PHYSICAL REVIEW C VOLUME 48, NUMBER 4 OCTOBER 1993

Some general constraints on identical band symmetries

Mike W. Guidry,[1,2] Michael R. Strayer,[2] Cheng-Li Wu,[1,3,4] and Da Hsuan Feng[4]
[1]*Department of Physics, University of Tennessee, Knoxville, Tennessee 37996*
[2]*Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831*
[3]*Joint Institute for Heavy Ion Research, Oak Ridge, Tennessee 37831*
[4]*Department of Physics and Atmospheric Science, Drexel University, Philadelphia, Pennsylvania 19104*
(Received 15 April 1993)

We argue on general grounds that nearly identical bands observed for superdeformation and less frequently for normal deformation must be explicable in terms of a symmetry having a microscopic basis. We assume that the unknown symmetry is associated with a Lie algebra generated by terms bilinear in fermion creation and annihilation operators. Observed features of these bands and the general properties of Lie groups are then used to place constraints on acceptable algebras. Additional constraints are placed by assuming that the collective spectrum is associated with a dynamical symmetry, and examining the subgroup structure required by phenomenology. We observe that requisite symmetry cannot be unitary, and that the simplest known group structures consistent with these minimal criteria are associated with the Ginocchio algebras employed in the fermion dynamical symmetry model. However, our arguments are general in nature, and we propose that they imply model-independent constraints on any candidate explanation for identical bands.

PACS number(s): 21.60.Ev

A well-established feature of nuclear structure physics is the occurrence of systematic behavior as particle number is changed. A common procedure is to separate the change of features with A into a slowly varying portion assumed to be described by a phenomenological liquid drop, and fluctuations that reflect the local shell structure and are typically described by some form of deformed mean-field calculation. Recently, appreciable attention has been given to the phenomenon of identical bands: rotational bands in different nuclei that exhibit similarities to a degree not expected from liquid drop behavior, and that occur with sufficient frequency to call into doubt any general explanation in terms of cancellations among polarizing effects of the single-particle orbitals. Such bands were first discussed for superdeformed nuclei [1,2]. Two examples are the twinned bands found in the rare earth region where bands in nuclei differing in A by one unit have almost identical transition energies [3–8], and the set of identical superdeformed bands in the mass-190 region, which exhibit almost identical variation of the second moment of inertia over a broad range of frequencies [9,10,2]. More recently, evidence has been presented that similar behavior also exists for normal deformation, though not as frequently as for superdeformation [11].

With this paper we wish to initiate a general discussion of microscopic symmetries that might account for strong relationships between rotational bands lying in different nuclides. As a first step in this direction we consider a subset of the general problem: What symmetries could be responsible for similarities between bands lying in even-even nuclei that differ by two protons or two neutrons? In Fig. 1 and Fig. 2 we show two examples of such regularities: superdeformed bands in some even-even Hg isotopes, and the low-spin members of the ground bands in some even-even isotopes of the actinide nuclei.

Traditional mean-field theories can account for identical band behavior if an appropriate cancellation occurs between various polarizing influences as the single-particle orbitals are filled. Under certain special conditions, such behavior has been demonstrated [12,13]. However, there is some consensus that identical bands are too widespread for accidental cancellations to explain them, and it is difficult to see how standard mean-field theories alone can account for identical bands. Several types of symmetry-based explanations have been suggested (for example, see Refs. [14–16]), but since these suggestions are basically phenomenological, they properly should be viewed as an alternative language to describe such bands and not microscopic explanations. In particular, none of these approaches seems capable of answering in any simple fashion the question of why there exist certain identical bands, but at the same time there are many more bands in similar nuclei that are not identical.

We take the point of view that unexpected regularity is always a sign of unexpected symmetry, and that identical bands are without question a consequence of a symmetry; the challenge is to identify the symmetry and to relate it to microscopic nuclear structure. Only if the symmetry can be understood in terms of microscopic structure do we have a hope of understanding objectively why the symmetry is essentially unbroken for identical bands, but broken for those cases exhibiting no identical band behavior.

