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Analysis of the reaction $\pi^+ d \to pp$ to 500 MeV

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An energy-dependent and set of single-energy partial-wave analyses have been completed for the reaction $\pi^+ d \to pp$. Amplitudes are presented for pion laboratory kinetic energies from threshold to 500 MeV. These results are compared with those found in other recent analyses. We comment on the present database and make suggestions for future experiments.

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I. INTRODUCTION

The reaction $\pi d \to NN$ is fundamental to studies of the πNN system [1]. Historically, the $\pi d \leftrightarrow NN$ processes are perhaps best known for their role in determining [2] the spin and parity of the pion. The πd and $N\Delta$ channels are also responsible for most of the inelasticity in NN scattering at intermediate energies. In this study, we will concentrate on the reactions $\pi^+ d \leftrightarrow pp$. A number of partial-wave [3,4] and model-based [1] analyses have been performed on this database in the past. The more theoretical approaches have had difficulty in describing all observables [1], and have generally concentrated on limited kinematic regions. The partial-wave analyses have found some motivation from claims [5] of dibaryon resonances in pp scattering reactions.

The present analysis extends from threshold to 500 MeV in terms of the laboratory kinetic energy of the pion. This corresponds to laboratory kinetic energies between 287.5 MeV and 1287 MeV in the pp system. The value of $\sqrt{s}$ varies from 2.015 GeV to 2.437 GeV, and spans the range of energies typically associated with dibaryon candidates. The database was extended to 535 MeV in order to regularize the solution at the end point energy.

In the next section, we will make some comments on the database used in this analysis. In Sec. III we outline the general formalism for the $\pi^+ d \to pp$ reaction. Methods used in the partial-wave analysis are discussed in Sec. IV. Our main results are presented in Sec. V. Here we will also compare with the available data and other recent analyses. Finally, in Sec. VI, we summarize our findings and make some suggestions for future investigations.

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II. THE DATABASE

Early experimental studies of the reaction $\pi^+ d \to pp$ began to produce results in the 1950s. At this time, the first measurements of $\sigma_{\rm tot}$, $d\sigma/d\Omega$, A_{y0}, and iT_{11} became available. The trend of $\pi^+ d \to pp$ data accumulation since 1951 is displayed in Fig. 1. The rapid increase in the number and type of measurements in the early 1980s was motivated by a growing interest in the problem of exotics. This reaction was expected to give further information on the existence (or nonexistence) of dibaryon states suggested in analyses of NN elastic scattering data. Numerous high-quality pp polarization and $p{-}d$ polarization-transfer measurements were made. The total database more than doubled, and several partial-wave analyses [3,4] were carried out at this time. The present study has utilized a set of data which is again larger than those used in the previous analyses [3,4].

Our total set of experimental data [4a,4b,6-93] (4440 points) includes measurements of the differential cross section with unpolarized targets (1094 points), asymmetry in the case of one polarized proton (1749 points), spin-

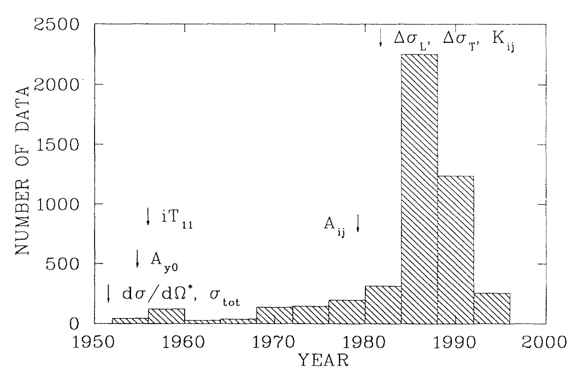

FIG. 1. Data accumulation from 1951 to the present. Arrows indicate the year when measurements of particular observables were first published.

correlation parameters for the two protons (844 points), the deuteron vector analyzing-power (155 points), and spin-correlation parameters of one proton and the deuteron (429 points). We have included unpolarized total cross sections (150 points) as well as total cross sections in pure proton spin states (19 points). Energy-angle distributions are given for $d\sigma/d\Omega$, A_{yy}, A_{zz}, A_{xy}, A_{y0}, and iT_{11} in Fig. 2.

As shown in Table I we have removed σ_{tot} (17 points), $d\sigma/d\Omega$ (66 points), and A_{y0} (18 points) data corresponding to 2.3% of the total. These measurements were the source of serious conflicts within the database, and were not included in the analysis. We will discuss the effect of

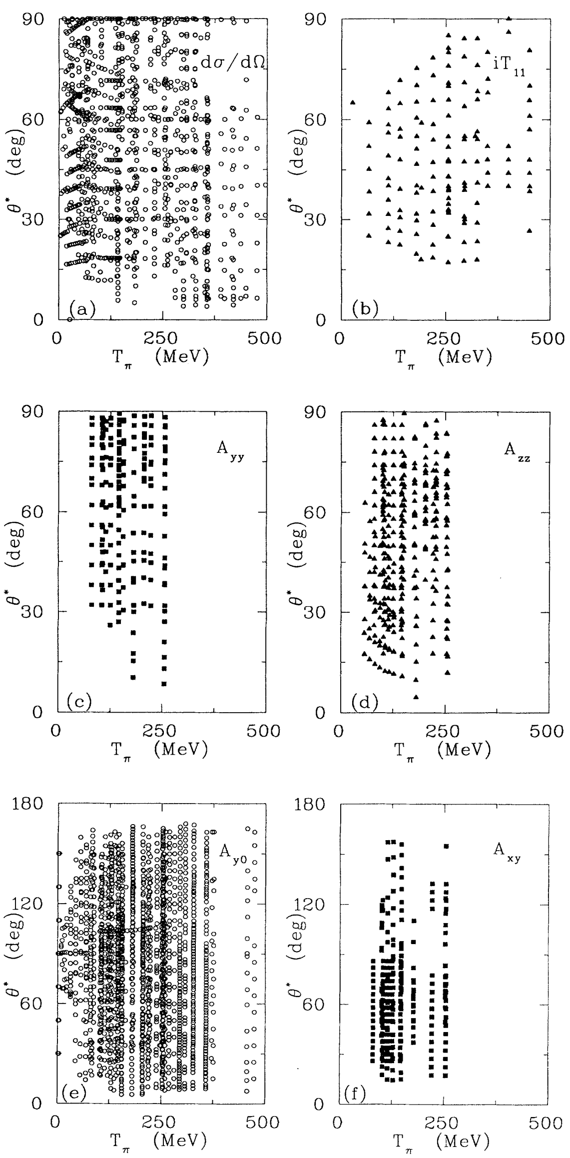

FIG. 2. Energy-angle distribution of total dataset. (a) Differential cross section $d\sigma/d\Omega$; (b) deuteron vector analyzing power, iT_{11}, spin correlation parameters of the two protons (c) A_{yy} and (d) A_{zz}; (e) proton analyzing power A_{y0}; (d) spin correlation parameter for two protons A_{xz}.

