}(K_f;p_f) iJ^{\mu}_{\beta\alpha;\delta\gamma}\left(K_f, \widetilde{k} - \frac{p}{2} + \frac{Q}{2}; [p+Q, -q_2]\right) S_{\gamma\epsilon}(\widetilde{k}+Q)$$
$$\times iM_{\epsilon\phi;\sigma\rho}\left(\widetilde{k} - \frac{p}{2} + \frac{Q}{2}, k'' - \frac{p}{2} + \frac{Q}{2}; p+Q\right) S_{\phi\delta}(\widetilde{k}-p) S_{\rho\lambda}(k''+Q)$$
$$\times iJ^{\nu}_{\lambda\kappa;\eta\xi}\left(k'' - \frac{p}{2} + \frac{Q}{2}, K_i; \left[p - \frac{q}{2}, q_1\right]\right) S_{\kappa\sigma}(k''-p)\Psi_{\xi\eta}(K_i;p_i). \tag{4.4d}$$

We evaluate the divergence of these amplitudes with respect to q_1. Separating out common overall factors,

$$q_{1\nu}[\mathcal{C}^{\mu\nu}(W_{31}) + \mathcal{C}^{\mu\nu}(W_{33}) + \mathcal{C}^{\mu\nu}(Y_{31}) + \mathcal{C}^{\mu\nu}(Y_{33})] = ie_p \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k''}{(2\pi)^4} \overline{\Psi}_{\alpha\beta}(K_f;p_f)$$
$$\times J^{\mu}_{\beta\alpha;\delta\gamma}\left(K_f, k'' - \frac{p}{2} + \frac{Q}{2}; [p+Q, -q_2]\right) \{WY\}_{\gamma\delta}, \tag{4.5}$$

we obtain

$$\{WY\}_{\gamma\delta} = i\left\{\left[1 - S(k''+Q)S^{-1}\left(k'' - \frac{q}{2}\right)\right]\Psi(K_i'';p_i)\right\}_{\gamma\delta}$$
$$- \int \frac{d^4k'}{(2\pi)^4} S_{\gamma\epsilon}(k''+Q) M_{\epsilon\rho;\xi\eta}\left(k'' - \frac{p}{2} + \frac{Q}{2}, k' - \frac{p}{2} + \frac{Q}{2}; p+Q\right) S_{\rho\delta}(k''-p)$$
$$\times \left\{\left[\underline{1} - S(k'+Q)S^{-1}\left(k' - \frac{q}{2}\right)\right]\Psi(K_i;p_i)\right\}_{\eta\xi}$$
$$- \int \frac{d^4k'}{(2\pi)^4} S_{\gamma\epsilon}(k''+Q)\left[V_{\epsilon\rho;\lambda\phi}\left(k'' - \frac{p}{2} - \frac{q}{4}, k' - \frac{p}{2} - \frac{q}{4}; p - \frac{q}{2}\right)\right.$$
$$\left.\underline{-V_{\epsilon\rho;\lambda\phi}\left(k'\prime - \frac{p}{2} + \frac{Q}{2}, k' - \frac{p}{2} + \frac{Q}{2}; p+Q\right)}\right] S_{\rho\delta}(k''-p)\Psi_{\phi\lambda}(K_i;p_i)$$
$$-i \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4\widetilde{k}}{(2\pi)^4} S_{\gamma\epsilon}(k''+Q) M_{\epsilon\phi;\sigma\rho}\left(k'' - \frac{p}{2} + \frac{Q}{2}, \widetilde{k} - \frac{p}{2} + \frac{Q}{2}; p+Q\right)$$
$$\times S_{\phi\delta}(k''-p) S_{\rho\tau}(\widetilde{k}+Q)\left[V_{\tau\kappa;\lambda\eta}\left(\widetilde{k} - \frac{p}{2} - \frac{q}{4}, k' - \frac{p}{2} - \frac{q}{4}; p - \frac{q}{2}\right)\right.$$
$$\left.\underline{-V_{\tau\kappa;\lambda\eta}\left(\widetilde{k} - \frac{p}{2} + \frac{Q}{2}, k' - \frac{p}{2} + \frac{Q}{2}; p+Q\right)}\right] S_{\kappa\sigma}(\widetilde{k}-p)\Psi_{\eta\lambda}(K_i;p_i). \tag{4.6}$$

Here Eq. (2.11) was used again. Using the wave equation for the scattering matrix, Eq. (2.1), the underlined parts of the above equation cancel and (4.6) reduces to

$$\{WY\}_{\gamma\delta} = i\Psi_{\gamma\delta}(K_i'';p_i) - i\left[S(k''+Q)S^{-1}\left(k'' - \frac{q}{2}\right)\right]_{\gamma\eta}\Psi_{\eta\delta}(K_i'';p_i)$$
$$+ \int \frac{d^4k'}{(2\pi)^4} S_{\gamma\epsilon}(k''+Q) M_{\epsilon\rho;\xi\eta}\left(k'' - \frac{p}{2} + \frac{Q}{2}, k' - \frac{p}{2} + \frac{Q}{2}; p+Q\right) S_{\rho\delta}(k''-p)$$
$$\times \left\{\left[S(k'+Q)S^{-1}\left(k' - \frac{q}{2}\right)\right]\Psi(K_i';p_i)\right\}_{\eta\xi}$$
$$- \int \frac{d^4k'}{(2\pi)^4} S_{\gamma\epsilon}[k''+Q)]\underline{V_{\epsilon\rho;\lambda\phi}\left(k'' - \frac{p}{2} - \frac{q}{4}, k' - \frac{p}{2} - \frac{q}{4}; p - \frac{q}{2}\right)\Psi_{\phi\lambda}(K_i';p_i)S_{\rho\delta}(k''-p)}$$
$$-i \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4\widetilde{k}}{(2\pi)^4} S_{\gamma\epsilon}(k''+Q) M_{\epsilon\phi;\sigma\rho}\left(k'' - \frac{p}{2} + \frac{Q}{2}, \widetilde{k} - \frac{p}{2} + \frac{Q}{2}; p+Q\right) S_{\phi\delta}(k''-p)$$
$$\times S_{\rho\tau}(\widetilde{k}+Q)\underline{V_{\tau\kappa;\lambda\phi}\left(\widetilde{k} - \frac{p}{2} - \frac{q}{4}, k' - \frac{p}{2} - \frac{q}{4}; p - \frac{q}{2}\right)\Psi_{\phi\lambda}(K_i';p_i)}S_{\kappa\sigma}(\widetilde{k}-p), \tag{4.7}$$

where $K_i'' = k'' - \frac{p}{2} - \frac{q}{4}$, $K_i' = k' - \frac{p}{2} - \frac{q}{4}$, and $p - \frac{q}{2} = p_i$. The other kinematical variables, K_f, K_i , p, Q, and q, are the same as in Sec. III. Using the bound state equation,

$$\int \frac{d^4k'}{(2\pi)^4} V_{\epsilon\rho;\lambda\phi}\left(k'' - \frac{p}{2} - \frac{q}{4}, k' - \frac{p}{2} - \frac{q}{4}; p - \frac{1}{2}\right) \Psi_{\phi\lambda}(K_i';p_i) = -i\left[S^{-1}\left(k'' - \frac{q}{2}\right)\Psi\left(k'' - \frac{p}{2} - \frac{q}{4};p_i\right)S^{-1}(k''-p)\right]_{\epsilon\rho},$$

the fourth term becomes

$$\begin{aligned}\{\text{4th term}\} &= iS_{\gamma\epsilon}(k''+Q)\left[S^{-1}\left(k'' - \frac{q}{2}\right)\Psi\left(k'' - \frac{p}{2} - \frac{q}{4};p_i\right)S^{-1}(k''-p)\right]_{\epsilon\rho} S_{\rho\delta}(k''-p)\\ &= iS_{\gamma\epsilon}(k''+Q)S^{-1}_{\epsilon\lambda}\left(k'' - \frac{q}{2}\right)\Psi_{\lambda\delta}\left(k'' - \frac{p}{2} - \frac{q}{4};p_i\right),\end{aligned}$$

and cancels the second term. Likewise, the fifth term cancels with the third term. Only the first term remains to contribute to Eq. (4.5), so we conclude that

$$\begin{aligned}q_{1\nu}[\mathcal{C}^{\mu\nu}(W_{31}) + \mathcal{C}^{\mu\nu}(W_{33}) + \mathcal{C}^{\mu\nu}(Y_{31}) + \mathcal{C}^{\mu\nu}(Y_{33})] &= -e_p \int \frac{d^4k}{(2\pi)^4}\int \frac{d^4k'}{(2\pi)^4}\overline{\Psi}_{\alpha\beta}(K_f;p_f)\\ &\times J^{\mu}_{\beta\alpha;\delta\gamma}\left(K_f, k' - \frac{p}{2} + \frac{Q}{2};[p+Q,-q_2]\right)\Psi_{\gamma\delta}(K_i;p_i). \qquad (4.8)\end{aligned}$$

The remaining four amplitudes in Fig. 12, $\mathcal{C}^{\mu\nu}(X_{31})$, $\mathcal{C}^{\mu\nu}(X_{33})$, $\mathcal{C}^{\mu\nu}(Z_{31})$ and $\mathcal{C}^{\mu\nu}(Z_{33})$, are listed in Appendix B. The divergence of these amplitudes can be obtained in a similar way, and the result is

$$\begin{aligned}q_{1\nu}[\mathcal{C}^{\mu\nu}(X_{31}) + \mathcal{C}^{\mu\nu}(X_{33}) + \mathcal{C}^{\mu\nu}(Z_{31}) + \mathcal{C}^{\mu\nu}(Z_{33})] &= e_p \int \frac{d^4k}{(2\pi)^4}\int \frac{d^4k'}{(2\pi)^4}\overline{\Psi}_{\alpha\beta}(K_f;p_f)\\ &\times J^{\mu}_{\beta\alpha;\delta\gamma}\left(k - \frac{p}{2} - \frac{Q}{2}, K_i';\left[p - \frac{q}{2}, -q_2\right]\right)\Psi_{\gamma\delta}(K_i';p_i). \qquad (4.9)\end{aligned}$$

From Eq. (4.3), Eq. (4.8), and Eq. (4.9) the divergence of the total amplitude given by the $W + X + Y + Z$ processes is

$$\begin{aligned}q_{1\nu}\mathcal{C}^{\mu\nu}(WXYZ) &= q_{1\nu}[\mathcal{C}^{\mu\nu}(W) + \mathcal{C}^{\mu\nu}(X) + \mathcal{C}^{\mu\nu}(Y) + \mathcal{C}^{\mu\nu}(Z)]\\ &= -\int \frac{d^4k}{(2\pi)^4}\int \frac{d^4k'}{(2\pi)^4}\overline{\Psi}_{\alpha\beta}(K_f;p_f)I^{\mu}_{\beta\alpha;\delta\gamma}\Psi_{\gamma\delta}(K_i;p_i), \qquad (4.10)\end{aligned}$$

where

$$I^{\mu}_{\beta\alpha;\delta\gamma} = e_pJ^{\mu}_{\beta\alpha;\delta\gamma}\left(k - \frac{p}{2} + \frac{q}{4}, k' - \frac{p}{2} + \frac{Q}{2};[p+Q,-q_2]\right) - e_pJ^{\mu}_{\beta\alpha;\delta\gamma}\left(k - \frac{p}{2} - \frac{Q}{2}, k' - \frac{p}{2} - \frac{q}{4};\left[p - \frac{q}{2}, -q_2\right]\right).$$

Note that I^{μ} does not vanish in general, and the violation of current conservation is now expressed in terms of the one-photon interaction current. We conclude that the contributions from the impulse, interaction current, and scattering matrix given by Eq. (4.2) are not enough to satisfy gauge invariance in Compton scattering. This failure is due to the nonlocality of the nuclear dynamics. A new, two-photon interaction current is needed to restore gauge invariance.