We emphasize that a description of identical bands in terms of a symmetry does not preclude an alternative description without explicit reference to symmetry. For example, angular momentum is associated with a sym-

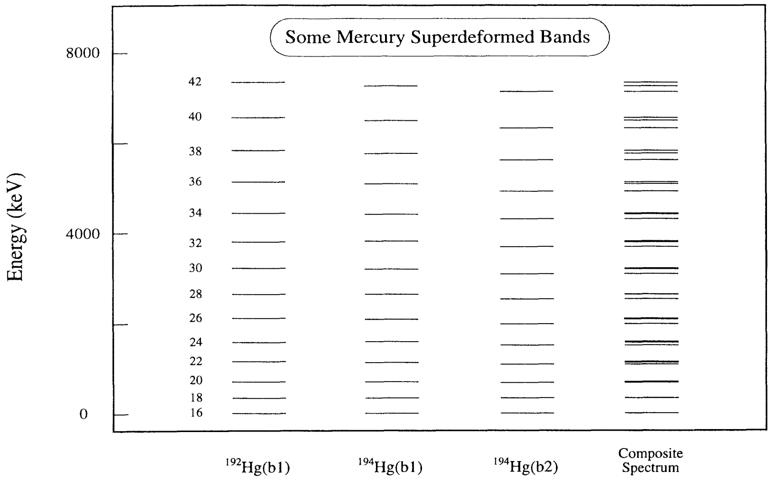

FIG. 1. Energies for some states in superdeformed bands in even-even Hg isotopes. Spins are not known for any superdeformed bands, but it is thought that they can be estimated with an uncertainty of 1–2 units. Therefore, there is a small ambiguity in the matching of states in adjacent nuclei in this figure. The spins assumed here are those adopted in the compilation of [28], where references to the experimental data may be found. The energies are normalized at the assigned 16^+ state for each nucleus. The right column shows the multiplet structure that results if each state of a given spin is interpreted as a member of a representation multiplet of a weakly broken symmetry.

metry, but angular momentum can be discussed in terms of differential operators and geometrical concepts without direct appeal to the angular momentum SU_2 algebra. Therefore, we believe it to be self-evident that any valid explanation of identical bands, whether formulated in the language of symmetry or not, will be *equivalent to* an explanation in terms of a yet to be found symmetry with a microscopic nuclear structure basis.

Let us note in this connection that a symmetry as an explanation of identical bands sidesteps the objection common in mean-field discussions of polarization effects that one can never achieve in a practical calculation the accuracy required to reproduce identical bands (say, 1 part in 10^3). The starting point in the symmetry approach is the assumption that identical bands are the *norm*, unless disturbed by symmetry breaking terms. We may illustrate this point of view by the following hypothetical, but historically conceivable, example. Suppose we did not know that angular momentum was associated with a symmetry, but we knew that it existed, and we calculated single-particle contributions to total angular momentum in the same way that we can calculate single-particle contributions to say a collective quadrupole moment (notice that this means we would not have available the simplifications brought about by Clebsch-Gordan coefficients, the Wigner-Eckart theorem, and so on).

Now suppose that rapid advances in experimental techniques allowed a much more precise measurement of large angular momenta associated with many high-spin collective states, and we found the unexpected result that there were whole sets of states that had exactly the same angular momentum (say, 1 part in 10^3) in each nucleus (the $2J+1$ magnetic substate degeneracy of an angular momentum J state). In this context, a similar consternation would arise: How could one understand all these identical states; in particular, how could one achieve a microscopic theory sufficiently precise that the 61 states that we secretly know to be the magnetic substates of a ^{238}U state of angular momentum 30 all get, to 1 part in 10^3, the same contributions from the sum over 238 single-particle angular momenta? (Remember, this sum must be done numerically, and must be based on some empirically determined single-particle angular momenta; there are no Clebsch-Gordan coefficients in our hypothetical example.) But we know the answer to this puzzle: Angular momentum is exactly conserved because of a

