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Longitudinal response functions of heavier nuclei

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The electronuclear longitudinal response is studied in a large class of nuclei. Medium weight as well as heavy nuclei are considered. Beyond mean field effects short range correlations are taken into account by means of a local density approach. They are responsible for a decrease of the response up to 15% in the quasielastic region. The amount of quenching is proportional to the average proton density reaching its maximum for medium heavy nuclei.

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I. INTRODUCTION

In the last years much experimental effort has been devoted to the separation of the inclusive electronuclear response functions. Today data are available in the quasielastic region for light [1–4], medium-weight [5–9], and heavy nuclei [10, 11]. In most cases they show common features such as a quenching of the longitudinal response in the peak region and an overall missing longitudinal strength compared to results in the plane wave impulse approximation (PWIA) and to the Coulomb sum rule [12], respectively. The disagreement is generally smaller for lighter nuclei, but can become much more sizable for the heavier ones. Many theoretical calculations of the longitudinal response functions exist for light and medium-weight nuclei. On the contrary for heavier nuclei only a few calculations have been performed [13] and nuclear matter (NM) results [14] are often used for comparison with these data.

It is the aim of the present work to study the longitudinal response for a large number of nuclei within a unique scheme. To this end we propose an ansatz which combines the nucleon-nucleon correlation effects of a NM calculation with the finite size and shell effects of a Hartree-Fock (HF) description of the nuclear systems. The approach is based on a local density approximation that in similar form has already been applied in the studies of nucleon momentum distributions [15] as well as for the Coulomb sum rule [16] in heavier nuclei. Such a model has the great advantage of allowing the study of systematic correlation effects as a function of the density distribution.

The paper is organized as follows. The model is described in Sec. II while in Sec. III results for various nuclei from ^{40}Ca to ^{238}U are presented. That section also contains a comparison with experimental data and a brief discussion on the effects due to a density-dependent proton form factor.

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II. RESPONSE OF NUCLEAR MATTER AND OF HEAVY NUCLEI

The longitudinal nuclear response function is given by

$$R(|\mathbf{q}|,\omega) = \sum_n |\langle n|\rho(\mathbf{q})|0\rangle|^2 \delta(\omega - E_n + E_0) \ , \qquad (1)$$

where the charge operator for pointlike nucleons is

$$\rho(\mathbf{q}) = \sum_{i=1}^{Z} e^{i\mathbf{q}\cdot\mathbf{r}_i} \ , \qquad (2)$$

where Z is the number of protons. Equation (1) shows that knowledge of nuclear initial and final state wave functions is necessary for the calculation of the response. Unfortunately, today, realistic evaluations of the response can be performed only in light nuclei, while we are far from being able to perform realistic calculations in more complex systems. In the present work we propose a less ambitious approach which allows one to estimate $R(|\mathbf{q}|,\omega)$ for a large number of medium-weight and heavy nuclei within a unified approximation.

One may consider the NM response as a good starting point for the calculation of $R(|\mathbf{q}|,\omega)$ of heavy nuclei, since NN correlations in the ground and final states can be taken into account properly [14]. However, a NM calculation misses the important finite size and shell effects of real nuclei. Aiming at incorporating the latter effects into the NM response we first separate off from it the pure Fermi gas (FG) part, obtaining

$$\delta R^{\rm NM}(|\mathbf{q}|,\omega,k_F) = R^{\rm NM}(|\mathbf{q}|,\omega,k_F) - R^{\rm FG}(|\mathbf{q}|,\omega) \ . \qquad (3)$$

The resulting quantity $\delta R^{\rm NM}$ represents that part of the response per proton due to the presence of dynamical correlations generated mainly by the short and medium range part of the potential [Pauli correlations are included in $R^{\rm FG}(|\mathbf{q}|,\omega)$]. A quite similar quantity can be defined for finite nuclei, namely,

$$\delta R^{A}(|\mathbf{q}|,\omega) = R^{A}(|\mathbf{q}|,\omega) - R^{\rm HF}(|\mathbf{q}|,\omega) \ , \qquad (4)$$

where now the single-particle response is evaluated within a Hartree-Fock (HF) approximation. Once again, $\delta R^A(|\mathbf{q}|,\omega)$ contains correlations which are not accounted for by the mean field description and are due to the residual interaction.