TABLE I. Number and type of observables used in the present analysis to 535 MeV in the pion laboratory kinetic energy. The number of excluded data is also given.

Observable	No. of data	Deleted data
$\sigma(\theta)$	1117	66[a]
σ_T	167	17[b]
$\Delta\sigma_L$	14	0
$\Delta\sigma_T$	5	0
A_{y0}	1767	18[c]
A_{xx}	62	0
A_{yy}	185	0
A_{zz}	340	0
A_{xz}	257	0
iT_{11}	155	0
K_{xx}	9	0
K_{yy}	10	0
K_{xz}	5	0
K_{zx}	5	0
ϵ	136	0
H_i	264[d]	0
Total	4541	101
No. of energies	586	40

[a]See Refs. [33], [46], [49], [61], [68], [70], and [90].
[b]See Refs. [16], [17], [33], [36], [61], [70], and [90].
[c]See Refs. [8], [33], and [40].
[d]Helicity amplitudes from Ref. [4b].

further data "pruning" in Sec. IV. We should also indicate that the ϵ data [4a,4b] (defined in Ref. [4a]) were not directly included in this analysis. Instead, we included the amplitudes (real and imaginary parts) constructed from this data [4b] in our fits.

As a result of detailed conversations with the author, Turpin, and with Jones, Strikman, and Wilkin, we decided to use a normalization factor of 1.00 (rather than the value of 1.07 proposed in Ref. [89a]). Since the measurement of a polarization of the deuteron, by the stripped proton, was done near the quasielastic peak (where the internal nucleons have momenta near zero in the deuteron), contributions from the d state are negligible [89b].

We have reviewed the systematic errors and the energy uncertainty associated with the present database. Many publications list only statistical errors in the data tables and discuss systematic errors in the text. A systematic uncertainty (typically 3%) was assigned in cases where we could not determine a value from the literature or communications with the authors. Several datasets were divided into a few parts due to different sources of systematic uncertainties [experiments using different kinds of targets (CH_2 and LH_2), a polarized proton beam, or target, etc.].

III. FORMALISM

The relations between partial-wave amplitudes and observables have been given in a number of previous theoretical and phenomenological studies. A translation guide for the various notations is given by Cantale *et* *al.* [4a,94]. Due to parity conservation, there are 6 independent helicity amplitudes for this reaction. Thus, for a reconstruction of the scattering amplitude at fixed values of the energy and scattering angle, one requires 11 independent measurements. The amplitude, $F_{\alpha\beta;\lambda}(\theta)$, is labeled by the helicities (α and β) of the two protons and (λ) the deuteron helicity. Here the angle θ is the center-of-mass scattering angle of the pion in the reaction $pp \to \pi^+ d$. Our notation [94] for the helicity amplitudes is given below:

$$
\begin{aligned}
F_{\frac{1}{2}\,\frac{1}{2};1} &\equiv H_1 = \frac{1}{4\pi} \sum_{J(\text{even})} (2J+1)\ h_1^J\ d^J_{0\,-1},\\
F_{\frac{1}{2}\,\frac{1}{2};0} &\equiv H_2 = \frac{1}{4\pi} \sum_{J(\text{even})} (2J+1)\ h_2^J\ d^J_{00},\\
F_{\frac{1}{2}\,\frac{1}{2};-1} &\equiv H_3 = \frac{1}{4\pi} \sum_{J(\text{even})} (2J+1)\ h_3^J\ d^J_{01},\\
F_{-\frac{1}{2}\,\frac{1}{2};0} &\equiv H_4 = \frac{1}{4\pi} \sum_{J(\text{odd})} (2J+1)\ h_4^J\ d^J_{01},\\
F_{\frac{1}{2}\,-\frac{1}{2};-1} &\equiv H_5 = \frac{1}{4\pi} \sum_{J} (2J+1)\ h_5^J\ d^J_{11},\\
F_{\frac{1}{2}\,-\frac{1}{2};1} &\equiv H_6 = \frac{1}{4\pi} \sum_{J} (2J+1)\ h_6^J\ d^J_{1\,-1},
\end{aligned}
\tag{1}
$$

where the $d^J_{\alpha\,\beta}$ are reduced rotation matrices, and $h_6^J = (-1)^{J+1}\, h_5^J$. The following symmetry relation

$$F_{\alpha\,\beta\,;\,\lambda} = (-1)^{\alpha+\beta+\lambda} F_{-\alpha\,-\beta\,;\,-\lambda},$$

is also obeyed by the above helicity amplitudes. The partial-wave amplitudes, $T^J_{L^{pp}S_{pp};L^\pi}$, are labeled by the values of L^{pp}, S_{pp}, and J corresponding to the pp state ${}^{2S_{pp}+1}L^{pp}_J$, and L^π for the $\pi^+ d$ state. The decomposition is given for even and odd values of J below. For J even we have

$$
\begin{aligned}
h_1^J &= \sqrt{\frac{J+1}{2J+1}} T^J_{J\,0;J-1} + \sqrt{\frac{J}{2J+1}} T^J_{J\,0;J+1}\\
&\quad - \sqrt{\frac{J}{2J+1}} T^J_{J-1\,1;J} + \sqrt{\frac{J+1}{2J+1}} T^J_{J+1\,1;J},\\
h_2^J &= \sqrt{\frac{2J}{2J+1}} T^J_{J\,0;J-1} - \sqrt{\frac{2J+2}{2J+1}} T^J_{J\,0;J+1},\\
h_3^J &= \sqrt{\frac{J+1}{2J+1}} T^J_{J\,0;J-1} + \sqrt{\frac{J}{2J+1}} T^J_{J\,0;J+1}\\
&\quad + \sqrt{\frac{J}{2J+1}} T^J_{J-1\,1;J} - \sqrt{\frac{J+1}{2J+1}} T^J_{J+1\,1;J},\\
h_4^J &= 0,\\
h_5^J &= \sqrt{\frac{J+1}{2J+1}} T^J_{J-1\,1;J} + \sqrt{\frac{J}{2J+1}} T^J_{J+1\,1;J}.
\end{aligned}
\tag{2}
$$

For J odd we have

$$
\begin{aligned}
h_1^J &= 0,\\
h_2^J &= 0,\\
h_3^J &= 0,\\
h_4^J &= \sqrt{\frac{2J}{2J+1}} T^J_{J\,1;J-1} - \sqrt{\frac{2J+2}{2J+1}} T^J_{J\,1;J+1},\\
h_5^J &= \sqrt{\frac{J+1}{2J+1}} T^J_{J\,1;J-1} + \sqrt{\frac{J}{2J+1}} T^J_{J\,1;J+1}.
\end{aligned}
\tag{3}
$$

In the next section, and in our figures, we use the notation ${}^{2S_{pp}+1}L^{pp}_J L^\pi$ to denote partial-wave amplitudes. The connection between pp and $\pi^+ d$ states is given in Table II.