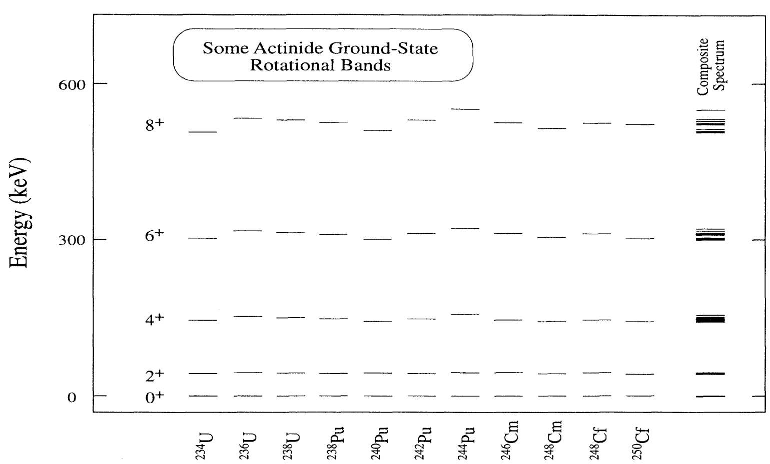

FIG. 2. Low-spin members of ground-state rotational bands in some actinide nuclei. The right column is as for Fig. 1.

symmetry, and the 61 states in our example are in fact exactly degenerate as a consequence of this symmetry.

One may argue that there is one difference between our angular momentum fable and the proposed identical band symmetry: the former symmetry is exact, the latter is not. However, this is not a substantial objection. Imagine the previous arguments carried out in the presence of a weak magnetic field (explicit symmetry breaking terms). Angular momentum is no longer exactly conserved, but the symmetry properties of angular momentum still provide a clear understanding of the physics in terms of splittings of the multiplet degeneracies, provided the symmetry breakings are not so large that the multiplet structure is completely obscured. The qualitative similarity of the right column of Fig. 2 with the splitting of angular momentum degeneracies by a magnetic field should be carefully noted.

Thus, we propose that identical bands are describable in terms of an unknown symmetry that is weakly broken, in analogy to angular momentum symmetry in the presence of a small magnetic field. Nonidentical bands then correspond to a stronger breaking of this symmetry. If the symmetry can be identified and given a microscopic interpretation, it is possible that the pattern of symmetry breaking (large in some cases, small in others) can be understood, and this would constitute a microscopic explanation of identical bands. We emphasize that it is *essential* for the symmetry to have a microscopic interpretation, precisely because it cannot be an exact symmetry. Only if one can explain why identical bands occur in some cases and not in others can one claim to have understood them, and this means that the symmetry breaking terms and their systematic behavior must be understood explicitly, not by hypothesis.

In this paper we address the nature of this conjectured identical band symmetry. Our approach is not to propose a specific symmetry, but to use general properties of group algebras and the phenomenology of identical bands to delimit the class of symmetries that could provide an acceptable solution.

Guided by examples from varied fields of physics, we will assume that the candidate symmetry is describable by a Lie group, with a corresponding Lie algebra. Further, we will assume that the appropriate Lie algebra is generated by forms bilinear in the fermion operators that create and annihilate nucleons in definite shell model orbits. There are four such independent combinations:

$$a_i^\dagger a_j, \qquad a_i a_j^\dagger, \qquad a_i^\dagger a_j^\dagger, \qquad a_i a_j, \tag{1}$$

where $a^\dagger$ and a create and annihilate particles obeying Fermi-Dirac statistics and i and j denote all required quantum numbers. Thus, we assume that the generators of the desired symmetry can be described microscopically by a shell model having one- and two-body residual interactions.