While it is evident that the two single-particle responses ($R^{\rm FG}$ and $R^{\rm HF}$) are quite different, it is reasonable to assume a larger similarity for $\delta R^{\rm NM}$ and $\delta R^{\rm A}$, since presumably short and medium range correlations in finite nuclei and in nuclear matter at the same density are not too different. This fact suggests the following local density ansatz for the longitudinal response per proton of medium-weight and heavy nuclei:

$$\delta R^A(|\mathbf{q}|,\omega) = \int d\mathbf{r} \frac{k_F^3(\mathbf{r})}{3\pi^2}\, \delta R^{\rm NM}(|\mathbf{q}|,\omega,k_F(\mathbf{r})) \ , \quad (5)$$

and therefore

$$R^A(|\mathbf{q}|,\omega) = R^{\rm HF}(|\mathbf{q}|,\omega) + \int d\mathbf{r} \frac{k_F^3(\mathbf{r})}{3\pi^2}\, \delta R^{\rm NM}(|\mathbf{q}|,\omega,k_F(\mathbf{r})) \ . \quad (6)$$

The r dependence of the Fermi momentum k_F can be taken from the Thomas-Fermi approximation ($k_F(\mathbf{r}) = [3\pi^2\rho(\mathbf{r})]^{1/3}$), where $\rho(\mathbf{r})$ is the HF density of the protons normalized to the proton number [$\int d\mathbf{r}\,\rho(\mathbf{r}) = Z$].

This ansatz has the advantage of taking into account in a transparent way finite size and shell effects as well as correlation effects. However, such a separation has to be interpreted with care. In principle, δR does not only contain correlation effects but also interference terms between correlation and single-particle parts. Fortunately, in a local density approximation it does not seem to be important what kind of single-particle model is used for the interference terms. This is implicitly shown in Ref. [15], where nuclear momentum distributions are calculated with an ansatz similar to the present one. In fact, there it turns out that the obtained momentum distributions are very similar to the ones obtained in conventional many-body calculations which include dynamical correlations.

For an explicit evaluation of the term R^A one needs an expression for $R^{\rm HF}$ and the longitudinal NM response as a function of the Fermi momentum k_F, i.e., of the density. Indeed to make the calculations simpler one would need a parametrization of $\delta R^{\rm NM}$ as function of k_F. In the following we describe how we get $R^{\rm HF}$ and a parametrization of $\delta R^{\rm NM}$ within the PWIA. This approximation is particularly suitable if one wants to focus mainly on the properties of the response functions which are a consequence of the ground state properties of the system. Moreover, at values of momentum transfer around 2.5 fm^{-1} it represents a rather good approximation especially in the quasielastic peak region [18]. A discussion about possible final state interaction (FSI) effects is postponed to the end of this section.

The longitudinal NM response per proton in PWIA is given by

$$R^{\rm NM}_{\rm PWIA}(|\mathbf{q}|,\omega,k_F) = \frac{2\pi M^*}{|\mathbf{q}|}\int_{|\mathbf{p}|^*_-}^{|\mathbf{p}|^*_+} d|\mathbf{p}|\,|\mathbf{p}|\,n^{\rm NM}(|\mathbf{p}|,k_F) \ , \quad (7)$$

with

$$|\mathbf{p}|^*_\pm = \frac{M^*}{|\mathbf{q}|}\left|\omega \pm \frac{|\mathbf{q}|^2}{2M^*}\right| \ , \quad (8)$$