The various observables for the $\pi^+ d \to pp$ reaction are given in terms of helicity amplitudes [94,95] below:

$$
\begin{aligned}
I_0 &\equiv t^{00}_{00} = \sum_i H_i^2,\\
\frac{d\sigma}{d\Omega} &= \sigma_g \cdot I_0,\\
\sigma_{\text{tot}} &= 2\pi\sigma_g \int_0^{\frac{\pi}{2}} I_0 \sin\theta d\theta,\\
A_{y0} &\equiv \sqrt{2} i t^{11}_{00} = 2\text{Im}\big(H_1H_5^* + H_2H_4^* + H_3H_6^*\big)/I_0,\\
A_{xx} &= \big[-H_2^2 + H_4^2 + 2\text{Re}(H_1H_3^* - H_5H_6^*)\big]/I_0,\\
A_{yy} &= \big[-H_2^2 - H_4^2 + 2\text{Re}(H_1H_3^* + H_5H_6^*)\big]/I_0,\\
A_{zz} &= \big(-H_1^2 - H_2^2 - H_3^2 + H_4^2 + H_5^2 + H_6^2\big)/I_0,\\
A_{xz} &= 2\text{Re}\big(-H_1H_5^* - H_2H_4^* - H_3H_6^*\big)/I_0,\\
iT_{11} &\equiv it^{00}_{11} = \sqrt{\frac{3}{2}}\text{Im}\big(H_1H_2^* + H_2H_3^* - H_4H_5^* + H_4H_6^*\big)/I_0,\\
T_{20} &\equiv t^{00}_{20} = \frac{1}{\sqrt{2}}\big(H_1^2 - 2H_2^2 + H_3^2 - 2H_4^2 + H_5^2 + H_6^2\big)/I_0,\\
T_{21} &\equiv t^{00}_{21} = \sqrt{\frac{3}{2}}\text{Re}\big(H_1H_2^* - H_2H_3^* + H_4H_5^* - H_4H_6^*\big)/I_0,\\
T_{22} &\equiv t^{00}_{22} = \sqrt{3}\text{Re}\big(H_1H_3^* + H_5H_6^*\big)/I_0,
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
K_{xx} &\equiv P_x^x = \sqrt{2}\mathrm{Re}\big(H_1H_4^* + H_2H_5^* + H_2H_6^* + H_3H_4^*\big)/I_0,\\
K_{yy} &\equiv P_y^y = \sqrt{2}\mathrm{Re}\big(H_1H_4^* - H_2H_5^* + H_2H_6^* - H_3H_4^*\big)/I_0,\\
K_{zz} &\equiv P_z^z = \big(H_1^2 - H_3^2 - H_5^2 + H_6^2\big)/I_0,\\
K_{xz} &\equiv P_z^x = 2\mathrm{Re}\big(-H_1H_5^* + H_3H_6^*\big)/I_0,\\
K_{zx} &\equiv P_x^z = \sqrt{2}\mathrm{Re}\big(-H_1H_2^* - H_2H_3^* + H_4H_5^* + H_4H_6^*\big)/I_0,\\
K_{yzz} &\equiv P_{zz}^y = 2\mathrm{Im}\big(-H_1H_5^* + 2H_2H_4^* - H_3H_6^*\big)/I_0,\\
K_{zyz} &\equiv P_{yz}^z = \frac{3}{\sqrt{2}}\mathrm{Im}\big(H_1H_2^* - H_2H_3^* + H_4H_5^* + H_4H_6^*\big)/I_0,\\
K_{zxy} &\equiv P_{xy}^z = 3\mathrm{Im}\big(-H_1H_3^* + H_5H_6^*\big)/I_0,\\
K_{xyz} &\equiv P_{yz}^x = \frac{3}{\sqrt{2}}\mathrm{Im}\big(-H_1H_4^* + H_2H_5^* + H_2H_6^* - H_3H_4^*\big)/I_0,\\
K_{yxz} &\equiv P_{xz}^y = \frac{3}{\sqrt{2}}\mathrm{Im}\big(H_1H_4^* + H_2H_5^* - H_2H_6^* - H_3H_4^*\big)/I_0,\\
K_{xxy} &\equiv P_{xy}^x = 3\mathrm{Im}\big(H_1H_6^* - H_3H_5^*\big)/I_0,\\
K_{yxx} - K_{yyy} &\equiv P_{xx}^y - P_{yy}^y = 6\mathrm{Im}\big(-H_1H_6^* - H_3H_5^*\big)/I_0.
\end{aligned}
$$

In these relations, $A^2 \equiv A^*A$. The factor σ_g is equal to $\frac{1}{6}\left(\frac{\hbar c}{k_\pi}\right)^2$, where k_π is the pion momentum in the center-of-mass frame. The superscript $*$ denotes complex conjugation. Our notation is given first in the above relations; the second designation is due to Blankleider and Afnan [94], where the t's are spherical harmonics. The observables $d\sigma/d\Omega$ and $\sigma_{\rm tot}$ are the usual unpolarized cross sections. The other observables involve the polarization of a single proton (A_{y0}), two protons (A_{xx}, A_{yy}, A_{zz}, and A_{xz}), the deuteron (iT_{11}, T_{20}, T_{21}, and T_{22}), one proton and the deuteron (K_{xx}, K_{yy}, K_{zz}, K_{xz}, and K_{zx}), or two protons and the deuteron (K_{yzz}, K_{zyz}, K_{zxy}, K_{xyz}, K_{yxz}, K_{xxy}, and $K_{yxx} - K_{yyy}$).

TABLE II. Notation conversion table.

${}^{2S_{pp}+1}L_J^{pp}$	${}^{S_\pi+S_d+1}L_J^\pi$	Notation	$J^{\rm Parity}$
${}^1S_0$	${}^3P_0$	${}^1S_0P$	0^+
${}^3P_1$	${}^3S_1$	${}^3P_1S$	1^-
	${}^3D_1$	${}^3P_1D$	
${}^1D_2$	${}^3P_2$	${}^1D_2P$	2^+
	${}^3F_2$	${}^1D_2F$	
${}^3P_2$	${}^3D_2$	${}^3P_2D$	2^-
${}^3F_2$		${}^3F_2D$	
${}^3F_3$	${}^3D_3$	${}^3F_3D$	3^-
	${}^3G_3$	${}^3F_3G$	
${}^1G_4$	${}^3F_4$	${}^1G_4F$	4^+
	${}^3H_4$	${}^1G_4H$	
${}^3F_4$	${}^3G_4$	${}^3F_4G$	4^-
${}^3H_4$		${}^3H_4G$	