We now attempt to identify the minimal set of operators that can (1) satisfy the above assumptions, (2) provide sufficient degrees of freedom to account for the observed features of identical bands, and (3) close a Lie algebra, thereby defining a symmetry. First, let us note that the bands in question are strongly rotational, and manifestly are dominated by collective quadrupole degrees of freedom. Therefore, we require that our operator set contain the five components of a quadrupole tensor

$$Q_\mu \simeq [a_j^\dagger a_j]^2_\mu \qquad \mu = 1, 2, \ldots, 5, \tag{2}$$

where the symbol $\simeq$ indicates that we leave open details such as multiplicative factors or possible sums over internal indices for now, and the square brackets denote standard angular momentum coupling. Since the states of the bands carry definite angular momentum, we require the three components of an angular momentum operator,

$$L_\mu \simeq [a_j^\dagger a_j]^1_\mu \qquad \mu = 1, 2, 3\ , \tag{3}$$

and since we deal with states of specific particle number, we need a particle number operator

$$n \simeq [a_j^\dagger a_j]^0_0\ . \tag{4}$$

Therefore, the required algebra must involve at least nine multipole operators of the general form given by Q, L, and n. However, this is not sufficient to define a minimal set. The most striking feature of identical bands is that they occur in different nuclei. *Thus the symmetry that we seek must have irreducible representations containing states that differ in particle number.* Symmetries with this property are well known in nuclear physics. For example, isotopic spin, quasispin and pairing vibrations, and pairing rotations all connect states that lie in different nuclei. However, these represent simple symmetries that link a restricted number of states in neighboring nuclei. The symmetry that we seek to describe identical bands must be much more comprehensive, because it is required to simultaneously link many states in a band in one nucleus to a corresponding number of states in other nuclei. The observation that the symmetry must be able to connect states in nuclei differing in neutron or proton number by two means that we must include pairing operators in our set of generators. The simplest operators consistent with nuclear structure phenomenology are the two S-pair operators

$$S^\dagger \simeq [a_j^\dagger a_j^\dagger]^0_0, \qquad S \simeq [a_j a_j]^0_0\ . \tag{5}$$

However, the requirement of closing the algebra means that we cannot stop here. Commutation of the S-pair operators with the quadrupole operators will lead inevitably to terms involving D-pair operators, since the commutators involve the coupling of angular momentum 0 to angular momentum 2; therefore, we must also include ten angular momentum 2 pairing operators in our set:

$$D_\mu^\dagger \simeq [a_j^\dagger a_j^\dagger]^2_\mu\ , \qquad D_\mu \simeq [a_j a_j]^2_\mu\ , \qquad \mu = 1, 2, \ldots, 5\ . \tag{6}$$

Notice that these considerations imply that identical bands cannot be explained as a consequence of a unitary symmetry. Therefore, pseudo-SU$_3$, interacting boson model IBM, or other unitary symmetries cannot, as a matter of principle, provide an explanation of identical bands. If one obtains within such models a moment of inertia that is independent of the particle number, this is a consequence of particular assumptions or constraints; it is not a consequence of the symmetry.

The assemblage of 21 operators, Q, L, n, S, $S^\dagger$, D, and $D^\dagger$, constitutes a *minimal set* capable of satisfying our requirements. This is of course a necessary but perhaps not sufficient condition. For example, it is possible that a particular microscopic choice spanning the preceding set can be closed under commutation, but the matrix elements associated with the corresponding closed algebra might still be inadequate to accommodate experimental observations. Alternatively, a particular choice for this set may not lead to a closed algebra because commutations involving D pairs could lead to operators with even higher angular momentum. However, we reiterate that our specific goal is to identify the *minimal set* as a starting point for constructing theories of identical bands.