C. Two-photon interaction currents and the WT identity

In Refs. [24, 26] the need for an interaction current was demonstrated in the following way. First, the matrix element of the impulse (one-body) current for an exclusive processes, such as the charge form factor of a composite system, was derived. Then it was shown that the divergence of the impulse current was not zero, but was related to the two-body force. This relation can be regarded as a constraint on the interaction currents; they must be constructed so that their divergence cancels the divergence of the impulse current. Finally, an explicit form of the interaction current operator was derived, and the result was shown to satisfy the constraints. Physically, interaction currents [as shown in Fig. 2(a)] exist because a photon can interact with charged constituents which are present within the nonlocal region $d = |x_1 - x_1'| \sim |x_2 - x_2'|$ over which the force extends.

We now extend this idea to cases involving two photons, Fig. 2(b), where both of the photons couple to the charged constituents of the system. The nonlocality will lead to a new two-body current, a *two-photon interaction* current

$$J^{\mu\nu}_{\alpha\beta;\delta\gamma}(K_f, K_i;[p_i, q_1, q_2]).$$

The variable $p_i = p_1 + p_2$ is the total four-momentum of the two nucleons before absorbing (or emitting) the photons, and K_f (K_i) [sometimes denoted simply by K' (K)] is the relative momentum of the final (initial) state. The total amplitude for Compton scattering is then given by

$$\mathcal{C}^{\mu\nu}(\text{total}) = \mathcal{C}^{\mu\nu}(W) + \mathcal{C}^{\mu\nu}(X) + \mathcal{C}^{\mu\nu}(Y) + \mathcal{C}^{\mu\nu}(Z) + \mathcal{C}^{\mu\nu}(J_{2\gamma}), \tag{4.11}$$

where

$$\mathcal{C}^{\mu\nu}(J_{2\gamma}) = -\int \frac{d^4k}{(2\pi)^4}\int \frac{d^4k'}{(2\pi)^4}\overline{\Psi}_{\alpha\beta}(K_f;p_f)J^{\mu\nu}_{\beta\alpha;\delta\gamma}(K_f,K_i;[p_i,q_1,q_2])\Psi_{\gamma\delta}(K_i;p_i). \tag{4.12}$$

In order for the total amplitude to be gauge invariant, $q_{1\nu}\mathcal{C}^{\mu\nu}(\text{total}) = 0$, we require that the two-photon interaction current satisfy the following two-body WT identity:

$$\begin{aligned} -I^{\mu}_{\beta\alpha;\delta\gamma} &= q_{1\nu}J^{\mu\nu}_{\beta\alpha;\delta\gamma}(K',K;[p_i,q_1,q_2]) \\ &= e_p\left[J^{\mu}_{\beta\alpha;\delta\gamma}\left(K'-\frac{q_1}{2},K;[p_i,-q_2]\right) - J^{\mu}_{\beta\alpha;\delta\gamma}\left(K',K+\frac{q_1}{2};[p_i+q_1,-q_2]\right)\right]. \end{aligned} \tag{4.13}$$

In conclusion, if the nuclear forces are nonlocal, the impulse current, one-photon interaction current, and scattering matrix contributions to Compton scattering are not enough to satisfy gauge invariance. *A two-photon interaction current is needed to restore gauge invariance, and its divergence must satisfy the two-body WT identity, Eq. (4.13).* In the next section we will derive the explicit form of the two-photon interaction current for several cases.

V. TWO-PHOTON INTERACTION CURRENT

In this section we find the explicit form for the two-photon interaction currents for three simple models of the NN force: (a) a two-pion exchange model with baryon resonances, (b) a covariant separable potential model, and (c) the charged one-pion exchange force. The results for the current operators will be shown to satisfy the two-body WT identity, Eq. (4.13). We use the method of Feynman diagrams [39] for models (a) and (c), and the method of minimal substitution [26, 28] for model (b).

A. Two-pion exchange model

One of the most familiar nonlocal NN forces is the (correlated) two-pion exchange interaction [31] with intermediate excited baryons (Δ and N^*), illustrated in Fig. 1(b). This mechanism is a possible explanation for the intermediate-range attraction approximated by the "σ" exchange used in the one-boson exchange model [29]. The nonlocality of this interaction is caused by the propagation of the intermediate baryon resonances. Δ is particularly important, as demonstrated by its role in inclusive electron scattering just above the quasifree region [1].

In general, the two-photon interaction current associated with this nonlocal mechanism is described by 22 diagrams (including the coupling of two photons to each pion and two-photon contact terms). Here we will simplify the discussion by considering only the exchange of neutral pions between the neutron and proton, giving a simple model which still includes the essence of the nonlocality caused by the propagation of the baryon resonances. (Hereafter we will use the symbol "N^*" to denote both Δ and all other baryon resonances which might appear in the intermediate states.)

The nonlocal potential, Fig. 13, is given by

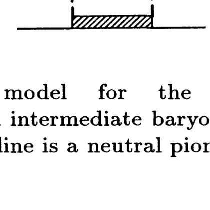

FIG. 13. A model for the nonlocal potential $V_{N^*}(K',K;p)$, with intermediate baryon resonances (shaded lines). The dashed line is a neutral pion.

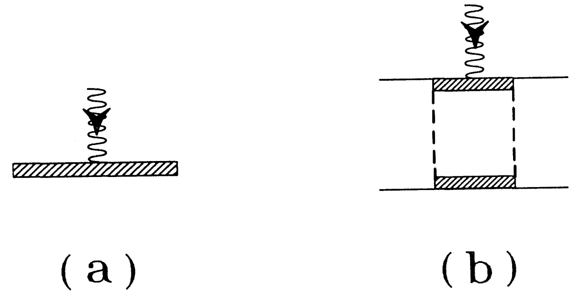

FIG. 14. Electromagnetic interactions involving baryon resonances: (a) γN^* coupling and (b) the one-photon interaction current generated by the presence of the γN^* coupling inside of the nonlocal potential.

$$V_{N^*}(K',K;p)_{\alpha\beta;\delta\gamma} = \int \frac{dk^4}{(2\pi)^4}\left[\Gamma_a S_*\left(\frac{p}{2}+\frac{K'+K}{2}-k\right)\Gamma_c\right]_{\alpha\gamma}\left[\Gamma_b S_*\left(\frac{p}{2}-\frac{K'+K}{2}+k\right)\Gamma_d\right]_{\beta\delta}$$
$$\times\Delta\left(k+\frac{K'-K}{2}\right)\Delta\left(k-\frac{K'-K}{2}\right), \quad (5.1)$$

where $\Delta(k) = [k^2-\mu^2+i\epsilon]^{-1}$ is the propagator of the neutral pion with mass μ and Γ_n is the πNN^* vertex. Here, $K' = \frac{1}{2}(p_1'-p_2')$, $K = \frac{1}{2}(p_1-p_2)$, and $p = p_1+p_2 = p_1'+p_2'$ are the relative momentum of the two nucleons in the final state, in the initial state, and the total momentum, respectively. The propagator of the baryon resonance is represented by $S_*(p)$; we will not need its explicit form. The WT identity for the γN^* vertex, $\Lambda^\mu_{N^*}$ [see Fig. 14(a)], is given by

$$q_\mu\Lambda^\mu_{N^*} = S_*^{-1}(p+q) - S_*^{-1}(p). \quad (5.2)$$

The one-photon interaction current is given by evaluating the Feynman diagram in Fig. 14(b) to get

$$J^\mu_{N^*}(K',K;[p,q])_{\alpha\beta;\delta\gamma} = \int \frac{dk^4}{(2\pi)^4}\left[\Gamma_a S_*\left(\frac{p+q}{2}+\frac{K'+K}{2}+\frac{q}{4}-k\right)(e_{N^*}\Lambda^\mu_{N^*})\,S_*\left(\frac{p}{2}+\frac{K'+K}{2}-\frac{q}{4}-k\right)\Gamma_c\right]_{\alpha\gamma}$$
$$\times\left[\Gamma_b\;S_*\left(\frac{p}{2}-\frac{K'+K}{2}+\frac{q}{4}+k\right)\Gamma_d\right]_{\beta\delta}\Delta\left(k+\frac{K'-K}{2}-\frac{q}{4}\right)\Delta\left(k-\frac{K'-K}{2}+\frac{q}{4}\right). \quad (5.3)$$

Note that $p = p_1+p_2$ and $p+q = p_1'+p_2'$, where q is defined to be the *absorbed* momentum. The charge of the baryon resonance is denoted by e_{N^*} ($e_{N^*} = 1$ in this model).

We now examine the two-body WT identity for this one-photon interaction current. Using Eq. (5.2) with the appropriate kinematical variables, $q_\mu\Lambda^\mu_{N^*} = S_*^{-1}(\frac{p+q}{2}+\frac{K'+K}{2}+\frac{q}{4}-k) - S_*^{-1}(\frac{p}{2}+\frac{K'+K}{2}-\frac{q}{4}-k)$, we get

$$q_\mu J^\mu_{N^*}(K',K;[p,q])_{\alpha\beta;\delta\gamma} = e_{N^*}\int\frac{dk^4}{(2\pi)^4}\left[\Gamma_a S_*\left(\frac{p}{2}+\frac{[K'-q/2]+K}{2}-k\right)\Gamma_c\right]_{\alpha\gamma}$$
$$\times\left[\Gamma_b S_*\left(\frac{p}{2}-\frac{[K'-q/2]+K}{2}+k\right)\Gamma_d\right]_{\beta\delta}$$
$$\times\Delta\left(k+\frac{[K'-q/2]-K}{2}\right)\Delta\left(k-\frac{[K'-q/2]-K}{2}\right)$$
$$-e_{N^*}\int\frac{dk^4}{(2\pi)^4}\left[\Gamma_a S_*\left(\frac{[p+q]}{2}+\frac{K'+[K+q/2]}{2}-k\right)\Gamma_c\right]$$
$$\times\left[\Gamma_b S_*\left(\frac{[p+q]}{2}-\frac{K'+[K+q/2]}{2}+k\right)\Gamma_d\right]_{\beta\delta}$$
$$\times\Delta\left(k+\frac{K'-[K+q/2]}{2}\right)\Delta\left(k-\frac{K'-[K+q/2]}{2}\right)$$
$$= e_{N^*}\left[V_{N^*}\left(K'-\frac{q}{2},K;p\right)_{\alpha\beta;\delta\gamma} - V_{N^*}\left(K',K+\frac{q}{2};p+q\right)_{\alpha\beta;\delta\gamma}\right] \quad (5.4)$$

This is the desired result, satisfying the two-body WT identity, Eq. (2.11).