where the nucleon effective mass M^* is generated in the NM calculation. Analogously, one has

$$R^{\rm HF}_{\rm PWIA}(|\mathbf{q}|,\omega,k_F) = Z\,\frac{2\pi M^*}{|\mathbf{q}|}\int_{|\mathbf{p}|^*_-}^{|\mathbf{p}|^*_+} d|\mathbf{p}|\,|\mathbf{p}|\,n^{\rm HF}(|\mathbf{p}|) \ , \quad (9)$$

with $\int n^{\rm HF}(|\mathbf{p}|)\,d|\mathbf{p}| = 1$. Similar to the response in Eq. (3), the NM nucleon momentum distribution $n^{\rm NM}$ can also be split in an independent particle ($n^{\rm FG}$) and a correlation part ($\delta n^{\rm NM}$), so that Eq. (7) can be rewritten as

$$R^{\rm NM}_{\rm PWIA}(|\mathbf{q}|,\omega,k_F) = \frac{2\pi M^*}{|\mathbf{q}|}\int_{|\mathbf{p}|^*_-}^{|\mathbf{p}|^*_+} d|\mathbf{p}|\,|\mathbf{p}|\,\left[n^{\rm FG}(|\mathbf{p}|) + \delta n^{\rm NM}(|\mathbf{p}|,k_F)\right] \ . \quad (10)$$

From Eqs. (3) and (10) one has

$$\delta R^{\rm NM}_{\rm PWIA}(|\mathbf{q}|,\omega,k_F) = \frac{2\pi M^*}{|\mathbf{q}|}\int_{|\mathbf{p}|^*_-}^{|\mathbf{p}|^*_+} d|\mathbf{p}|\,|\mathbf{p}|\,\delta n^{\rm NM}(|\mathbf{p}|,k_F) \ . \quad (11)$$

Having the previous expression for δR, a parametrization as function of k_F has to be found for $\delta n^{\rm NM}(|\mathbf{p}|)$. Following Ref. [15] we take the form of the lowest-order cluster (LOC) approximation developed in Ref. [17]. A comparison between $\delta R^{\rm NM}_{\rm PWIA}(|\mathbf{q}|,\omega,k_F = 1.33\ {\rm fm}^{-1})$ calculated once with the exact $\delta n^{\rm NM}$ [19] and once with the LOC parametrization of $\delta n^{\rm NM}$ is shown in Fig. 1. One sees that the results are very similar. Thus we may use the LOC approximation in order to get the correct density dependence of $\delta n^{\rm NM}$

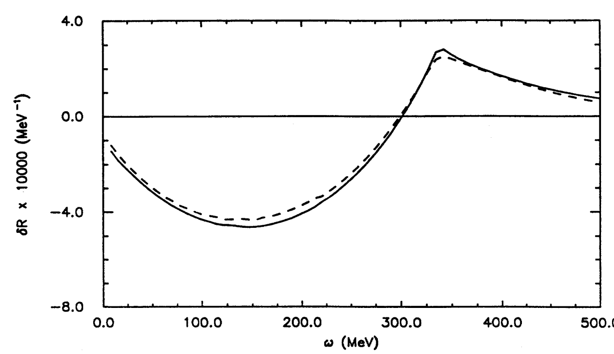

FIG. 1. $\delta R^{\rm NM}_{\rm PWIA}(|\mathbf{q}|,\omega,k_F)$ at $|\mathbf{q}| = 500$ MeV/c and $k_F = 1.33$ fm^{-1} obtained from Eq. (11) using $\delta n_{\rm NM}(|\mathbf{p}|,k_F)$ of Ref. [19] (solid line) and the LOC parametrization of Ref. [15] (dashed line, short range parameter $\beta = 1.1$ fm$^{-1}, M^* = 0.81M$).