Further relations, for total cross sections in pure spin states, are given by

$$
\begin{aligned}
\Delta\sigma_L &= \sigma(\rightleftarrows) - \sigma(\rightrightarrows)\\
&= -2\int A_{zz}\frac{d\sigma}{d\Omega}d\Omega\\
&= -4\pi\sigma_g\int_0^{\frac{\pi}{2}}\big(I_0 - 2H_1^2 - 2H_2^2 - 2H_3^2\big)\sin\theta d\theta,\\
\Delta\sigma_T &= \sigma(\uparrow\downarrow) - \sigma(\uparrow\uparrow)\\
&= -\int\big(A_{xx} + A_{yy}\big)\frac{d\sigma}{d\Omega}d\Omega\\
&= -4\pi\sigma_g\int_0^{\frac{\pi}{2}}\big(H_2^2 - 2\mathrm{Re}H_1^*H_3\big)\sin\theta d\theta,\\
\Delta\sigma_L^d &= \int T_{20}\frac{d\sigma}{d\Omega}d\Omega\\
&= \frac{\sigma_{\rm tot}}{\sqrt{2}} - \frac{3}{\sqrt{2}}\cdot 2\pi\sigma_g\int_0^{\frac{\pi}{2}}\big(H_2^2 + H_4^2\big)\sin\theta d\theta,\\
\Delta\sigma_T^d &= \int T_{22}\frac{d\sigma}{d\Omega}d\Omega\\
&= 2\sqrt{3}\pi\sigma_g\int_0^{\frac{\pi}{2}}\mathrm{Re}\big(H_1^*H_3 + H_5^*H_6\big)\sin\theta d\theta.
\end{aligned}
\tag{5}
$$

Various consistency conditions among the observables [95] were checked during the analysis. In particular, some observables are symmetric ($d\sigma/d\Omega$, A_{xx}, A_{yy}, A_{zz}, T_{20}, T_{22}, and K_{yzz}) when $(\theta) \to (\pi - \theta)$, others ($iT_{11}$, T_{21}, K_{yy}, and K_{yxz}) are antisymmetric.

IV. PARTIAL-WAVE ANALYSIS

Energy-dependent analyses have been published by the Kyoto [3a] and SPB [3b] groups. The last single-energy partial-wave analyses for $\pi^+d \to pp$ were published by the Geneva [4a,4b] and Queen Mary College [4c] groups. These results have generally covered a narrow energy interval in the delta isobar region (14 points from 9 to 256 MeV in [4c] and 3 points from 80 to 146 MeV in [4a,4b]). The analysis of Ref. [3b] covered a somewhat larger energy interval (70 to 450 MeV). It should be noted that some constraints from model-based calculations were included in the analysis of Ref. [4c].

In the present energy-dependent analysis, 13 searched partial-waves were parametrized with 52 varied parameters. Amplitudes coupling to pp states with $J \leq 6$ were considered. Of the amplitudes listed in Table II, the ^1G_4H and ^3H_4G partial waves were found to be negligible and were not searched. Two higher partial waves, the ^3H_5G and ^1I_6H, were found to be significant and were included in the analysis. The chosen form allowed both nonresonant and "resonancelike" amplitudes, with appropriate threshold behavior. The following form was used:

$$A = v^{L^\pi + \frac{1}{2}} \frac{N}{D_R + iD_I}, \tag{6}$$

where we have defined

$$\begin{aligned} v &= \sqrt{\frac{2T_\pi}{\mu}}, \\ N &= \sum_{n=1}^{4} B_n Z^{n-1}, \\ D_I &= \sum_{n=1}^{4} C_n Z^{n-1}, \\ D_R &= 1 + C_0 Z, \\ Z &= \frac{T_\pi}{200\ \text{MeV}}, \end{aligned} \tag{7}$$

with μ and T_π being the pion mass and kinetic energy, respectively. The searched parameters were B_n and C_n. A maximum of 7 parameters were used in any single wave. In order to fix an overall phase, the ^3P_1S partial wave was kept real ($C_n = 0$). This particular partial wave was chosen for its smooth energy dependence.

Initial values for the parameters were obtained by fitting each partial wave to the updated single-energy solutions of Bugg [4c] below 260 MeV. A different phase convention was used in Ref. [4c]. Therefore, a rotation was required to produce a real ^3P_1S amplitude. This initial fit was then searched against the database. The energy range was extended (with the addition of further parameters, where necessary) until a final solution was obtained. A Coulomb barrier suppression was added but produced no meaningful improvement in the fit. Even the lowest energy cross sections, at a few MeV, appeared to be well represented by the forms used in this analysis. No attempt was made to impose the unitarity constraints implied by analyses of NN elastic scattering. The NN reaction cross sections should exceed the $pp \to \pi^+ d$ cross section in all partial waves. This aspect of the solution was examined by comparing reaction cross sections from the recent NN fit [96] (SP93), from the Virginia Polytechnic Institute and State University (VPI) group, against the $pp \to \pi^+ d$ cross sections. The largest contribution, $^1D_2$, is shown in Fig. 3. Very small violations were found in $^1S_0$, which has no inelasticity in the VPI analysis [96], and in $^3P_1$ near threshold. We feel that the

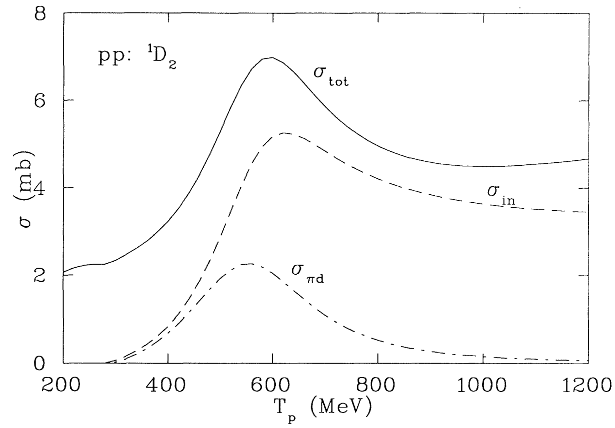

FIG. 3. Partial cross section of $^1D_2$ pp state. Solid curve is the pp total cross section from SP93 [94], dashed curve is the pp inelastic cross section from SP93 [94], and dot-dashed curve is the $pp \to \pi^+ d$ total cross section from the present analysis.

NN elastic analysis is totally insensitive to these very small effects.

The solution presented here produced a χ^2 of 7065 for the 4440 data and 546 experiments below 535 MeV. (An "experiment" being a measurement, possibly an angular distribution, at a particular energy.) The analysis extended from 1.59 MeV to 535 MeV. Stability of the fit against changes in the database was tested by "pruning" all data which were more than 3 standard deviations away from the fit. This resulted in the elimination of 135 data points with a consequent decrease in χ^2 to 5297. The "pruned" data were distributed throughout the database. Upon searching, χ^2 was reduced to 5265 for the 4305 remaining data. No significant change in the solution was found as a result of this (rather severe) alteration of the database. The solution reported in this paper was obtained against the original dataset, prior to the above pruning.