The two-photon interaction current can be derived by evaluating the diagrams in Fig. 15. Note that processes in which one of the photons couples to an external leg and the other couples to a baryon resonance should not be considered a two-photon interaction current because all such diagrams are already included in the amplitudes for the (*impulse*) $\times$ (*one-photon interaction current*), i.e., $\mathcal{C}^{\mu\nu}(W_{13})$, $\mathcal{C}^{\mu\nu}(W_{31})$, $\mathcal{C}^{\mu\nu}(X_{13})$, and $\mathcal{C}^{\mu\nu}(X_{31})$. The result for the two-photon interaction current is therefore

$$J^{\mu\nu}_{N^*}(K',K;[p,q_1,q_2])_{\alpha\beta;\delta\gamma} = -\int\frac{dk^4}{(2\pi)^4}\Delta\left(k+\frac{K'-K}{2}-\frac{u}{4}\right)\Delta\left(k-\frac{K'-K}{2}+\frac{u}{4}\right)$$
$$\times\left[\Gamma_b S_*\left(\frac{p}{2}-\frac{K'+K}{2}+\frac{u}{4}+k\right)\Gamma_d\right]_{\beta\delta}$$
$$\times\left[\Gamma_a S_*\left(\frac{p+u}{2}+\frac{K'+K}{2}+\frac{u}{4}-k\right)\Sigma^{\mu\nu}\,S_*\left(\frac{p}{2}+\frac{K'+K}{2}-\frac{u}{4}-k\right)\Gamma_c\right]_{\alpha\gamma}, \quad (5.5)$$

where

$$\Sigma^{\mu\nu} = (e_{N^*}\Lambda^{\mu}_{N^*})\, S_*\left(\frac{p}{2}+\frac{K'+K}{2}-\frac{u}{4}-k+q_1\right)(e_{N^*}\Lambda^{\nu}_{N^*}) + (e_{N^*}\Lambda^{\nu}_{N^*})\, S_*\left(\frac{p}{2}+\frac{K'+K}{2}-\frac{u}{4}-k-q_2\right)(e_{N^*}\Lambda^{\mu}_{N^*}),$$

and $u = q_1 - q_2$. The first and second terms in $\Sigma^{\mu\nu}$ correspond to the direct [Fig. 15(a)] and the crossed [Fig. 15(b)] processes, respectively. The divergence of the current operator, $q_{1\nu}J^{\mu\nu}_{N^*}(K',K;[p,q_1,q_2])$, can be simply obtained by using Eq. (5.2). Omitting the common factors we have

$$\begin{aligned} q_{1\nu}J^{\mu\nu}_{N^*}(\text{direct}) \sim & \left[\Gamma_a S_*\left(\frac{p+u}{2}+\frac{K'+K}{2}+\frac{u}{4}-k\right)(e_{N^*}\Lambda^{\mu}_{N^*})\, S_*\left(\frac{p}{2}+\frac{K'+K}{2}-\frac{u}{4}-k\right)\Gamma_c\right] \\ & -\left[\Gamma_a S_*\left(\frac{p+u}{2}+\frac{K'+K}{2}+\frac{u}{4}-k\right)(e_{N^*}\Lambda^{\mu}_{N^*})\, S_*\left(\frac{p}{2}+\frac{K'+K}{2}-\frac{u}{4}-k+q_1\right)\Gamma_c\right] \end{aligned} \quad (5.6a)$$

and

$$\begin{aligned} q_{1\nu}J^{\mu\nu}_{N^*}(\text{crossed}) \sim & \left[\Gamma_a S_*\left(\frac{p}{2}+\frac{K'+K}{2}-\frac{u}{4}-k-q_2\right)(e_{N^*}\Lambda^{\mu}_{N^*})\, S_*\left(\frac{p}{2}+\frac{K'+K}{2}-\frac{u}{4}-k\right)\Gamma_c\right] \\ & -\left[\Gamma_a S_*\left(\frac{p+u}{2}+\frac{K'+K}{2}+\frac{u}{4}-k\right)(e_{N^*}\Lambda^{\mu}_{N^*})\, S_*\left(\frac{p}{2}+\frac{K'+K}{2}-\frac{u}{4}-k\right)\Gamma_c\right]. \end{aligned} \quad (5.6b)$$

Note that the first term in Eq. (5.6a) cancels the second term in Eq. (5.6b). Finally, we get

$$\begin{aligned} q_{1\nu}J^{\mu\nu}_{N^*}(K',K;[p,q_1,q_2]) = & -e_{N^*}\int\frac{dk^4}{(2\pi)^4}\Delta\left(k+\frac{K'-[K+q_1/2]}{2}+\frac{q_2}{4}\right)\Delta\left(k-\frac{K'-[K+q_1/2]}{2}-\frac{q_2}{4}\right) \\ & \times\left[\Gamma_b S_*\left(\frac{[p+q_1]}{2}+k-\frac{K'+[K+q_1/2]}{2}-\frac{q_2}{4}\right)\Gamma_d\right] \\ & \times\left[\Gamma_a S_*\left(\frac{[p+q_1]-q_2}{2}-k+\frac{K'+[K+q_1/2]}{2}-\frac{q_2}{4}\right)(e_{N^*}\Lambda^{\mu}_{N^*})\right. \\ & \left.\times S_*\left(\frac{[p+q_1]}{2}-k+\frac{K'+[K+q_1/2]}{2}+\frac{q_2}{4}\right)\Gamma_c\right] \\ & +e_{N^*}\int\frac{dk^4}{(2\pi)^4}\Delta\left(k+\frac{[K'-q_1/2]-K}{2}+\frac{q_2}{4}\right)\Delta\left(k-\frac{[K'-q_1/2]-K}{2}-\frac{q_2}{4}\right) \\ & \times\left[\Gamma_b S_*\left(\frac{p}{2}+k-\frac{[K'-q_1/2]+K}{2}-\frac{q_2}{4}\right)\Gamma_d\right] \\ & \times\left[\Gamma_a S_*\left(\frac{p-q_2}{2}-k+\frac{[K'-q_1/2]+K}{2}-\frac{q_2}{4}\right)(e_{N^*}\Lambda^{\mu}_{N^*})\right. \\ & \left.\times S_*\left(\frac{p}{2}-k+\frac{[K'-q_1/2]+K}{2}+\frac{q_2}{4}\right)\Gamma_c\right] \\ = & \, e_{N^*}\left[J^{\mu}_{N^*}\left(K'-\frac{q_1}{2},K;[p,-q_2]\right)-J^{\mu}_{N^*}\left(K',K+\frac{q_1}{2};[p+q_1,-q_2]\right)\right]. \end{aligned} \quad (5.7)$$

This is the desired result satisfying the two-body WT identity for the two-photon interaction current, Eq. (4.13).

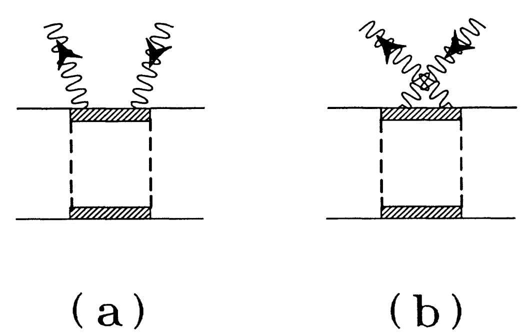

FIG. 15. Diagrams for the two-photon interaction current associated with the nonlocal potential: (a) direct process and (b) crossed process.

B. Interaction currents for a covariant separable potential

As the second example of a nonlocal force we consider a covariant separable potential. With a separable kernel the BS equation has analytic solutions for both scattering amplitudes and bound state wave functions, and such a model has been used for both two-nucleon systems [40] and three-nucleon systems [41]. Such a model therefore provides an explicitly soluble scheme for Compton scattering. The covariant separable interaction has also been successfully applied to the description of the structure of mesons as quark-antiquark ($q\bar{q}$) composite systems [42]. The model gives an analytical solution for the dynamical quark mass generated by the spontaneous breaking of chiral symmetry. Here $q\bar{q} \leftrightarrow$ meson duality [43] seems to support a factorizable form of the interaction, particularly for electromagnetic observables including vector meson processes. The physics of hadron resonances, such as the $\pi N \to \Delta \to \pi N$ process, may also be efficiently described with an effective interaction of separable form.

In this section we give the explicit form for the one-photon and two-photon interaction currents associated with separable interactions. The derivation of the one-photon interaction current is given in Ref. [26]. The derivation of the two-photon interaction current is fairly lengthy and will be published in a separate paper [44].