Obviously, the ansatz of Eq. (11) does not account for the correlations in the final state. In Fig. 2 we show the effect of FSI's in the case of NM comparing $\delta R^{\rm NM}$ [Eq. (3)] and $\delta R^{\rm NM}_{\rm PWIA}$ [Eq. (11)] at a momentum transfer of 500 MeV/c. They have been calculated making use of the NM momentum distribution of Ref. [19] and the NM longitudinal response of Ref. [14], respectively. In both works the same realistic NN interaction is used. One notes that correlations are mostly effective in the quasielastic peak region ($\omega_{\rm peak} = q^2/2M^*, M^* = 0.81$ [20], i.e., $\omega_{\rm peak} = 165$ MeV for $|\mathbf{q}| = 500$ MeV/c) and for the high-energy tail. While they reduce the response in the peak region, they lead to an increase at high energies. Comparing the two curves of Fig. 2 one finds that δR is more affected by ground than by final state correlations. Thus our limitation to consider only initial state correlations in the local density approximation seems to be reasonable.

Though we do not consider final state correlations in the local density approximation we give a rough estimate of them. To this aim in Fig. 3 we show the relative effects of FSI's on the longitudinal response of ^{2}H and ^{4}He [22] and compare them to the corresponding NM result. The curves in Fig. 3 represent the ratio $r = (R - R_{\rm PWIA})/R_{\rm PWIA}$. In the case of deuteron R is the exact response obtained with the Argonne v_{14} [21] potential model (q = 450 MeV/c). In the case of ^{4}He R is the response reconstructed from a Green's function Monte Carlo calculation of its Laplace transform (q=400 MeV/c, v_{14} Argonne potential). The corresponding PWIA responses are obtained using the exact and variational Monte Carlo momentum distributions of ^{2}H and ^{4}He, respectively. In the case of ^{4}He an average separation average equal to the difference between ^{3}H and ^{4}He binding energies has been used. For nuclear matter the PWIA result is obtained using the NM momentum distribution of Ref. [19] and $M^* = 0.81$ [14]. The FSI leads to a reduction in the peak region for all three cases. In general, it seems that FSI effects in NM are qualitatively rather similar but less pronounced than in real nuclei, though the results for ^{4}He are at smaller momentum transfer ($|\mathbf{q}|$ = 400 MeV/c) than those for NM ($|\mathbf{q}|$ = 500 MeV/c) and FSI effects tend to be less im-

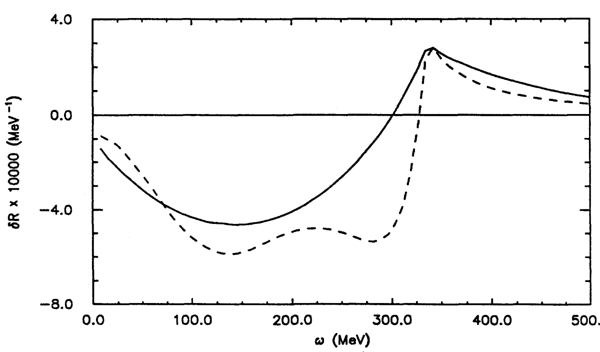

FIG. 2. $\delta R^{\rm NM}(|\mathbf{q}|, \omega, k_F)$ of Eq. (3) at $|\mathbf{q}|$ = 500 MeV/c and k_F =1.33 fm^{-1} (dashed line). The solid line is the same as in Fig. 1.

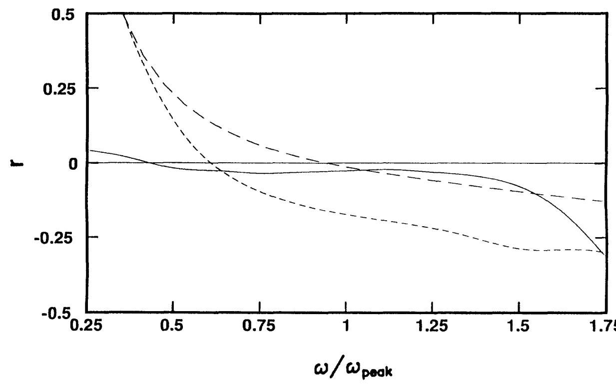

FIG. 3. Relative effects of FSI's [$r = (R - R_{\rm PWIA})/R_{\rm PWIA}$] as function of $\omega/\omega_{\rm peak}$ for deuteron (long dashed line), ^{4}He (short dashed line), and nuclear matter (solid line) at $|\mathbf{q}|$ = 400–500 MeV/c.