Single energy analyses were done at 25-MeV intervals, using a binning width of 25 MeV. Starting values for the partial-wave amplitudes, as well as their (fixed) energy derivatives, were obtained from the energy-dependent fit. The scattering database was supplemented with a constraint on each varied amplitude. Constraint errors were taken to be 0.007. This was added, in quadrature, to 5% of the amplitude. Such constraints were essential to prevent the solutions from "running away" when the bin was relatively empty of scattering data, as was the case at some of the higher energies. These errors were generous enough that they afforded little constraint for those solutions where sufficient data existed within the bin.

Single-energy analyses are done in order to reveal "structure" which may be missing from the energy-dependent fit. Plots of the amplitudes reveal no such missing structure. Results of the single-energy analyses are summarized in Table III. Two of the single-energy solutions at 100 and 450 MeV receive anomalously large contributions to χ^2 from both the data and amplitude constraints. The 450-MeV result has a large χ^2 due

TABLE III. Single-energy (binned) fits and χ^2 values. N_{prm} is the number of amplitudes (real + imaginary) varied in the fit. χ^2_D is the contribution from data, χ^2_C is due to the amplitude constraints, and χ^2_E is given by the energy-dependent fit, SP93.

T_π (MeV)	Range (MeV)	N_{data}	N_{prm}	χ^2_C	χ^2_D	χ^2_E
25	12.8–35.4	145	7	0.9	386.9	411.6
50	37.6–60.7	155	11	0.9	165.1	178.9
75	62.9–87.3	424	15	1.8	595.1	614.7
100	91.0–112.0	616	15	27.1	1188.3	1419.4
125	113.8–137.1	517	15	11.0	642.4	695.1
150	140.0–158.3	632	17	7.4	687.9	732.1
175	165.0–187.3	282	17	7.5	374.4	438.8
200	191.3–210.3	191	17	4.2	115.8	155.9
225	217.9–235.9	229	19	3.8	221.3	269.8
250	238.9–262.0	481	19	5.9	602.1	700.2
275	264.9–285.1	109	19	2.9	206.1	277.6
300	294.0–307.4	210	19	2.5	213.9	255.9
325	318.9–330.0	160	19	4.0	134.1	251.2
350	341.4–360.3	185	19	2.1	189.2	218.1
375	371.4–375.7	26	19	1.5	32.0	44.5
400	390.0–400.0	28	21	1.1	16.2	34.4
425	417.0–420.0	28	21	0.3	50.7	64.9
450	437.6–456.5	48	21	18.9	75.7	224.3
475	473.8–487.4	23	23	1.6	22.7	35.0
500	495.9–506.5	45	23	6.7	42.5	120.5

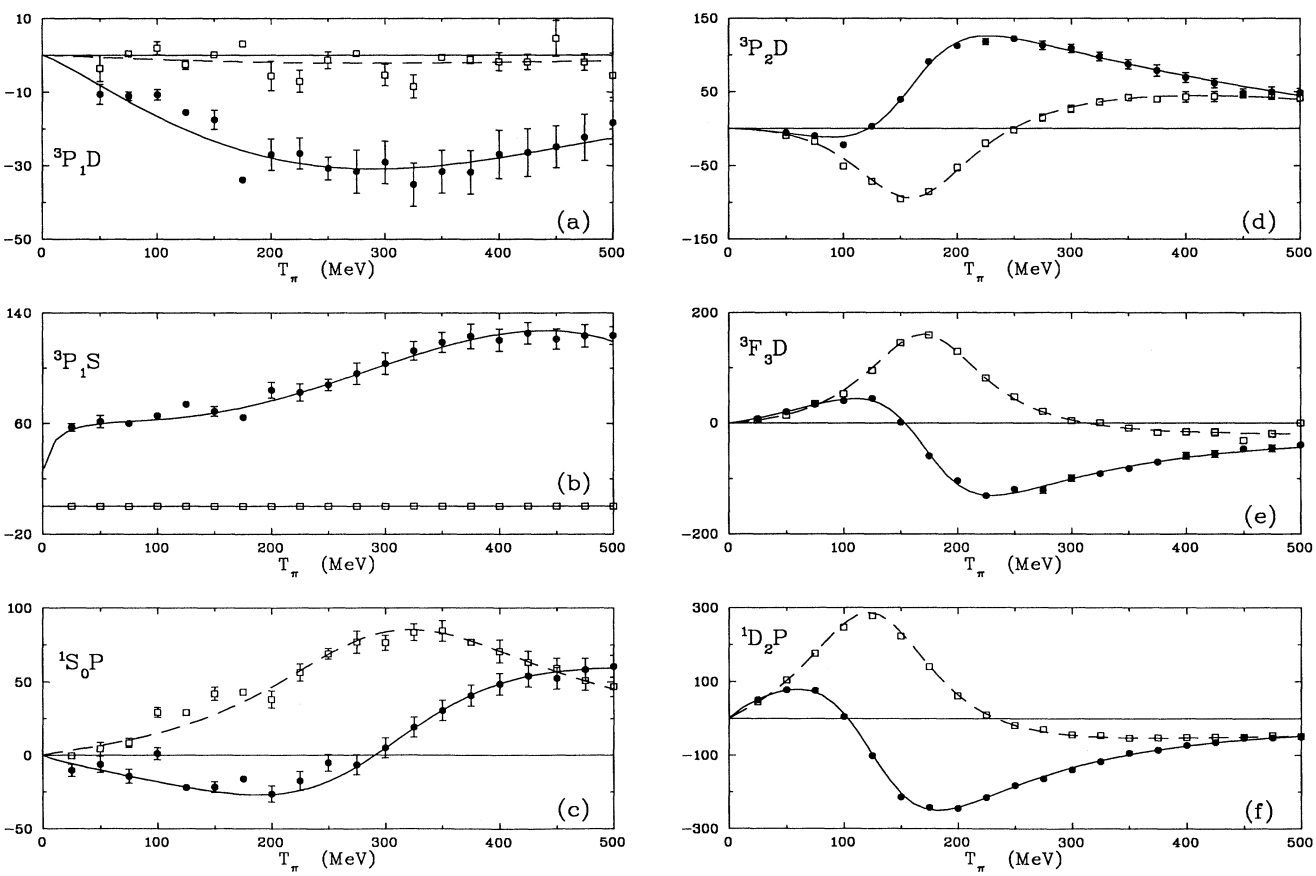

FIG. 4. Partial-wave amplitudes from 0 to 500 MeV. Solid curves are the real parts of amplitudes; dashed curves are the imaginary parts. Single-energy solutions are plotted as black circles (real part) and open squares (imaginary part). All amplitudes have been multiplied by a factor of 10^3.