We will use a simple rank-one model for the covariant separable interaction:

$$\mathcal{V}_{\alpha\beta;\delta\gamma}(K',K;p) = F_{\alpha\beta}(K')F^{\dagger}_{\delta\gamma}(K), \tag{5.8}$$

where the vertices are given by $F_{\alpha\beta}(K') = f(K'^2)\Omega_{\alpha\beta}$ and $F^{\dagger}_{\delta\gamma}(K) = f^{\dagger}(K^2)\Omega^{\dagger}_{\delta\gamma}$. The form factors $f(K'^2)$ and $f^{\dagger}(K^2)$ depend on the relative momentum of the two nucleons in the final state, $K' = \frac{1}{2}(p_1' - p_2')$, and the initial state, $K = \frac{1}{2}(p_1 - p_2)$, where p_1' (p_1) and p_2' (p_2) are the respective four-momentum of the first particle with charge e_1 and the second particle with charge e_2. The 4×4 matrix $\Omega_{\alpha\beta}$ operating on the nucleon spinors is assumed to be independent of any momenta. The dagger attached to the form factor means that the Hermitian conjugate is to be taken if a quantum mechanical operator is inserted as the argument. In coordinate space (Fig. 16), the separable interaction is given by

$$\mathcal{V}_{\alpha\beta;\delta\gamma}(x_1',x_2';x_1,x_2) = \Delta_{\alpha\beta}(x_1',x_2')\Delta^{\dagger}{}_{\delta\gamma}(x_1,x_2), \tag{5.9}$$

where

$$\Delta_{\alpha\beta}(x_1',x_2') = \int\int \frac{d^4p_1'd^4p_2'}{(2\pi)^8} f\left(\left[\frac{p_1'-p_2'}{2}\right]^2\right) \times \Omega_{\alpha\beta} e^{ip_1' x_1'} e^{ip_2' x_2'}$$

and

$$\Delta^{\dagger}_{\delta\gamma}(x_1,x_2) = \int\int \frac{d^4p_1d^4p_2}{(2\pi)^8} f^{\dagger}\left(\left[\frac{p_1-p_2}{2}\right]^2\right) \times \Omega^{\dagger}_{\alpha\beta} e^{-ip_1 x_1} e^{-ip_2 x_2}$$

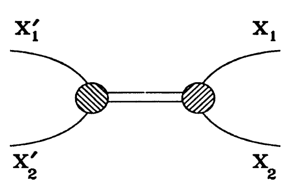

FIG. 16. A rank-1 separable potential $\mathcal{V}(x_1',x_2';x_1,x_2)$, where the shaded circle represents the vertex $\Delta(x_1',x_2')$ and the conjugate vertex $\Delta^{\dagger}(x_1,x_2)$.

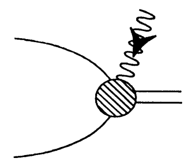

FIG. 17. The matrix element for one-photon annihilation at the vertex $\Delta(x_1',x_2')$.

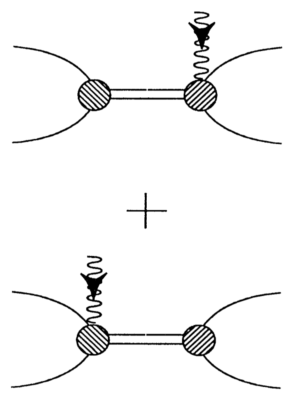

FIG. 18. Terms which make up the one-photon interaction current, J^{μ}, associated with the separable potential $\mathcal{V}(x_1',x_2';x_1,x_2)$.

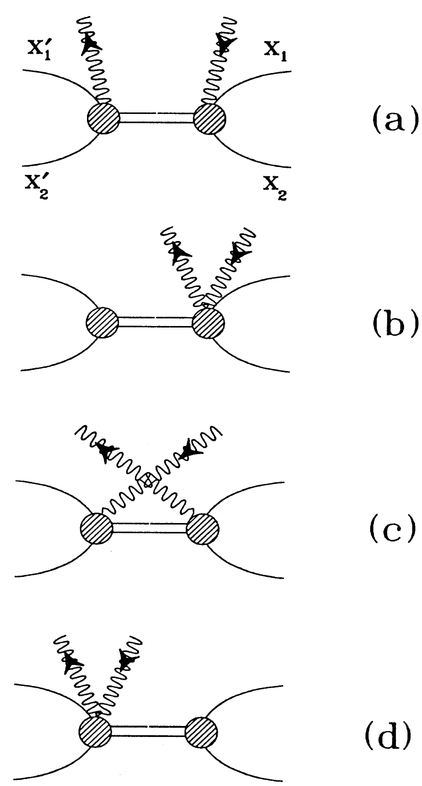

FIG. 19. Terms which make up the two-photon interaction current, $J^{\mu\nu}$, associated with the separable potential.

are the Fourier transforms of the vertex functions, $F_{\alpha\beta}(K')$ and $F^{\dagger}_{\delta\gamma}(K)$. Note that the momentum variables in the form factors can be replaced by the derivative operators, $f\left([\frac{p_1-p_2}{2}]^2\right) \to f\left(-[\frac{\partial_1-\partial_2}{2}]^2\right)$. We now introduce the photon field, $\mathcal{A}^{\mu}(x_n)$, through minimal substitution, $\partial^{\mu}_n \to \overline{\partial^{\mu}_n} = \partial^{\mu}_n + ie_n\mathcal{A}^{\mu}(x_n)$, where e_n, x_n and ∂^{μ}_n are the charge, coordinate, and derivative operator of the nth particle. The line over the derivative operator will indicate minimal substitution of the quantized photon field given by

$$\mathcal{A}_{\mu}(x) = \int \frac{d^4q}{(2\pi)^4}\left\{a_{\mu}(q)e^{-iqx} + a^{\dagger}_{\mu}(q)e^{+iqx}\right\}, \tag{5.10}$$

where $a^{\dagger}_{\mu}(q)$ and $a_{\mu}(q)$ are the creation and annihilation operators of a photon with momentum q and the polarization ϵ_{μ}. The presence of the photon field modifies the vertices,

$$\Delta_{\alpha\beta}(x'_1,x'_2) \to \overline{\Delta_{\alpha\beta}(x'_1,x'_2)} = \int\int \frac{d^4p'_1d^4p'_2}{(2\pi)^8} f\left(\overline{-\left[\frac{\partial'_1-\partial'_2}{2}\right]^2}\right)\Omega_{\alpha\beta}e^{ip'_1x'_1}e^{ip'_2x'_2}, \tag{5.11a}$$

$$\Delta^{\dagger}_{\delta\gamma}(x_1,x_2) \to \overline{\Delta^{\dagger}_{\delta\gamma}(x_1,x_2)} = \int\int \frac{d^4p_1d^4p_2}{(2\pi)^8} f^{\dagger}\left(\overline{-\left[\frac{\partial_1-\partial_2}{2}\right]^2}\right)\Omega^{\dagger}_{\alpha\beta}e^{-ip_1x_1}e^{-ip_2x_2}, \tag{5.11b}$$

and each form factor involves the photon field operator. The matrix element of the one-photon absorption (Fig. 17) at the vertex is given by $\langle O|\overline{\Delta(x'_1,x'_2)}|a^{\dagger}_{\mu}(q)\rangle$. The presence of the photon field modifies the potential, and the variation is given by

$$\begin{aligned}\delta\mathcal{V}(x'_1,x'_2;x_1,x_2) &= \overline{\Delta(x'_1,x'_2)\Delta^{\dagger}(x_1,x_2)} - \Delta(x'_1,x'_2)\Delta^{\dagger}(x_1,x_2)\\ &= \overline{\Delta(x'_1,x'_2)}\left\{\overline{\Delta^{\dagger}(x_1,x_2)} - \Delta^{\dagger}(x_1,x_2)\right\} + \left\{\overline{\Delta(x'_1,x'_2)} - \Delta(x'_1,x'_2)\right\}\Delta^{\dagger}(x_1,x_2).\end{aligned} \tag{5.12}$$

This change of action also defines the one-photon interaction current, J^{μ}, and the two-photon interaction current, $J^{\mu\nu}$, through the coupling of photon field:

$$\delta\mathcal{V} = \mathcal{A}_{\mu}J^{\mu} + \mathcal{A}_{\mu}\mathcal{A}_{\nu}J^{\mu\nu} + \cdots . \tag{5.13}$$

The one-photon interaction current with momentum transfer q can be derived from

$$J_{\mu}(x'_1,x'_2;x_1,x_2;[q]) = -(2\pi)^4\langle O|\delta\mathcal{V}|a^{\dagger}_{\mu}(q)\rangle \tag{5.14}$$

and

$$\begin{aligned}\langle O|\delta\mathcal{V}|a^{\dagger}_{\mu}(q)\rangle = {}&\langle O|\left\{\overline{\Delta(x'_1,x'_2)} - \Delta(x'_1,x'_2)\right\}|a^{\dagger}_{\mu}(q)\rangle\langle O|\Delta^{\dagger}(x_1,x_2)|O\rangle\\ &+\langle O|\Delta(x'_1,x'_2)|O\rangle\langle O|\left\{\overline{\Delta^{\dagger}(x_1,x_2)} - \Delta^{\dagger}(x_1,x_2)\right\}|a^{\dagger}_{\mu}(q)\rangle .\end{aligned}$$

The two terms in Eq. (5.14) are illustrated in Fig. 18; the details of the derivation can be found in Ref. [26]. For a system with one proton ($e_1 = e_p$) and one neutron ($e_2 = e_n = 0$), the one-photon interaction current obtained from the separable potential is given by

$$J^{\mu}(K',K;[q]) = e_p\left\{\frac{[4K'-q]^{\mu}}{(q\cdot[4K'-q])}\left[\mathcal{V}\left(K'-\frac{q}{2},K\right) - \mathcal{V}(K',K)\right] - \frac{[4K+q]^{\mu}}{(q\cdot[4K+q])}\left[\mathcal{V}\left(K',K+\frac{q}{2}\right) - \mathcal{V}(K',K)\right]\right\}. \tag{5.15}$$

We point out that the current operator itself is expressed in terms of the separable potential, and it is clear that the operator satisfies the two-body WT identity, Eq. (2.11). The two-photon interaction current is obtained from,

$$J_{\mu\nu}(x'_1,x'_2;x_1,x_2;[q_1,q_2]) = (2\pi)^8\langle a_{\mu}(q_2)|\delta\mathcal{V}|a^{\dagger}_{\nu}(q_1)\rangle, \tag{5.16}$$

where q_1 (q_2) and ϵ_{ν} (ϵ_{μ}) are the absorbed (emitted) momentum and polarization of the incoming (outgoing) photon. The matrix element has four terms,

$$\begin{aligned}\langle a_\mu(q_2)|\delta\mathcal{V}|a^\dagger_\nu(q_1)\rangle = &\langle a_\mu(q_2)|\overline{\Delta(x'_1,x'_2)}|O\rangle\langle O|\left\{\overline{\Delta^\dagger(x_1,x_2)}-\Delta^\dagger(x_1,x_2)\right\}|a^\dagger_\nu(q_1)\rangle\\ &+\langle O|\overline{\Delta(x'_1,x'_2)}|O\rangle\langle a_\mu(q_2)|\left\{\overline{\Delta^\dagger(x_1,x_2)}-\Delta^\dagger(x_1,x_2)\right\}|a^\dagger_\nu(q_1)\rangle\\ &+\langle O|\overline{\Delta(x'_1,x'_2)}|a^\dagger_\nu(q_1)\rangle\langle a_\mu(q_2)|\left\{\overline{\Delta^\dagger(x_1,x_2)}-\Delta^\dagger(x_1,x_2)\right\}|O\rangle\\ &+\langle a_\mu(q_2)|\left\{\overline{\Delta(x'_1,x'_2)}-\Delta(x'_1,x'_2)\right\}|a^\dagger_\nu(q_1)\rangle\langle O|\Delta^\dagger(x_1,x_2)|O\rangle,\end{aligned} \tag{5.17}$$