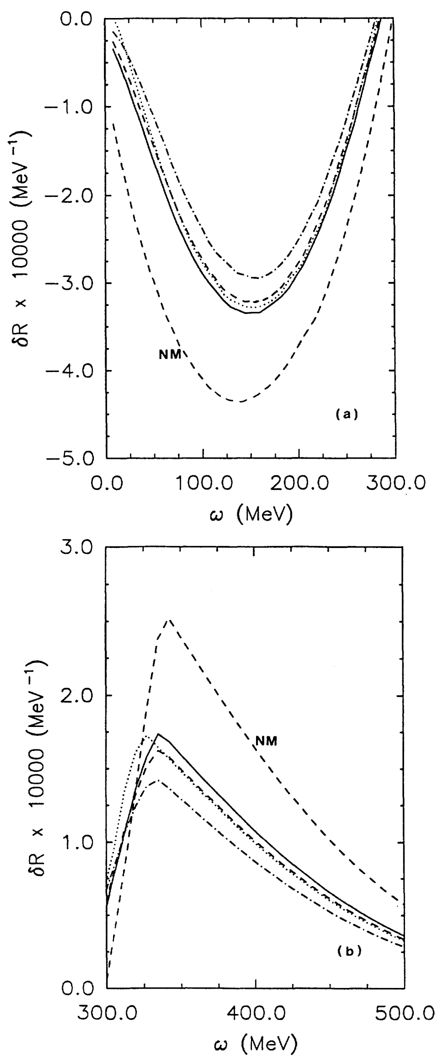

FIG. 4. $\delta R^A(|\mathbf{q}|, \omega, k_F)$ at $|\mathbf{q}|$ =500 MeV/c for several nuclei in peak (a) and high-energy region (b). ^{16}O, dot-dashed line; ^{40}Ca, dashed line; ^{56}Fe, solid line; ^{208}Pb, dotted line. The NM result $\delta R^{\rm NM}_{\rm PWIA}$ is also shown.

portant with increasing $|\mathbf{q}|$. Thus we do think that the simple inclusion of NM-FSI effects in our final results for the responses will probably represent a lower estimate of these effects in medium-weight and heavy nuclei.

III. RESULTS AND DISCUSSION

In Fig. 4, $\delta R^A(|\mathbf{q}|,\omega)$ is shown for several nuclei with A ranging from 16 to 208. As in NM the effect of correlations consists mainly in a shift of strength from the peak region to higher energies. In complete analogy with the results for the Coulomb sum rule [16] the size of this effect grows from ^{16}O to ^{56}Fe and then decreases with further increasing A. As was pointed out in Ref. [16] this behavior is related to the average proton density (cf. Table I of Ref. [16]). One has the strongest effects for the nuclei with the highest densities, and one finds rather similar results for nuclei with the same average density, e.g., for ^{40}Ca and ^{208}Pb. Neutron-proton asymmetry effects have been neglected for δR^A since results for symmetric nuclear matter have been used for the local density approximation. These effects should be proportional to $(\frac{N-Z}{A})^2$ and therefore less important. However, in the dominant HF part of the total response asymmetry effects have been taken into account.