From this analysis, we may conclude that minimal algebras with a chance to describe identical bands require at least 21 generators. We may now inspect standard classifications for candidate Lie algebras (see Wybourne [17], Table 7.1). It is interesting that the properties of Lie algebras ensure that there are only a few possibilities, and that the simplest of these saturate our deduced minimal operator set: the Lie algebras isomorphic to Sp_6 and to SO_7 contain exactly 21 generators. The next fewest number of generators among the standard Lie algebras is associated with the group SU_5, but unitary groups are excluded by the physical conditions that we have imposed. The next simplest possibility is isomorphic to SO_8 and contains 28 generators, 7 beyond the minimal set. Therefore we conclude, simply on the basis of counting generators, that the most fruitful group structures to explore in search of a minimal description of identical bands are the groups Sp_6 and SO_7, with the next most favorable possiblility being the group SO_8. We note that this exhausts all possibilities with fewer than 36 generators.

There is no guarantee that 21 arbitrarily chosen operators will satisfy the physical conditions that we have imposed and at the same time close an Sp_6 or SO_7 algebra; to the contrary, these independent conditions set highly restrictive conditions on the microscopic structure of the operator set, and it is not clear *a priori* that they can be fulfilled simultaneously. Therefore, it is encouraging that algebras satisfying these conditions are known already. The Sp_6 algebra introduced by Ginocchio [18] that is an integral part of the coupling scheme employed in the fermion dynamical symmetry model [19] is a fermion algebra in which the 21 generators have the schematic microscopic structure and physical interpretation required in the preceding discussion of phenomenology: a quadrupole operator, an angular momentum operator, a particle-number operator, and $L = 0, 2$ pairing operators. Furthermore, it has been demonstrated that this algebra harbors a dynamical symmetry $\mathrm{Sp}_6 \supset \mathrm{SU}_3 \supset \mathrm{SO}_3$ that may be interpeted physically as producing axially symmetric rotational bands.

Ginocchio has introduced an SO_8 algebra that involves operators of the form that we have discussed, plus 7 additional generators, that has also been used in the fermion dynamical symmetry model. This group supports a dynamical symmetry $\mathrm{SO}_8 \supset \mathrm{SO}_6 \supset \mathrm{SO}_3$, with the physical interpretation of producing γ-unstable rotational bands. Although it is not obvious that such a symmetry is by itself useful for describing identical bands, it is known that a coupling of the SO_8 and Sp_6 Ginocchio symmetries produces collective modes that have axially symmetric equilibrium deformations, but with a degree of softness to fluctuations into the γ plane [20,21]. Such a coupled symmetry mode might conceivably be relevant to the present discussion. Finally, for completeness we note that Ginocchio also introduced an SO_7 symmetry through a subgroup of his SO_8 group, but the physical interpretation for the 21 generators of this particular group is not consistent with the phenomenological requirements we have placed on our minimal set of generators [18,22].

Let us now ask whether there may be more constraints that we can place than those already enumerated. In particular, let us consider the *subgroup structure.* Since the states in question represent nondegenerate collective modes of a many-body Hamiltonian, we will assume that what we seek is *a dynamical symmetry:* the Hamiltonian can be written as a polynomial in the invariants (Casimirs) of a subgroup chain of the highest group. Since the nature of the collective modes will then depend on the nature of the subgroup chain, this will place further strong constraints on acceptable symmetries.

Let us now see what general statements we can make about the subgroups. First, the foregoing arguments would indeed lead to identical bands, but they would be *too identical,* since those arguments concern the highest symmetry and would produce completely degenerate multiplets. But the states of an identical rotational band are not degenerate [they have a $J(J+1)$ spacing]. These states of a rotational band must be members of a multiplet of a subgroup of the highest symmetry because of the following: (1) there are strong transitions between the states, suggesting that they are members of an irreducible representation (irrep) connected by raising and lowering operators of the relevant group; (2) each of these irreps lies in a single nucleus, so by hypothesis the corresponding group cannot be the highest group, which has irreps spanning different nuclei; therefore, the irreps must belong to a subgroup of the highest group.