where the first, second, third, and fourth terms are illustrated by the diagrams in Figs. 19(a)–19(d). The result for the two-photon interaction current is

$$J^{\mu\nu}(K',K;[q_1,q_2]) = \mathcal{O}^{\mu\nu}_1+\mathcal{O}^{\mu\nu}_2+\widetilde{\mathcal{O}}^{\mu\nu}_1+\widetilde{\mathcal{O}}^{\mu\nu}_2, \tag{5.18}$$

where

$$\mathcal{O}^{\mu\nu}_1 = e_p^2\Big[\mathcal{V}\left(K'+\tfrac{q_2}{2},K+\tfrac{q_1}{2}\right)-\mathcal{V}\left(K'+\tfrac{q_2}{2},K\right)+\mathcal{V}(K',K)-\mathcal{V}\left(K',K+\tfrac{q_1}{2}\right)\Big]G^{\mu\nu}_a,$$

$$\mathcal{O}^{\mu\nu}_2 = e_p^2\left[\mathcal{V}\left(K',K+\tfrac{q_1}{2}\right)G^{\mu\nu}_b+\mathcal{V}\left(K',K-\tfrac{q_2}{2}\right)G^{\mu\nu}_c+\mathcal{V}\left(K',K+\frac{q_1-q_2}{2}\right)G^{\mu\nu}_d-\mathcal{V}(K',K)\,G^{\mu\nu}_e\right],$$

and $\widetilde{\mathcal{O}}^{\mu\nu}_1=\mathcal{O}^{\nu\mu}_1(q_1\leftrightarrow q_2)$, $\widetilde{\mathcal{O}}^{\mu\nu}_2=\mathcal{O}^{\nu\mu}_2(q_1\leftrightarrow q_2)$. The kinematical factors ($G^{\mu\nu}$) are defined by

$$\begin{aligned}G^{\mu\nu}_a &= \frac{[4K'+q_2]^\mu[4K+q_1]^\nu}{([4K'+q_2]\cdot q_2)\,([4K+q_1]\cdot q_1)},\\ G^{\mu\nu}_b &= \frac{[4K+2q_1-q_2]^\mu[4K+q_1]^\nu}{([4K+2q_1-q_2]\cdot q_2)\,([4K+q_1]\cdot q_1)},\\ G^{\mu\nu}_c &= \frac{[4K-2q_2+q_1]^\mu[4K-q_2]^\nu}{([4K-2q_2+q_1]\cdot q_1)\,([4K-q_2]\cdot q_2)},\\ G^{\mu\nu}_d &= \frac{1}{(2K+q_1-q_2)^2-(2K)^2}\\ &\quad\times\left[2g^{\mu\nu}+\frac{[4K-2q_2+q_1]^\nu[4K-q_2]^\mu}{([4K-2q_2+q_1]\cdot q_1)}-\frac{[4K+2q_1-q_2]^\mu[4K+q_1]^\nu}{([4K+2q_1-q_2]\cdot q_2)}\right],\\ G^{\mu\nu}_e &= \frac{1}{(2K+q_1-q_2)^2-(2K)^2}\\ &\quad\times\left[2g^{\mu\nu}+\frac{[4K-2q_2+q_1]^\nu[4K-q_2]^\mu}{([4K-q_2]\cdot q_2)}-\frac{[4K+2q_1-q_2]^\mu[4K+q_1]^\nu}{([4K+q_1]\cdot q_1)}\right].\end{aligned}$$

Here $\mathcal{O}^{\mu\nu}_1,\mathcal{O}^{\mu\nu}_2,\widetilde{\mathcal{O}}^{\mu\nu}_1$, and $\widetilde{\mathcal{O}}^{\mu\nu}_2$ correspond to the four processes in Figs. 19(a)–19(d), respectively. Finally, we evaluate the divergence of this two-photon interaction current

$$\begin{aligned}q_{1\nu}J^{\mu\nu}(K',K;[q_1,q_2]) &= e_p^2\frac{[4K'-2q_1+q_2]^\mu}{(2K'+q_2-q_1)^2-(2K'-q_1)^2}\left\{\mathcal{V}\left(K'-\tfrac{q_1}{2},K\right)-\mathcal{V}\left(K'+\frac{q_2-q_1}{2},K\right)\right\}\\ &+e_p^2\frac{[4K+2q_1-q_2]^\mu}{(2K+q_1-q_2)^2-(2K+q_1)^2}\left\{\mathcal{V}\left(K',K+\frac{q_1-q_2}{2}\right)-\mathcal{V}\left(K',K+\tfrac{q_1}{2}\right)\right\}\\ &+e_p^2\frac{[4K'+q_2]^\mu}{(2K'+q_2)^2-(2K')^2}\left\{\mathcal{V}\left(K'+\tfrac{q_2}{2},K+\tfrac{q_1}{2}\right)-\mathcal{V}\left(K',K+\tfrac{q_1}{2}\right)\right\}\\ &+e_p^2\frac{[4K-q_2]^\mu}{(2K-q_2)^2-(2K)^2}\left\{\mathcal{V}\left(K',K-\tfrac{q_2}{2}\right)-\mathcal{V}\left(K'-\tfrac{q_1}{2},K-\tfrac{q_2}{2}\right)\right\}\\ &= e_p\left\{J^\mu\left(K'-\tfrac{q_1}{2},K;[p,-q_2]\right)-J^\mu\left(K',K+\tfrac{q_1}{2};[p+q_1,-q_2]\right)\right\}.\end{aligned} \tag{5.19}$$

This is just the two-body WT identity, Eq. (4.13).

C. Two-photon interaction current for the one-pion exchange process

Finally, we derive the two-photon interaction current for the one-pion exchange (OPE) mechanism. Only the charge exchange potential shown in Fig. 20(a) can contribute to the interaction current, and in order to keep the presentation simple, and to ensure that it is consistent with our assumption that $e_n = 0$, we work directly with the charge changing $\pi n p$ coupling (we do not use the isospin formalism), given by

$$\delta \mathcal{L} = g_+ \overline{\Psi}_n \gamma^5 \Psi_p \pi + p \leftrightarrow n, \tag{5.20}$$

where π annihilates an incoming π^- particle (or creates π^+), Ψ_p annihilates an incoming proton, and $\overline{\Psi}_n$ creates an outgoing neutron. From the isospin theory we know that $g_+ = \sqrt{2} g_{\pi NN}$, where $g^2{}_{\pi NN}/4\pi = 13.5$ is the familiar neutral pion coupling constant, but we will not need this result here.

The OPE charge exchange potential obtained from (5.20) is

$$V_{\beta\alpha;\gamma\delta}(K', K; p) = -g_+^2 \gamma^5_{\beta\gamma} \gamma^5_{\alpha\delta}\, \Delta(K + K'). \tag{5.21}$$

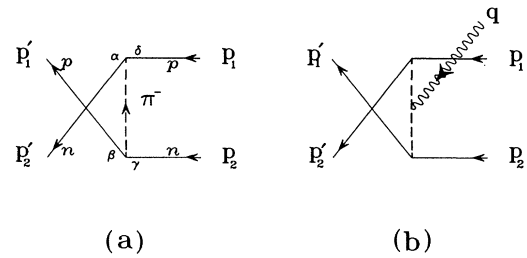

FIG. 20. (a) The OPE charge exchange NN interaction. Note that it is an exchange interaction and hence is nonlocal in the proton momentum transfer $p_1' - p_1$. (b) The OPE interaction current, where momentum conservation $p_1 + p_2 + q = p_1' + p_2'$ is understood.

where $K = \frac{1}{2}(p_1 - p_2)$, $K' = \frac{1}{2}(p_1' - p_2')$ and $p = (p_1 + p_2)$. Note that this potential is not a function of the proton momentum transfer, $K' - K$, and in this sense it is a nonlocal potential which can be described by the formalism developed in this paper.

The one-photon interaction current generated by this OPE charge exchange is shown in Fig. 20(b), and is

$$J^\mu_{\beta\alpha;\gamma\delta}(K', K; [p, q]) = -2g_+^2 e_p \gamma^5_{\beta\gamma} \gamma^5_{\alpha\delta}\, \Delta\left(K + K' - \frac{q}{2}\right) [K + K']^\mu \Delta\left(K + K' + \frac{q}{2}\right), \tag{5.22}$$

where the electromagnetic coupling of the negatively charged pion gives a Feynman factor of

$$j^\mu_{\gamma\pi}(k', k) = i e_p (k' + k)^\mu, \tag{5.23}$$

where k and k' are the four-momenta of the pion before and after the interaction. The famous Z graphs must not be added as a separate contribution because, in the relativistic formalism, the nucleon propagators include the negative energy components which generate the Z graph, and hence these contributions are automatically included in the nucleon current terms. Note that the one-photon interaction current (5.22) satisfies the correct WT identity (2.11),

$$\begin{aligned} q_\mu J^\mu(K', K; [p, q]) &= -g_+^2 e_p\, \gamma^5 \otimes \gamma^5 \left[\Delta\left(K + K' - \frac{q}{2}\right) - \Delta\left(K + K' + \frac{q}{2}\right)\right] \\ &= e_p \left[V\left(K' - \frac{q}{2}, K; p\right) - V\left(K', K + \frac{q}{2}; p + q\right)\right]. \end{aligned} \tag{5.24}$$