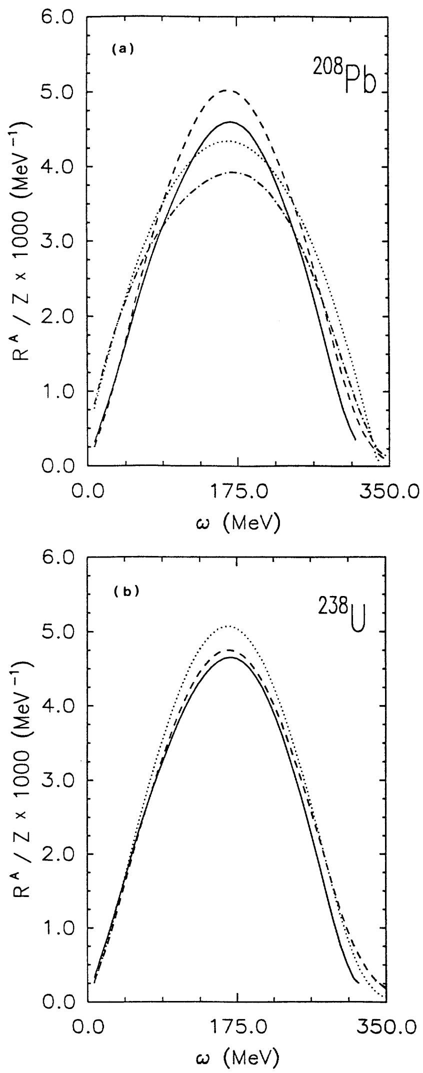

FIG. 5. Longitudinal response function per proton of ^{208}Pb (a) and ^{238}U (b) at $|\mathbf{q}|$ =500 MeV/c. In (a) the dotted line is $R^{\rm FG}(|\mathbf{q}|,\omega)$ with $k_F = 1.33$ fm^{-1}, the dashed line is $R^{\rm HF}(|\mathbf{q}|,\omega)$, and the dash-dotted and solid lines are $R^A(|\mathbf{q}|,\omega)$ of Eq. (6) with $R^{\rm FG}$ and $R^{\rm HF}$, respectively. In (b) the dotted line is $R^{\rm HF}(|\mathbf{q}|,\omega)$, and the dashed line is $R^A(|\mathbf{q}|,\omega)$ of Eq. (6), where $\delta R^{\rm NM}_{\rm PWIA}$ is used in the integral. In the solid line the quantity $(\delta R^{\rm NM} - \delta R^{\rm NM}_{\rm PWIA})$ embodying FSI's has been added. $M^* = 0.81M$ for all curves.

Figure 5 shows results of various approximations for the longitudinal response of ^{208}Pb and ^{238}U. Let us consider first the effect of different single-particle models [see Fig. 5(a)]. One sees that the HF calculation leads to a narrower peak than the FG model. This is clearly due