Second, the displayed spectra in Fig. 2 are shown with respect to the ground state of each nucleus, and those in Fig. 1 with respect to an assumed 16^+ state in each nucleus. Actually, each spectrum is offset by the mass difference of the reference state: the 0^+ states in, say, $^{236}\mathrm{U}$ and $^{238}\mathrm{U}$ are not at the same energy, but differ by the $^{238}\mathrm{U}$ and $^{236}\mathrm{U}$ ground-state mass difference. Thus, the highest symmetry must be broken by terms that introduce a nontrivial particle number dependence into the total Hamiltonian in order to account for the ground-state mass variation with A, but these terms must have only weak influence on the rotational spacings within the bands. This means that the dynamical symmetry must involve subgroups with irreps that correspond to rotational bands for which the moments of inertia do not depend on the particle number. To be specific, the dynamical symmetry must produce a band of collective states that has a spectrum $\alpha J(J+1)$, where α depends at most weakly on the particle number, while at the same time yielding a ground-state mass that has a nontrivial particle-number dependence. Since $J(J+1)$ is the eigenvalue of the angular momentum SO_3 Casimir operator, this implies a dynamical symmetry $G \supset \cdots G' \supset \cdots SO_3$, where G is the highest symmetry addressed in the preceding discussion, G' is the subgroup of G that has irreps corresponding to a band of collective even-spin states connected by strong $E2$ transitions and that numbers among its generators the three of angular momentum, and SO_3 is the angular momentum group generated by this subgroup of G' generators (dots imply that there could in principle be intervening groups in the chain). Although our argument is general, we note that an obvious candidate for the group G' is SU_3.

These are general statements, based on attributes of the observed bands and the properties of Lie groups and their associated dynamical symmetries. However, there is again a known example that can fulfill these conditions. The $Sp_6 \supset U_1 \times U_3 \supset SU_3 \supset SO_3$ dynamical symmetry chain of the Ginocchio Sp_6 algebra is employed in the Fermion dynamical symmetry model (FDSM) and corresponds to a Hamiltonian that is capable of describing ground-state mass differences [23,24], but at the same time produces collective rotational bands that can satisfy the preceding conditions [19,25,26]. In particular, the rotational bands correspond to irreps of SU_3 lying in individual nuclei that have a symmetry-limit spectrum

$$E = \text{const} + \alpha J(J+1),$$

where α is a function of the effective interaction of the truncated space, and only depends weakly on particle number if the effective interaction parameters have a small particle-number dependence and symmetry breaking is negligible [27]. Therefore, at least one example exists that satisfies our conditions schematically and has the potential to satisfy them quantitatively, but we emphasize that our arguments have been of a general nature and should apply to any candidate for a dynamical symmetry to describe identical bands. Thus the FDSM example might represent only one member of a general class of dynamical symmetries that lead naturally to identical bands.

In conclusion, we have proposed that any valid explanation of identical bands will be equivalent to a description in terms of a yet-to-be-identified microscopic symmetry. We have employed empirical observations and general principles of group theory and microscopic nuclear structure to restrict the classes of theories that one should investigate in searching for a candidate symmetry. We conclude that a minimal theory must be based on a Lie algebra with at least 21 generators and that this algebra cannot be unitary. Thus, explanations of identical bands in terms of Elliott SU_3, pseudo-SU_3, or IBM symmetries are excluded on fundamental grounds. If such theories give rise to identical bands, they do so for reasons that are not dictated by the symmetries of the theory. We have specified the schematic physical interpretation required of these generators and the schematic subgroup structure demanded by the phenomenology. We have pointed out that the simplest known symmetries consistent with these minimal requirements are derived from the Ginocchio algebras that are utilized in the fermion dynamical symmetry model. This provides both a candidate symmetry for identical bands and an existence proof for theories satisfying the minimal requirements. It is our hope that the present discussion will encourage investigation of whether there are additional symmetries that meet these conditions, and whether such symmetries can provide a quantitative and microscopic interpretation of identical bands.