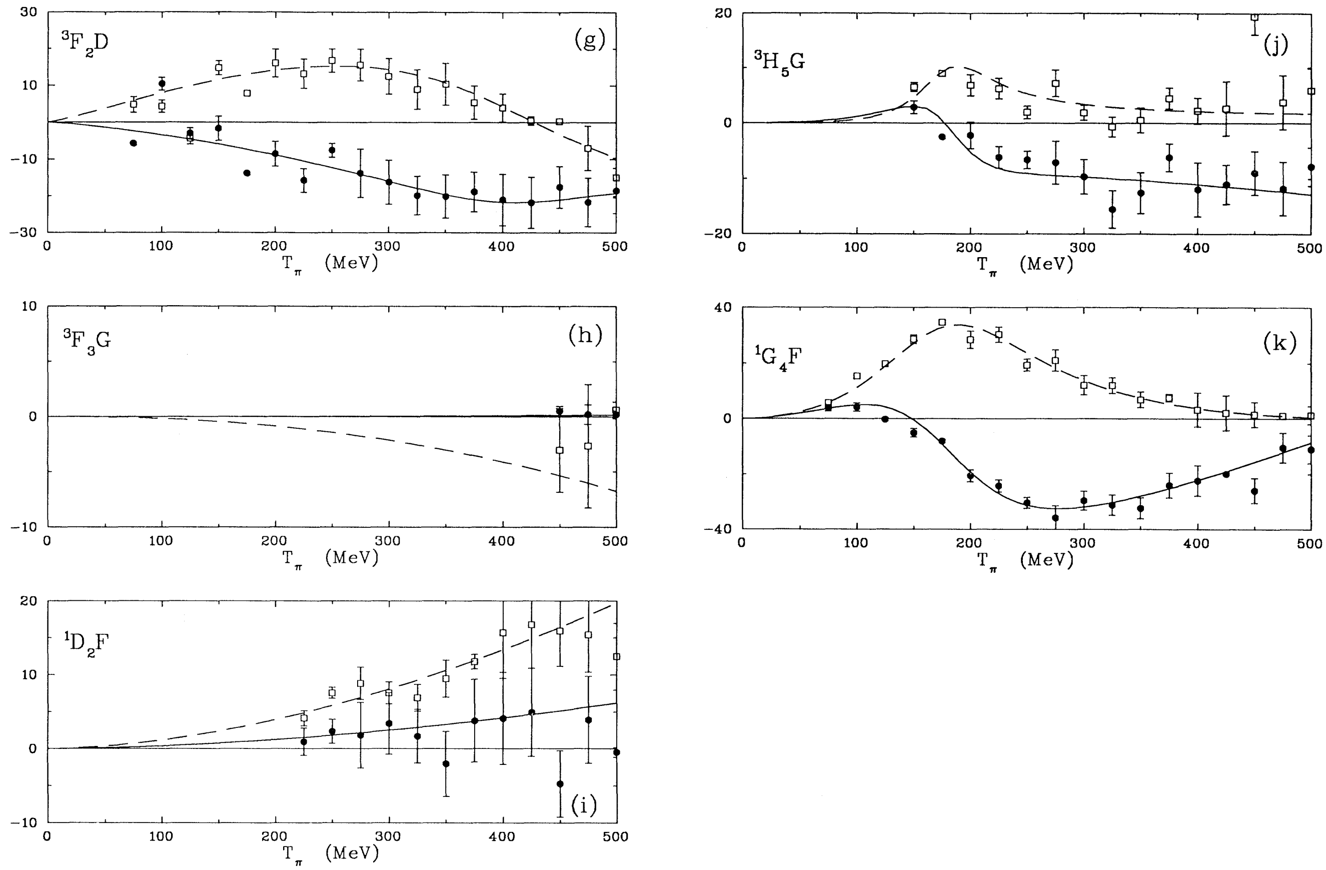

FIG. 4. (*Continued*).

mainly to the iT_{11} measurements of Ref. [20]. We have not found an explanation for this. The present solution is capable of describing the remaining iT_{11} measurements between 26 and 400 MeV. The second troublesome single-energy analysis is at 100 MeV. Here the situation is more complicated, as 317 data are analyzed in this case. Here, the problem is partly due to inconsistencies between different measurements. For example, the measurements of A_{zz} at 100.3 MeV [19] are in disagreement with others at 93.8 MeV [14] and 102.3 MeV [81].

V. RESULTS AND COMPARISONS

Our results for the partial-wave amplitudes, defined in Eqs. (2) and (3), are displayed in Fig. 4. Clearly, the

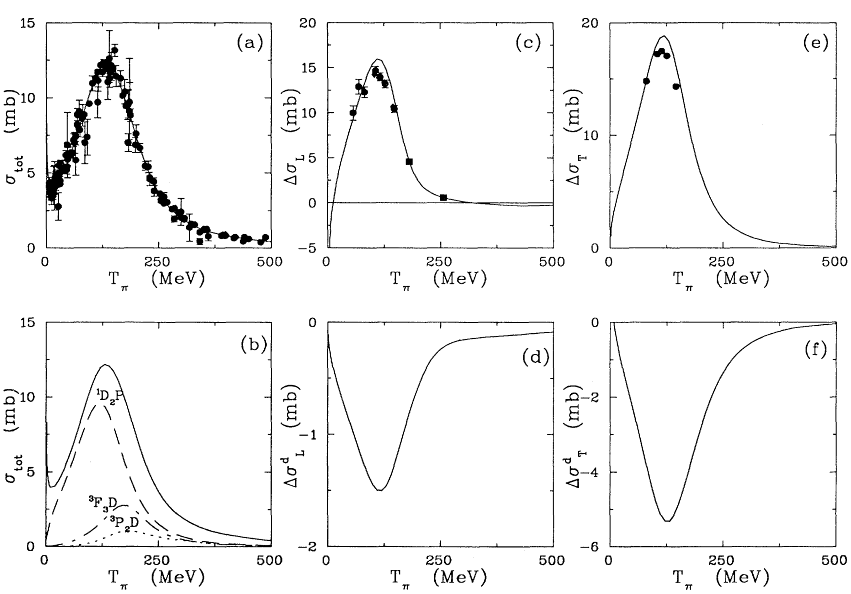

FIG. 5. Total cross section for the reaction $\pi^+ d \to pp$. (a) Unpolarized total cross section. (b) Contributions of the dominant ^1D_2P (dashed curve), ^3F_3D (dot-dashed curve), and ^3P_2D (dotted curve) amplitudes to the total unpolarized cross section. Total cross sections in pure proton spin states are (c) $\Delta\sigma_L$ and (d) $\Delta\sigma_T$, total cross sections in pure deuteron spin states are (e) $\Delta\sigma^d_L$ and (f) $\Delta\sigma^d_T$. Data have not been normalized.