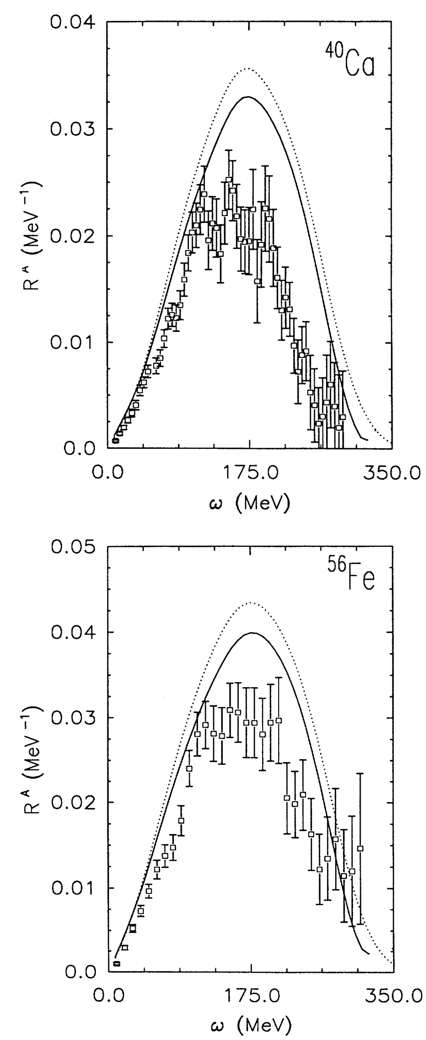

FIG. 6. Longitudinal response functions of ^{40}Ca and ^{56}Fe at $|\mathbf{q}|$ =500 MeV/c. Dashed line, $R^{\rm HF}$; solid line, R^A with inclusion of both short range correlations and FSI effects. The proton form factor $\overline{G}^2(q^2)$ [23] has been used. Experimental data from Ref. [8].

to density effects. In fact, in the FG model the width of the response is proportional to k_F, which is larger than the average $k_F(\mathbf{r})$ of a finite nucleus. Furthermore, Fig. 5(b) shows that short range correlations reduce the peak height by about 7%. Effects due to correlations become much more important in the tail region; there they can even double the strength. These results are similar to those obtained previously in a phenomenological correlated pair model [24]. In Ref. [24] it is shown that the redistribution of the strength is mainly due to the tensor component of the correlation function. The further inclusion of NM-FSI effects ($\delta R^{\mathrm{NM}} - \delta R^{\mathrm{NM}}_{\mathrm{PWIA}}$) reduces the peak height by an additional 3–4% and quenches the strength at high energies considerably. A correct treatment of FSI's, however, might lead to strong reductions in the peak region as well.

A comparison with experimental data is shown in Fig. 6 for medium-weight nuclei and in Fig. 7 for the heavier ones. The results for R^A have been multiplied by the proton form factor of Ref. [23]. Contributions of the neutrons to the response have been neglected, setting the neutron form factor to zero. Even if correlation and FSI effects tend to reduce the disagreement between theory and experiment it is clear that the quenching of the measured responses is still sizable. The only exception is ^{238}U, where one has a satisfactory agreement between theory and experiment.

It has been argued that one of the possible reasons for the quenching of the longitudinal response could be a modification of the proton form factor due to medium effects [25]. Of the many works which have been performed in this direction we only consider here a recent one [26], where within the vector dominance model the nucleon form factor can be written as a sum of a bag and a density-dependent meson cloud terms. Within our local density approach the inclusion of a density-dependent proton form factor in the longitudinal response function is straightforward. One has

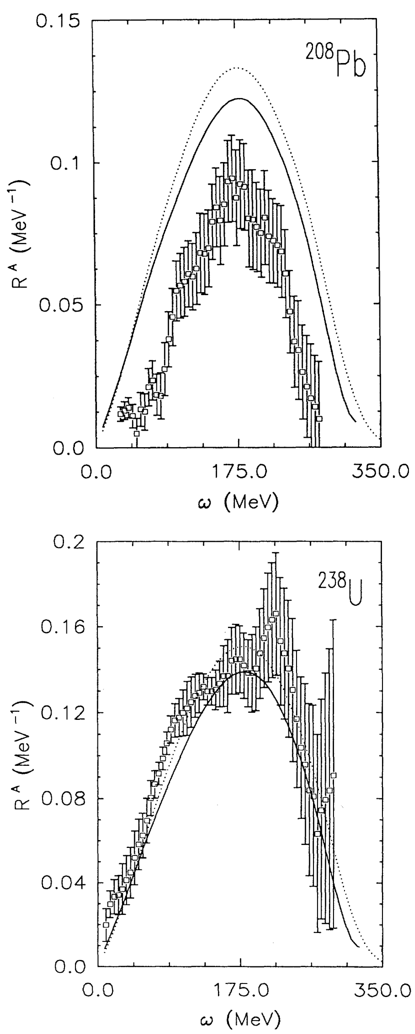

FIG. 7. Longitudinal response functions of ^{208}Pb and ^{238}U at $|\mathbf{q}|$ =500 MeV/c. Curves as in Fig. 6. Experimental data from Refs. [11, 27] (^{208}Pb) and [10] (^{238}U).