Next, the two-photon interaction current can be derived from the diagram shown in Fig. 21. The result is given by

$$\begin{aligned} J^{\mu\nu}(K', K; p_1, p_2; [p, q_1, q_2]) = \mathcal{N}\, \{&\Delta(k_3)[k_3 + k_2]^\mu \Delta(k_2)[k_2 + k_1]^\nu \Delta(k_1) \\ &+ \Delta(k_3)[k_3 + k_2']^\nu \Delta(k_2')[k_2' + k_1]^\mu \Delta(k_1) + \Delta(k_3)(-2g^{\mu\nu})\Delta(k_1)\}, \end{aligned} \tag{5.25}$$

where $\mathcal{N} = -g_+^2 e_p^2\, \gamma^5 \otimes \gamma^5$, and the new momenta are $k_1 = p_2 - p_1'$, $k_3 = p_2' - p_1$, $k_2 = p_2 - p_1' + q_1$, and $k_2' = p_2 - p_1' - q_2$. The first, second, and third terms of Eq. (5.25) correspond to the direct [Fig. 21(a)], crossed [Fig. 21(b)], and contact [Fig. 21(c)] terms, familiar processes in the Compton scattering from a free pion. The two-photon contact term contributes a Feynman factor of

$$j^{\mu\nu}_{\gamma\gamma\pi}(k', k) = 2 i g^{\mu\nu} e_p^2, \tag{5.26}$$

for the $\gamma\gamma\pi$ coupling. We assume that the pion is point-like, so that the vertices given by Eqs. (5.23) and (5.26) do not have form factors. By using the WT identity for the $\gamma\pi$ vertex,

$$q_\mu j^\mu_{\gamma\pi}(k', k) = i e_p [\Delta^{-1}(k') - \Delta^{-1}(k)], \tag{5.27}$$

we can evaluate the divergence of the two-photon interaction current:

$$\begin{aligned} q_{1\nu}J^{\mu\nu}(K',K;p_1,p_2;[p,q_1,q_2]) &= \mathcal{N}\{\Delta(k_3)[k_3+k_2]^\mu\Delta(k_2)[\Delta^{-1}(k_2)-\Delta^{-1}(k_1)]\Delta(k_1) \\ &\quad +\Delta(k_3)[\Delta^{-1}(k_3)-\Delta^{-1}(k_2')]\Delta(k_2')[k_2'+k_1]^\mu\Delta(k_1) \\ &\quad -2\Delta(k_3)[k_3-k_1+q_2]^\mu\Delta(k_1)\} \\ &= \mathcal{N}\{\Delta(k_1-q_2)[2k_1-q_2]^\mu\Delta(k_1)-\Delta(k_3)[2k_3+q_2]^\mu\Delta(k_3+q_2)\}\,. \end{aligned} \tag{5.28}$$

Each term in the curly brackets can be expressed in terms of the one-photon interaction current. We obtain

$$q_{1\nu}J^{\mu\nu}(K',K;p_1,p_2;[p,q_1,q_2]) = e_p\left[J^\mu\left(K'-\frac{q_1}{2},K;[p,-q_2]\right)-J^\mu\left(K',K+\frac{q_1}{2};[p+q_1,-q_2]\right)\right] \tag{5.29}$$

in precise agreement with Eq. (4.13). We note that contributions from the direct, crossed, and contact terms are all crucial to get the result, just as they are for Compton scattering from a free pion. In the derivation, we assumed that the pion is pointlike, and the use of the WT identity, Eq. (5.27), was crucial. If we had form factors $F_{\gamma\pi}(q)$ and $F_{\gamma\gamma\pi}(q)$ for the $\gamma\pi$ and $\gamma\gamma\pi$ vertices, we could still obtain a gauge invariant result if we defined new off-shell currents with the form factors confined to the transverse parts of the current. A complete discussion of how to carry this out for one-photon interactions can be found in Ref. [24]. The method can be extended to the treatment of two-photon interaction currents.

VI. SUMMARY AND DISCUSSION

There are four principal results in this paper.

(1) We derived a condition which the impulse (IMP) and final state interaction (FSI) processes must satisfy if current is to be conserved. (We use the terminology "FSI" to denote both final state interactions in electrodisintegration and also rescattering process in Compton scattering.) If the forces are nonlocal, the conventional amplitudes including IMP and FSI processes do not conserve current. In both electrodisintegration and Compton scattering this violation is expressed in terms of the quantity $\mathcal{T}(k',k;p,q)$. [See Eq. (2.10) and Eqs. (3.6a) and (3.6b)]. This quantity depends on

$$V\left(k'-\frac{p}{2}+\frac{q}{2},k-\frac{p}{2}+\frac{q}{2};p+q\right)-V\left(k'-\frac{p}{2},k-\frac{p}{2};p\right)\,,$$

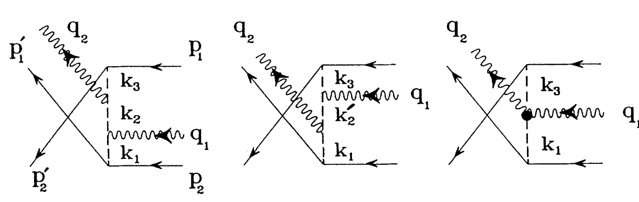

FIG. 21. Diagrams for the two-photon interaction current derived from the one-pion exchange mechanism: (a) the direct process, (b) the crossed process and (c) the contact term.

and these two terms correspond to momentum insertion by the photon before and after the nuclear interaction, as shown in Fig. 6. These two processes cancel for local interactions, but not for nonlocal ones.

(2) In addition to IMP and FSI processes, we introduced one-photon interaction currents associated with nonlocal forces. These interaction currents are generated by the coupling of photon(s) to the charged constituents in the nonlocal region ($d=|x_1'-x_1|\sim|x_2'-x_2|$) of the interaction $V(x_1',x_2';x_1,x_2)$. The contribution from the interaction currents [Fig. 5(c)] and interaction currents with rescattering [Fig. 5(d)], restore gauge invariance in electrodisintegration. The replacement of the impulse current by the combined current $\mathcal{J}^\mu$, Eq. (4.1), includes all of these contributions systematically.

(3) We used this combined current to analyze the reaction mechanism for Compton scattering from the deuteron. The total amplitude generated from this current, corresponding to the 16 diagrams shown in Fig. 10, is not gauge invariant. Gauge invariance can be restored if we introduce a new two-photon interaction current [Fig. 2(b)] which satisfies a *two-body WT identity*. This identity for the two-photon interaction current is given in Eq. (4.13).

(4) We derived the explicit form of the two-photon interaction currents from three simple models of nonlocal forces. In the first model the nonlocality arises from the two-pion exchange process with excited baryons in the intermediate state. The interaction current arises from the coupling of the photon to the excited baryons. In the second model the nonlocality arises from a covariant separable potential and the interaction current is derived by minimal substitution of the photon field into the form factors of the separable potential. In a third example the nonlocality arises from one-pion charge exchange and the interaction current is derived from the diagrams which describe Compton scattering from the virtual, off-shell exchanged pion. The interaction currents derived from all three of these models satisfy the two-body WT identities for the one-photon interaction current, Eq. (2.11), and for the two-photon interaction current, Eq. (4.13).

Throughout this paper we focused on the effect of the nonlocality in nuclear dynamics and assumed the nuclear force to be of a charge non-exchange type (except for the OPE current). Inclusion of charge exchange interactions is extremely important in calculating realistic amplitudes for Compton scattering and electrodisintegration. In future work we will extend these techniques to a general treatment of charge exchange interactions.

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APPENDIX A

Here we prove Eq. (4.3). The amplitudes in Fig. 11 are given by

$$\mathcal{C}^{\mu\nu}(W_{11}) = ie_p^2 \int \frac{d^4k}{(2\pi)^4} \overline{\Psi}_{\alpha\beta}(K_f;p_f)\,[\gamma^\mu S(k+Q)\gamma^\nu]_{\beta\gamma}\,\Psi_{\gamma\delta}(K_i;p_i) S^{-1}_{\delta\alpha}(k-p), \tag{A1}$$

$$\begin{aligned}\mathcal{C}^{\mu\nu}(W_{13}) = ie_p \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4}\, [\overline{\Psi}(K_f;p_f)\gamma^\mu S(k+Q)]_{\alpha\beta} \\ \times iJ^\nu_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}+\frac{Q}{2}, k'-\frac{p}{2}+\frac{q}{2}; \left[p+\frac{q}{2}, q_1\right]\right) \Psi_{\gamma\delta}(K'_i;p_i),\end{aligned} \tag{A2}$$

$$\begin{aligned}\mathcal{C}^{\mu\nu}(Y_{11}) = ie_p^2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4}\, [\overline{\Psi}(K_f;p_f)\gamma^\mu S(k+Q)]_{\alpha\beta} \\ \times iM_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}+\frac{Q}{2}, k'-\frac{p}{2}+\frac{Q}{2}; p+Q\right) [S(k'+Q)\gamma^\nu \Psi(K'_i;p_i)]_{\gamma\delta}\,,\end{aligned} \tag{A3}$$

$$\begin{aligned}\mathcal{C}^{\mu\nu}(Y_{13}) = ie_p \int \frac{d^4k}{(2\pi)^4}\, [\overline{\Psi}(K_f;p_f)\gamma^\mu S(k+Q)]_{\alpha\beta} \\ \times \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k''}{(2\pi)^4} iM_{\beta\alpha;\lambda\phi}\left(k-\frac{p}{2}+\frac{Q}{2}, k''-\frac{p}{2}+\frac{Q}{2},; p+Q\right) S_{\phi\epsilon}(k''+Q) S_{\rho\lambda}(k''-p) \\ \times iJ^\mu_{\epsilon\rho;\delta\gamma}\left(k''-\frac{p}{2}+\frac{Q}{2}, k'-\frac{p}{2}-\frac{q}{4}; \left[p-\frac{q}{2}, q_1\right]\right) \Psi_{\gamma\delta}(K'_i;p_i),\end{aligned} \tag{A4}$$

$$\mathcal{C}^{\mu\nu}(X_{11}) = ie_p^2 \int \frac{d^4k}{(2\pi)^4} \overline{\Psi}_{\alpha\beta}(K_f;p_f)\,[\gamma^\nu S(k-Q)\gamma^\mu]_{\beta\gamma}\,\Psi_{\gamma\delta}(K_i;p_i) S^{-1}_{\delta\alpha}(k-p), \tag{A5}$$

$$\begin{aligned}\mathcal{C}^{\mu\nu}(X_{13}) = ie_p \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4}\, [\overline{\Psi}(K_f;p_f)\gamma^\nu S(k-Q)]_{\alpha\beta} \\ \times iJ^\mu_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}-\frac{Q}{2}, k'-\frac{p}{2}-\frac{q}{4}; \left[p-\frac{q}{2}, -q_2\right]\right) \Psi_{\gamma\delta}(K'_i;p_i),\end{aligned} \tag{A6}$$

$$\begin{aligned}\mathcal{C}^{\mu\nu}(Z_{11}) = ie_p^2 \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4}\, [\overline{\Psi}(K_f;p_f)\gamma^\nu S(k-Q)]_{\alpha\beta} \\ \times iM_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}-\frac{Q}{2}, k'-\frac{p}{2}-\frac{Q}{2}; p-Q\right) [S(k'-Q)\gamma^\mu \Psi(K'_i;p_i)]_{\gamma\delta}\,,\end{aligned} \tag{A7}$$

$$\begin{aligned}\mathcal{C}^{\mu\nu}(Z_{13}) = ie_p \int \frac{d^4k}{(2\pi)^4}\, [\overline{\Psi}(K_f;p_f)\gamma^\nu S(k-Q)]_{\alpha\beta} \int \frac{d^4k'}{(2\pi)^4} \int \frac{d^4k''}{(2\pi)^4} \\ \times iM_{\beta\alpha;\lambda\phi}\left(k-\frac{p}{2}-\frac{Q}{2}, k''-\frac{p}{2}-\frac{Q}{2}; p-Q\right) S_{\phi\epsilon}(k''-Q) S_{\rho\lambda}(k''-p) \\ \times iJ^\mu_{\epsilon\rho;\delta\gamma}\left(k''-\frac{p}{2}-\frac{Q}{2}, k'-\frac{p}{2}-\frac{q}{4}; \left[p-\frac{q}{2}, -q_2\right]\right) \Psi_{\gamma\delta}(K'_i;p_i).\end{aligned} \tag{A8}$$