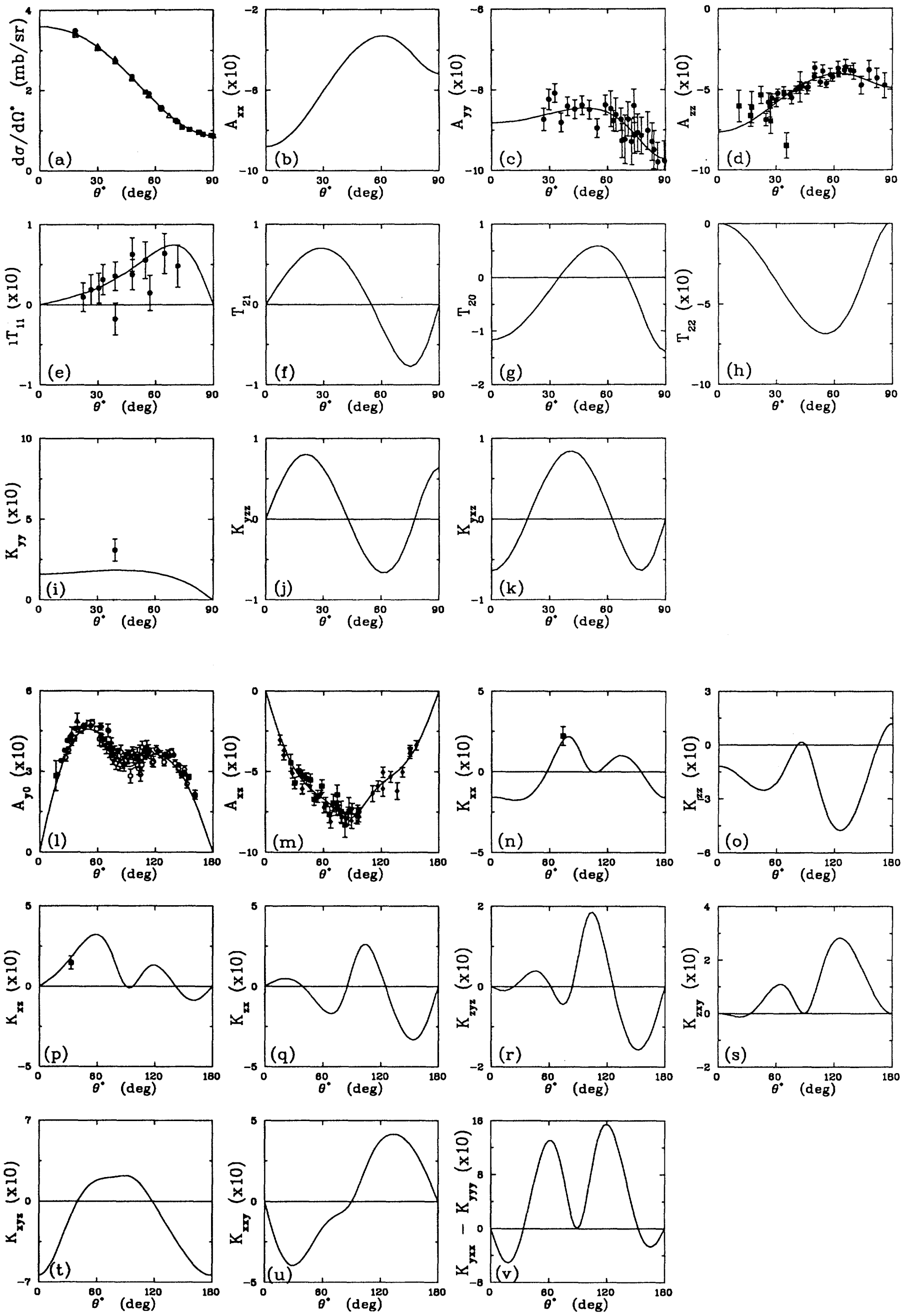

FIG. 6. Predictions for observables at $T_\pi = 145$ MeV. Data have been normalized. (a) $d\sigma/d\Omega$, (b) A_{xx}, (c) A_{yy}, (d) A_{zz}, (e) iT_{11}, (f) T_{21}, (g) T_{20}, (h) T_{22}, (i) K_{yy}, (j) K_{yzz}, (k) K_{yxz}, (l) A_{y0}, (m) A_{xz}, (n) K_{xx}, (o) K_{zz}, (p) K_{xz}, (q) K_{zx}, (r) K_{zyz}, (s) K_{zxy}, (t) K_{xyz}, (u) K_{xxy}, (v) $K_{yxx} - K_{yyy}$.

$\pi^+ d \to pp$ reaction is dominated by only a few amplitudes in this energy range. In particular, these are the ^1D_2P, ^3F_3D, ^3P_2D, and ^3P_1S. The total cross sections for pure proton and deuteron spin states are given, along with the unpolarized total cross section, in Fig. 5. In Fig. 5(b) we also show contributions to unpolarized total cross section from the 3 largest partial waves—the ^1D_2P, ^3F_3D, and ^3P_2D.

As discussed in Sec. IV, the energy-dependent solution gives a good overall fit to the data. For the whole database, the χ^2/(degree of freedom) was 1.59. We found that this value could be reduced to 1.22, through the removal of 135 data more than 3 standard deviations from the solution, with negligible effect on the partial-wave amplitudes. Some of the data conflicts are apparent in Fig. 6, where we have given predictions for all observables at T_π=145 MeV.

The results of this analysis are more striking when presented in an Argand plot, as given in Fig. 7. The ^1D_2P, ^3F_3D, and ^1G_4F partial waves show a correlation [97]. While different phase conventions have been used in previous analyses, it is possible to rotate these results to a common convention. In Fig. 8 we compare several of the dominant amplitudes determined in Refs. [3a,4c]. The values due to Bugg *et al.* [4c] are updated results of a previously published [4c] analysis. In this repre-

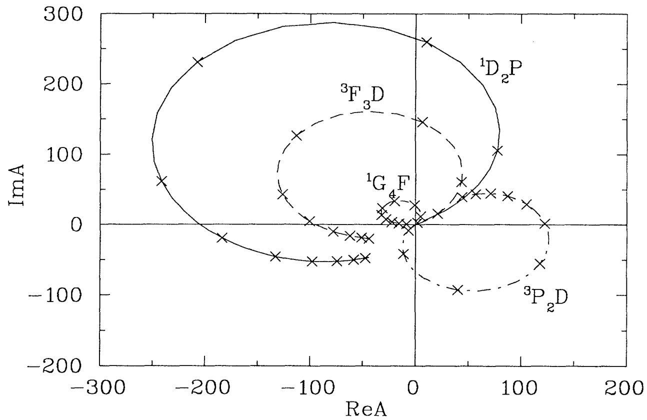

FIG. 7. Argand plot of dominant partial-wave amplitudes. The X points denote 50 MeV steps. All amplitudes have been multiplied by a factor of 10^3.

sentation the results of Bugg *et al.* are fairly consistent with the present analysis. The older results of Ref. [3a], while reasonably consistent with our results for the ^3P_2D and ^3F_3D partial waves, show larger deviations for ^1D_2P and ^1G_4F. In Fig. 9 we show how these differences are manifested in particular observables. It is useful to note whether these variations are due to differences in data fitting or instead reflect the ability to predict future measurements. In Fig. 9(a), for example, the displayed data were included in the present analysis, and the analysis of Ref. [4c], but was not used in the fit of Ref. [3a]. In Fig. 9(c), however, all but 3 [39] of the displayed points were included in the analyses of Refs. [3a,4c]. (In Ref. [3a], a preliminary version of some [87] of these data were used.) Here, the curves from different analyses show little deviation.