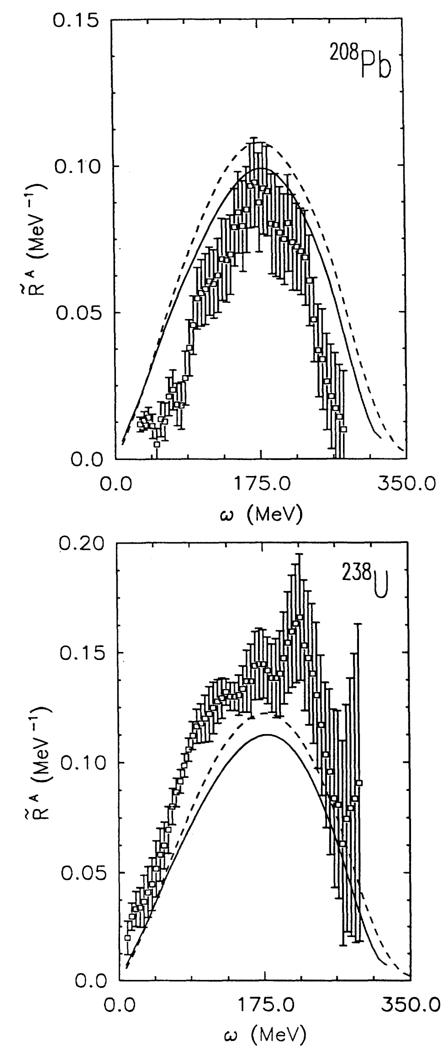

FIG. 8. Longitudinal response functions of ^{208}Pb and ^{238}U at $|\mathbf{q}|$ =500 MeV/c with inclusion of the density-dependent proton form factor of Ref. [26]. Dashed line, R^{HF}; solid line, R^A with inclusion of both short range correlations and FSI effects. Experimental data from Refs. [11, 27] (^{208}Pb) and [10] (^{238}U).

$$\tilde{R}^A(|\mathbf{q}|,\omega) = \int d\mathbf{r}\, \frac{k_F^3(\mathbf{r})}{3\pi^2 Z}\, F_L^2(\rho(\mathbf{r}),q^2)\, \left[R_{\mathrm{HF}}(|\mathbf{q}|,\omega) + Z\,\delta R^{\mathrm{NM}}_{\mathrm{PWIA}}(|\mathbf{q}|,\omega,k_F(\mathbf{r}))\right] \;, \tag{12}$$

where $F_L^2(\rho(\mathbf{r}),q^2)$ is the proton form factor of Ref. [26]. In Fig. 8 we show the effect of this density-dependent form factor on the longitudinal responses of ^{208}Pb and ^{238}U. As expected, one finds a better agreement with experiment in the former case and a worse one in the latter. Of course, an improvement would also been obtained for ^{40}Ca and ^{56}Fe.

To summarize, the local density approach to the longitudinal response allows us to study the role of short range correlations in a large class of nuclei from medium weight to heavy ones. We have found that they are responsible for shifting the strength from the peak region to the high-energy tail. The amount of quenching in the quasielastic region and the corresponding increase of the high-energy strength are related to the average charge density. The effects become larger going from ^{16}O to ^{56}Fe and then decrease with further increasing A. The comparison with experimental data shows that the size of these effects is not large enough to explain the data, except for the case of ^{238}U. Other effects such as FSI's can be responsible for a further reduction of strength in the quasielastic region. The density-dependent proton form factor such as that of Ref. [26] in the vector dominance hypothesis has been included in our local density calculation of the response. This gives a further quenching of the response, which brings the theoretical description in good agreement with the data for ^{208}Pb, but spoils the good results in ^{238}U.

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