The divergence in the first four amplitudes is given by

$$q_{1\nu}\,[\mathcal{C}^{\mu\nu}(W_{11}) + \mathcal{C}^{\mu\nu}(Y_{11}) + \mathcal{C}^{\mu\nu}(W_{13}) + \mathcal{C}^{\mu\nu}(Y_{13})] = ie_p \int \frac{d^4k}{(2\pi)^4}\, [\overline{\Psi}(K_f;p_f)\gamma^\mu S(k+Q)]_{\alpha\beta} \quad \{\odot\}_{\beta\alpha}\,,$$

where

$$
\begin{aligned}
\{\odot\}_{\beta\alpha} = {} & e_p\left[S^{-1}(k+Q) - S^{-1}\left(k-\frac{q}{2}\right)\right]_{\beta\gamma}\Psi_{\gamma\delta}(K_i;p_i)S^{-1}_{\delta\alpha}(k-p)\\
&+ie_p\int\frac{d^4k'}{(2\pi)^4}\,M_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}+\frac{Q}{2};p+Q\right)\\
&\qquad\times\left\{\left[1-S(k'+Q)S^{-1}\left(k'-\frac{q}{2}\right)\right]\Psi(K_i';p_i)\right\}_{\gamma\delta}\\
&+i\int\frac{d^4k'}{(2\pi)^4}\,q_{1\mu}J^{\mu}_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}-\frac{q}{4};\left[p-\frac{q}{2},q_1\right]\right)\Psi_{\gamma\delta}(K_i';p_i)\\
&+i\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k''}{(2\pi)^4}\,M_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}+\frac{Q}{2};p+Q\right)S_{\phi\epsilon}(k''+Q)S_{\rho\delta}(k''-p)\\
&\qquad\times iq_{1\mu}J^{\mu}_{\epsilon\rho;\delta\gamma}\left(k''-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}-\frac{q}{4};\left[p-\frac{q}{2},q_1\right]\right)\Psi_{\gamma\delta}(K_i';p_i)\\
= {} & e_pS^{-1}_{\beta\gamma}(k+Q)\Psi_{\gamma\delta}(K_i;p_i)S^{-1}_{\delta\alpha}(k-p)-\overline{e_pS^{-1}_{\beta\gamma}\left(k-\frac{q}{2}\right)\Psi_{\gamma\delta}(K_i;p_i)S^{-1}_{\delta\alpha}(k-p)}\\
&+e_p\int\frac{d^4k'}{(2\pi)^4}\left\{\underline{\underline{iM_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}+\frac{Q}{2};p+Q\right)\Psi_{\gamma\delta}(K_i';p_i)}}\right.\\
&\left.\underline{-iM_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}+\frac{Q}{2};p+Q\right)\left[S(k'+Q)S^{-1}\left(k'-\frac{q}{2}\right)\Psi(K_i';p_i)\right]_{\gamma\delta}}\right\}\\
&+e_p\int\frac{d^4k'}{(2\pi)^4}i\left[\overline{V_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}-\frac{q}{4},k'-\frac{p}{2}-\frac{q}{4};p-\frac{q}{2}\right)}\right.\\
&\qquad\left.\underline{\underline{-V_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}+\frac{Q}{2};p+Q\right)}}\right]\Psi_{\gamma\delta}(K_i;p_i)\\
&+e_p\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k''}{(2\pi)^4}\,iM_{\beta\alpha;\sigma\phi}\left(k-\frac{p}{2}+\frac{Q}{2},k''-\frac{p}{2}+\frac{Q}{2};p+Q\right)S_{\phi\epsilon}(k''+Q)S^{-1}_{\rho\sigma}(k''-p)\\
&\qquad\times i\left[\underline{V_{\epsilon\rho;\delta\gamma}\left(k''-\frac{p}{2}-\frac{q}{4},k'-\frac{p}{2}-\frac{q}{4};p-\frac{q}{2}\right)}\right.\\
&\qquad\left.\underline{\underline{-V_{\epsilon\rho;\delta\gamma}\left(k''-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}+\frac{Q}{2};p+Q\right)}}\right]\Psi_{\gamma\delta}(K_i;p_i).
\end{aligned}
$$

The *overlined* terms cancel by the wave equation for the bound state,

$$
S^{-1}\left(k-\frac{q}{2}\right)\Psi\left(k-\frac{p}{2}-\frac{q}{4};p_i\right)S^{-1}(k-p)=i\int\frac{d^4k''}{(2\pi)^4}V_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}-\frac{q}{4},k''-\frac{p}{2}-\frac{q}{4};p_i\right)\Psi_{\gamma\delta}\left(k''-\frac{p}{2}-\frac{q}{4};p_i\right).
$$

The *underlined* terms also cancel. The double-underlined terms cancel if we use the scattering equation

$$
\begin{aligned}
&M_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}+\frac{Q}{2};p+Q\right)\\
&\quad=V_{\alpha\beta;\delta\gamma}\left(k-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}+\frac{Q}{2};p+Q\right)\\
&\qquad+i\int\frac{d^4k''}{(2\pi)^4}M_{\beta\alpha;\sigma\phi}\left(k-\frac{p}{2}+\frac{Q}{2},k''-\frac{p}{2}+\frac{Q}{2};p+Q\right)S_{\phi\epsilon}(k''+Q)S_{\rho\sigma}(k''-p)\\
&\qquad\times V_{\epsilon\rho;\delta\gamma}\left(k''-\frac{p}{2}+\frac{Q}{2},k'-\frac{p}{2}+\frac{Q}{2};p+Q\right).
\end{aligned}
$$

Only the first term remains, so that

$$
\begin{aligned}
&q_{1\nu}\left[\mathcal{C}^{\mu\nu}(W_{11})+\mathcal{C}^{\mu\nu}(Y_{11})+\mathcal{C}^{\mu\nu}(W_{13})+\mathcal{C}^{\mu\nu}(Y_{13})\right]\\
&\qquad=ie_p^2\int\frac{d^4k}{(2\pi)^4}\left[\overline{\Psi}(K_f;p_f)\gamma^{\mu}S(k+Q)\right]_{\alpha\beta}S^{-1}_{\beta\gamma}(k+Q)\Psi_{\gamma\delta}(K_i;p_i)S^{-1}_{\delta\alpha}(k-p)\,. \qquad \text{(A9)}
\end{aligned}
$$

Similarily, the divergence of the $X_{11}+Z_{11}+X_{13}+Z_{13}$ processes is given by

$$
q_{1\nu}\left[\mathcal{C}^{\mu\nu}(X_{11})+\mathcal{C}^{\mu\nu}(Z_{11})+\mathcal{C}^{\mu\nu}(X_{13})+\mathcal{C}^{\mu\nu}(Z_{13})\right]=ie_p\int\frac{d^4k'}{(2\pi)^4}\{\odot\}_{\alpha\beta}\left[S(k'+Q)\gamma^{\mu}\Psi(K_i';p_i)\right]_{\beta\alpha},
$$

where

$$
\begin{aligned}
\{\odot\}_{\alpha\beta} = & -e_p S^{-1}_{\alpha\gamma}(k'-p)\overline{\Psi}_{\gamma\delta}(K'_f;p_f)S^{-1}_{\delta\beta}(k'-Q) + \overline{e_p S^{-1}_{\alpha\gamma}(k'-p)\overline{\Psi}_{\gamma\delta}(K'_f;p_f)S^{-1}_{\delta\beta}\left(k'+\frac{q}{2}\right)} \\
& -e_p \int \frac{d^4k'}{(2\pi)^4}\left\{\underline{\underline{\overline{\Psi}_{\gamma\delta}(K'_f;p_f) iM_{\delta\gamma;\alpha\beta}\left(k-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{Q}{2};p-Q\right)}}\right. \\
& \qquad \left. - \underline{\left[\overline{\Psi}(K'_f;p_f)S^{-1}\left(k+\frac{q}{2}\right)S(k-Q)\right]_{\gamma\delta} iM_{\delta\gamma;\alpha\beta}\left(k-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{Q}{2};p-Q\right)}\right\} \\
& -ie_p \int \frac{d^4k'}{(2\pi)^4}\overline{\Psi}_{\gamma\delta}(K_f;p_f)\left[\overline{V_{\delta\gamma;\alpha\beta}\left(k-\frac{p}{2}+\frac{q}{4},k'-\frac{p}{2}+\frac{q}{4};p-Q\right)}\right. \\
& \qquad \left. \underline{\underline{-V_{\delta\gamma;\alpha\beta}\left(k-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{Q}{2};p-Q\right)}}\right] \\
& -e_p \int \frac{d^4k}{(2\pi)^4}\int \frac{d^4k''}{(2\pi)^4} i\overline{\Psi}_{\gamma\delta}(K_f;p_f) \\
& \times\left[\underline{V_{\delta\gamma;\sigma\phi}\left(k-\frac{p}{2}+\frac{q}{4},k''-\frac{p}{2}+\frac{q}{4};p+\frac{q}{2}\right)}\ \underline{\underline{-V_{\delta\gamma;\sigma\phi}\left(k-\frac{p}{2}-\frac{Q}{2},k''-\frac{p}{2}-\frac{Q}{2};p-Q\right)}}\right] \\
& \times S_{\phi\epsilon}(k''-Q)S_{\rho\sigma}(k''-p) iM_{\epsilon\rho;\alpha\beta}\left(k''-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{Q}{2};p-Q\right).
\end{aligned}
$$

As in the derivation of Eq. (A9), the underlined terms cancel, leaving the first term only. We have
$q_{1\nu}\left[\mathcal{C}^{\mu\nu}(X_{11})+\mathcal{C}^{\mu\nu}(Z_{11})+\mathcal{C}^{\mu\nu}(X_{13})+\mathcal{C}^{\mu\nu}(Z_{13})\right]$

$$
= -ie_p^2\int \frac{{}^4k'}{(2\pi)^4} S^{-1}_{\beta\gamma}(k'-p)\overline{\Psi}_{\gamma\delta}(K'_f;p_f)S^{-1}_{\delta\alpha}(k'-Q)\left[S(k'+Q)\gamma^\mu\Psi(K'_i;p_i)\right]_{\beta\alpha}\ . \qquad \text{(A10)}
$$

Equations (A9) and (A10) were used to complete the proof of Eq. (4.3) in Sec. IV.