Nuclear physics research at the University of Tennessee was supported by the U.S. Department of Energy through Contracts No. DE-FG05-87ER40361 and DE-FG05-93ER40770. Oak Ridge National Laboratory is managed by Martin Marietta Energy Systems, Inc. for the U.S. Department of Energy under Contract No. DE-AC05-84OR21400. Nuclear physics research at Drexel University was supported by the National Science Foundation. The Joint Institute for Heavy Ion Research has as member institutions the University of Tennessee, Vanderbilt University, and the Oak Ridge National Laboratory; it is supported by the member institutions and by the U.S. DOE through contract No. DE-AS05-76ER04936 with the University of Tennessee.

[1] P. J. Nolan and P. J. Twin, Annu. Rev. Nucl. Part. Sci. **38**, 533 (1988).
[2] R. V. F. Janssens and T. L. Khoo, Annu. Rev. Nucl. Part. Sci. **41**, 321 (1991).
[3] B. Haas *et al.*, Phys. Rev. Lett. **60**, 503 (1988).
[4] J. K. Johansson *et al.*, Phys. Rev. Lett. **63**, 2200 (1989).
[5] T. Byrsky *et al.*, Phys. Rev. Lett. **64**, 1650 (1990).
[6] K. Zuber *et al.*, Nucl. Phys. **A520**, 195c (1990).
[7] B. Haas *et al.*, Phys. Rev. C **42**, R1817 (1990).
[8] K. Zuber *et al.*, Phys. Lett. B **254**, 308 (1991).
[9] F. S. Stephens, Nucl. Phys. **A520**, 91c (1990).
[10] W. Satula *et al.*, Nucl. Phys. **A529**, 289 (1991).
[11] C. Baktash *et al.*, Nucl. Phys. **A557**, 145c (1993).
[12] I. Ragnarsson, Nucl. Phys. **A520**, 67c (1990).
[13] Z. Szymański, Nucl. Phys. **A520**, 1c (1990).
[14] D. Bonatsos, S. B. Drenska, P. P. Raychev, R. P. Roussev, and Yu. F. Smirnov, J. Phys. G **17**, L67 (1991).
[15] A. Gelberg, P. von Brentano, and R. F. Casten, J. Phys. G **16**, L143 (1990).
[16] R. D. Amado, R. Bijker, F. Cannata, and J. P. Dedonder, Phys. Rev. Lett. **67**, 2777 (1991).
[17] B. G. Wybourne, *Classical Groups for Physicists* (Wiley Interscience, New York, 1974).
[18] J. N. Ginocchio, Ann. Phys. **126**, 234 (1980).
[19] C.-L. Wu, D. H. Feng, X.-G. Chen, J.-Q. Chen, and M. W. Guidry, Phys. Rev. C **36**, 1157 (1987).
[20] H. Wu and M. Vallieres, Phys. Rev. C **39**, 1066 (1989).
[21] L.-A. Wu *et al.*, unpublished.
[22] R. F. Casten, C.-L. Wu, D. H. Feng, J. N. Ginocchio, and X.-L. Han, Phys. Rev. Lett. **56**, 2578 (1986).

[23] X.-L. Han, M. W. Guidry, D. H. Feng, K. X. Wang, and C.-L. Wu, Phys. Lett. B **192**, 253 (1987).

[24] X.-L. Han, C.-L. Wu, M. W. Guidry, and D. H. Feng, Phys. Rev. C **45**, 1127 (1992).

[25] M. W. Guidry, C.-L. Wu, D. H. Feng, J. N. Ginocchio, X.-G. Chen, and J.-Q. Chen, Phys. Lett. B **176**, 1 (1986).

[26] J.-Q. Chen, D. H. Feng, M. W. Guidry, and C. L. Wu, Phys. Rev. C **44**, 559 (1991).

[27] C.-L. Wu, D. H. Feng, and M. W. Guidry, Ann. Phys. **222**, 187 (1993).

[28] X.-L. Han and C.-L. Wu, At. Nucl. Data Tables, in press.