VI. SUMMARY AND CONCLUSIONS

In this work we have analyzed a $\pi^+d \to pp$ database which is larger than those used in previously published analyses. A good fit to this database was found. The dominant partial-wave amplitudes display a correlation which is particularly evident in the Argand plots of Fig. 7.

We should also comment on existing and future measurements of the $\pi^+d \to pp$ observables. As mentioned previously, a fit to the existing iT_{11} data at 450 MeV was problematic. Further measurements near and above this energy would be very useful. For many other observables, however, the situation is much worse. A glance at the energy-angle plots of Fig. 2 reveals clear boundaries in the density of measurements near 250 and 350 MeV. (The 250 MeV boundaries are due to the energy limit at Los Alamos, for studies of the reaction $pp \to \pi^+d$.) At low energies, below 60 MeV, little more than cross section and A_{y0} measurements exist. (The single iT_{11} measurement [88] below 60 MeV is now more than 35 years old.) Some new TRIUMF measurements [98] of iT_{11} at 25 and 60 MeV will soon be available. The CHAOS detector [99], constructed at TRIUMF, should also allow improved low-energy experiments. The analysis of new deuteron analyzing-power and $p-d$ polarization-transfer data is in progress at Saclay [100].

This reaction is now incorporated into the SAID program [101], which is maintained at Virginia Tech. Detailed information regarding the database, partial-wave amplitudes, and observables may be obtained either interactively, through the SAID system (for those who have access to TELNET), or directly from the authors.

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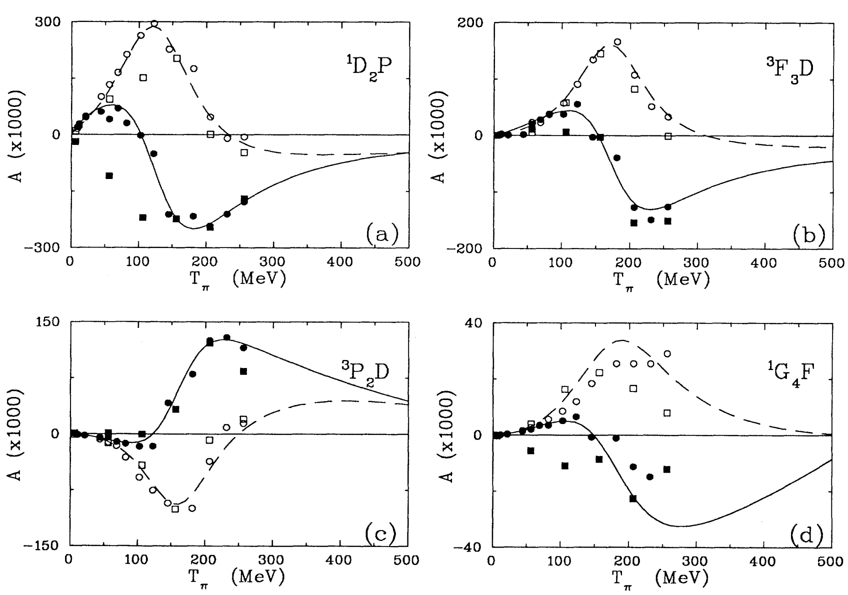

FIG. 8. Plots of the amplitudes (a) ^1D_2P, (b) ^3F_3D, (c) ^3P_2D, and (d) ^1G_4F. The solid (dashed) curves give the real (imaginary) part from the present solution. The circles and squares are single-energy results from Refs. [4c] (revised 1993 results) and Ref. [3a], respectively. Solid symbols give real parts; open symbols give imaginary parts.

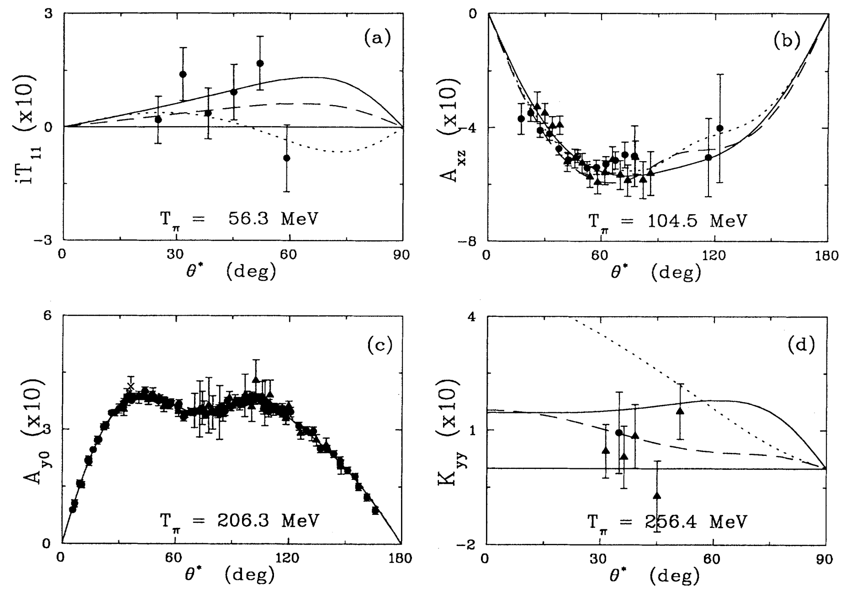

FIG. 9. Comparison of observables constructed from the present solution (solid line), the solution of Ref. [4c] (dashed line), and the solution of Ref. [3a] (dotted line). (a) iT_{11}, (b) A_{xz}, (c) A_{y0}, and (d) K_{yy}.

G. Jones, C. Lechanoine-LeLuc, E. L. Mathie, B. Mayer, M. W. McNaughton, L. C. Northcliffe, G. Pauletta, B. G. Ritchie, A. Saha, K. K. Seth, G. R. Smith, M. I. Strikman, S. E. Turpin, C. A. Whitten, and C. Wilkin for providing experimental data prior to publication and clarification of information already published. We are grateful to N. Hiroshige, G. Jones, A. V. Kravtsov, C. Lechanoine-LeLuc, and M. G. Ryskin for useful discussions concerning available partial-wave solutions. I.S. acknowledges the hospitality extended by the Physics Department of Virginia Tech. This work was supported in part by the U.S. Department of Energy Grant DE-FG05-88ER40454.

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$A_{xx} = A_{yy}$ (at 0 degrees),
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