APPENDIX B

We now derive Eq. (4.9). The four amplitudes are

$$
\begin{aligned}
\mathcal{C}^{\mu\nu}(X_{31}) = & ie_p\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}\left[\overline{\Psi}(K_f;p_f)\gamma^\nu S(k-Q)\right]_{\alpha\beta} \\
& \times iJ^\mu_{\beta\alpha;\delta\gamma}\left[k-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{q}{4};\left(p-\frac{q}{2},-q_2\right)\right]\Psi_{\gamma\delta}(K'_i;p_i),
\end{aligned}
\qquad \text{(B1)}
$$

$$
\begin{aligned}
\mathcal{C}^{\mu\nu}(Z_{31}) = & ie_p\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k''}{(2\pi)^4}\left[\overline{\Psi}(K_f;p_f)\gamma^\nu S(k-Q)\right]_{\alpha\beta} \\
& \times iM_{\beta\alpha;\delta\phi}\left(k-\frac{p}{2}-\frac{Q}{2},k''-\frac{p}{2}-\frac{Q}{2};p+Q\right)S_{\phi\epsilon}(k''-Q)S_{\rho\delta}(k''-p) \\
& \times iJ^\nu_{\epsilon\rho;\delta\gamma}\left[k''-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{q}{4};\left(p-\frac{q}{2},-q_2\right)\right]\Psi_{\gamma\delta}(K'_i;p_i),
\end{aligned}
\qquad \text{(B2)}
$$

$$
\begin{aligned}
\mathcal{C}^{\mu\nu}(X_{33}) = & i\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}\int\frac{d^4k''}{(2\pi)^4}\overline{\Psi}_{\alpha\beta}(K_f;p_f) \\
& \times iJ^\nu_{\beta\alpha;\delta\gamma}\left(k-\frac{p}{2}+\frac{q}{4},k''-\frac{p}{2}-\frac{Q}{2};[p-Q,q_1]\right)S_{\gamma\epsilon}(k''-Q)S_{\rho\delta}(k''-p) \\
& \times iJ^\mu_{\epsilon\rho;\delta\gamma}\left[k''-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{q}{4};\left(p-\frac{q}{2},-q_2\right)\right]\Psi_{\gamma\delta}(K'_i;p_i),
\end{aligned}
\qquad \text{(B3)}
$$

and

$$\mathcal{C}^{\mu\nu}(Z_{33}) = i\int \frac{d^4k}{(2\pi)^4}\int \frac{d^4k'}{(2\pi)^4}\int \frac{d^4k''}{(2\pi)^4}\int \frac{d^4\widetilde{k}}{(2\pi)^4}\overline{\Psi}_{\alpha\beta}(K_f;p_f)$$

$$\times iJ^{\nu}_{\beta\alpha;\eta\xi}\left(k-\frac{p}{2}+\frac{q}{4},\widetilde{k}-\frac{p}{2}-\frac{Q}{2};[p-Q,q_1]\right) S_{\xi\epsilon}(\widetilde{k}-Q)S_{\phi\eta}(\widetilde{k}-p)$$

$$\times iM_{\epsilon\phi;\sigma\rho}\left(\widetilde{k}-\frac{p}{2}-\frac{Q}{2},k''-\frac{p}{2}-\frac{Q}{2};p-Q\right) S_{\rho\lambda}(k''-Q)S_{\kappa\sigma}(k''-p)$$

$$\times iJ^{\mu}_{\lambda\kappa;\delta\gamma}\left[k''-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{q}{4};\left(p-\frac{q}{2},-q_2\right)\right]\Psi_{\gamma\delta}(K_i';p_i). \tag{B4}$$

To evaluate the divergence of these amplitudes, we separate out a common factor,

$$q_{1\nu}\left[\mathcal{C}^{\mu\nu}(X_{31})+\mathcal{C}^{\mu\nu}(X_{33})+\mathcal{C}^{\mu\nu}(Z_{31})+\mathcal{C}^{\mu\nu}(Z_{33})\right] = ie_p\int \frac{d^4k}{(2\pi)^4} \tag{6.1}$$

where the curly bracket is given by

$$\{XZ\}_{\delta\gamma} = e_p\int \frac{d^4k'}{(2\pi)^4}\left\{\overline{\Psi}(K_f;p_f)\left[S^{-1}\left(k+\frac{q}{2}\right)S(k-Q)-1\right]\right\}_{\alpha\beta} iJ^{\mu}_{\beta\alpha;\delta\gamma}\left[k-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{q}{4};\left(p-\frac{q}{2},-q_2 i\right)\right]$$

$$+e_p\int \frac{d^4k'}{(2\pi)^4}\int \frac{d^4k''}{(2\pi)^4}\left\{\overline{\Psi}(K_f;p_f)\left[S^{-1}\left(k+\frac{q}{2}\right)S(k-Q)-1\right]\right\}_{\epsilon\rho}$$

$$\times iM_{\rho\epsilon;\sigma\phi}\left(k-\frac{p}{2}-\frac{Q}{2},k''-\frac{p}{2}-\frac{Q}{2};p-Q\right) S_{\phi\beta}(k''-Q)S_{\alpha\sigma}(k''-p)$$

$$\times iJ^{\nu}_{\beta\alpha;\delta\gamma}\left[k''-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{q}{4};\left(p-\frac{q}{2},-q_2\right)\right]$$

$$+\int \frac{d^4k'}{(2\pi)^4}\int \frac{d^4k''}{(2\pi)^4}\overline{\Psi}_{\epsilon\rho}(K_f;p_f)\; iq_{1\nu}J^{\nu}_{\rho\epsilon;\sigma\phi}\left(k-\frac{p}{2}+\frac{q}{4},k''-\frac{p}{2}-\frac{Q}{2};[p-Q,q_1]\right)$$

$$\times S_{\phi\beta}(k''-Q)S_{\alpha\sigma}(k''-p)\; iJ^{\mu}_{\beta\alpha;\delta\gamma}\left[k''-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{q}{4};\left(p-\frac{q}{2},-q_2\right)\right]$$

$$+\int \frac{d^4k'}{(2\pi)^4}\int \frac{d^4k''}{(2\pi)^4}\int \frac{d^4\widetilde{k}}{(2\pi)^4}\overline{\Psi}_{\kappa\lambda}(K_f;p_f)iq_{1\nu}J^{\nu}_{\lambda\kappa;\eta\xi}\left(k-\frac{p}{2}+\frac{q}{4},\widetilde{k}-\frac{p}{2}-\frac{Q}{2};[p-Q,q_1]\right)$$

$$\times S_{\xi\epsilon}(\widetilde{k}-Q)S_{\phi\eta}(\widetilde{k}-p)\; iM_{\epsilon\phi;\sigma\rho}\left(\widetilde{k}-\frac{p}{2}-\frac{Q}{2},k''-\frac{p}{2}-\frac{Q}{2};p-Q\right)$$

$$\times S_{\rho\beta}(k''-Q)S_{\alpha\sigma}\left[k''-p)\; iJ^{\mu}_{\beta\alpha;\delta\gamma}\left(k''-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{q}{4}\right);\left(p-\frac{q}{2},-q_2\right)\right].$$

Use the two-body WT identity for the interaction current,

$$q_\mu J^\mu\left(k',k;[p,q]\right) = e_p\left[V\left(k'-\frac{q}{2},k;p\right)-V\left(k',k+\frac{q}{2};p+q\right)\right],$$

to get the following expression:

$$\{XZ\}_{\delta\gamma} = e_p\int \frac{d^4k'}{(2\pi)^4}\{\odot\}_{\alpha\beta}\, iJ^{\mu}_{\beta\alpha;\delta\gamma}\left[k-\frac{p}{2}-\frac{Q}{2},k'-\frac{p}{2}-\frac{q}{4};\left(p-\frac{q}{2},-q_2\right)\right],$$

where

$$
\begin{aligned}
\{\odot\}_{\alpha\beta} = & \left\{ \overline{\Psi}(K_f;p_f) \left[\underline{S^{-1}\left(k+\frac{q}{2}\right) S(k-Q) - 1} \right] \right\}_{\alpha\beta} \\
& + \int \frac{d^4k''}{(2\pi)^4} \left\{ \overline{\Psi}(K_f;p_f) \left[\overline{S^{-1}\left(k+\frac{q}{2}\right) S(k-Q) - \underline{\underline{1}}} \right] \right\}_{\epsilon\rho} \\
& \quad \times iM_{\rho\epsilon;\sigma\phi}\left(k-\frac{p}{2}-\frac{Q}{2}, k''-\frac{p}{2}-\frac{Q}{2}; p-Q\right) S_{\phi\beta}(k''-Q) S_{\alpha\sigma}(k''-p) \\
& + \int \frac{d^4k''}{(2\pi)^4} i\overline{\Psi}_{\epsilon\rho}(K_f;p_f) \left[\underline{\underline{V_{\rho\epsilon;\sigma\phi}\left(k-\frac{p}{2}-\frac{Q}{2}, k''-\frac{p}{2}-\frac{Q}{2}; p-Q\right)}} \right. \\
& \quad \left. - \underline{V_{\rho\epsilon;\sigma\phi}\left(k-\frac{p}{2}+\frac{q}{4}, k''-\frac{p}{2}+\frac{q}{4}; p+\frac{q}{2}\right)} \right] S_{\phi\beta}(k''-Q) S_{\alpha\sigma}(k''-p) \\
& + \int \frac{d^4k''}{(2\pi)^4} \int \frac{d^4\widetilde{k}}{(2\pi)^4} i\overline{\Psi}_{\kappa\lambda}(K_f;p_f) \left[\underline{\underline{V_{\lambda\kappa;\eta\xi}\left(k-\frac{p}{2}-\frac{Q}{2}, \widetilde{k}-\frac{p}{2}-\frac{Q}{2}; p-Q\right)}} \right. \\
& \quad \left. - \overline{V_{\lambda\kappa;\eta\xi}\left(k-\frac{p}{2}+\frac{q}{4}, \widetilde{k}-\frac{p}{2}+\frac{q}{4}; p+\frac{q}{2}\right)} \right] S_{\xi\epsilon}(\widetilde{k}-Q) S_{\phi\eta}(\widetilde{k}-p) \\
& \quad \times iM_{\epsilon\phi;\sigma\rho}\left(\widetilde{k}-\frac{p}{2}-\frac{Q}{2}, k''-\frac{p}{2}-\frac{Q}{2}; p-Q\right) S_{\rho\beta}(k''-Q) S_{\alpha\sigma}(k''-p) \,.
\end{aligned}
$$

Note that the underlined, double-underlined and overlined terms cancel themselves. We get $\{\odot\}_{\alpha\beta} = -\overline{\Psi}_{\alpha\beta}(K_f;p_f)$, and this proves Eq. (4.9).

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