Charge radius of the neutron: Discussion of the differences between experimental values

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The analyses of low-energy (< 150 eV) total neutron-atom cross sections, giving the best current but contradictory values for the charge radius of the neutron at present, are reconsidered. It is confirmed that the discrepancy between previous estimates of the crucial electron scattering length is due to different treatments of resonance contributions. In particular the discrepancy arises because the b_{ne} value is not extracted via the energy dependence of the atomic form factor but is essentially determined from the difference between the neutron-atom total cross sections above 1 eV and the zero energy value given by the coherent scattering length. A consistent resonance analysis of available data favors a value of the neutron charge radius which is less negative than the corresponding Foldy value.

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I. INTRODUCTION

Low-energy neutron-atom scattering has been found useful to determine the neutron-electron scattering length b_{ne} and therefrom the mean square charge radius of the neutron $\langle r^2\rangle_n^{\mathrm{ch}}$. The relation of b_{ne} with the electromagnetic structure of the neutron has provoked considerable effort to determine b_{ne} from high precison experiments [1–12]. The most relevant experimental values are summarized in Table I.

In a first estimate b_{ne} is expected to be given by the Foldy scattering length, $b_F = -1.468 \times 10^{-3}$ fm, corresponding to the anomalous magnetic moment of the neutron [13]. The experimental values summarized in Table I indicate deviations of b_{ne} from b_F on the order of 10%. The present experimental results are unsatisfactory in so far that they form two groups which do not overlap in their uncertainties. Analyses of the measured neutron-atom total cross section data by Koester *et al.* [4–6] yield b_{ne} values which are less negative than b_F. Contrary to this, Alexandrov *et al.* [7–12] find values of b_{ne} which are more negative than b_F from their analyses of neutron-atom total cross section measurements as well as of neutron diffraction measurements on single crystals. There is an obvious discrepancy between the b_{ne} values obtained from neutron diffraction [7,8] and those from total cross sections [4–6], while the b_{ne} values extracted from both groups from total cross section measurements [4–6,9–11] agree with each other within two standard deviations. However, this is a remarkable systematic difference which demands an explanation. It certainly has led to a vivid controversy [14–19]. Today, there is common agreement that the difference between the analyses of Alexandrov *et al.* [9–11] and Koester *et al.* [4–6], which are based on a practically equivalent data set, is with their different treatments of resonance corrections. Nikolenko and Popov [14] explain the difference by the neglect of interresonance interference terms in the analyses of Alexandrov *et al.* [9–11]. However, this explanation as well as the justifications given by Alexandrov [15–19], supporting his analyses of total neutron-atom cross sections, are not convincing. The situation with regard to the analyses of total neutron-atom cross section data is still unsatisfactory and requires an explanation.

In this paper we focus on the analyses of neutron-atom total cross section measurements with regard to the determination of b_{ne}. For this purpose we reconsider in

TABLE I. List of the best experimental values of the neutron-electron scattering length b_{ne}.

Authors	Reference	Year	b_{ne} (fm)
Hughes *et al.*	[1]	1953	$(-1.39 \pm 0.13) \times 10^{-3}$
Melkonian *et al.*	[2]	1959	$(-1.56 \pm 0.05) \times 10^{-3}$
Krohn and Ringo	[3]	1973	$(-1.30 \pm 0.03) \times 10^{-3}$
Koester *et al.*	[4–6]	1973–1988	$(-1.32 \pm 0.04) \times 10^{-3}$
Alexandrov *et al.*	[7,8]	1975–1985	$(-1.60 \pm 0.05) \times 10^{-3}$
Alexandrov *et al.*	[9–11]	1983–1989	$(-1.55 \pm 0.11) \times 10^{-3}$
Kopecky *et al.*	[25]	1992	$(-1.39 \pm 0.04) \times 10^{-3}$

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detail the resonance contributions, which generate the main difference between the analyses. In particular we discuss the consequences of consistent application of the different methods to the experimental data on Bi and Pb [5,6]. From our analyses of these neutron-atom total cross section measurements, consistent values for b_{ne} are obtained which are less negative than b_F.

In Sec. II we give a brief description of the notation and the evaluation of neutron-atom cross sections. The central point of our paper, the detailed consideration of resonance contributions, is outlined in Sec. III. Of essential importance are the conclusions given in Sec. IV. We are well aware that many of the arguments have been already given previously. However, because of several misleading statements in recent works, it seems important to recall them together here for clarity.

II. BASIC RELATIONS OF NEUTRON-ATOM SCATTERING

To extract information about the electromagnetic properties of the neutron from total neutron-atom cross sections, careful analyses of data taken with high precision experiments are required. Details of the relations for low-energy neutron-atom scattering have been presented in many previous papers [3–12] and in the review article of Sears [20]. Thus only a brief sketch of that is made herein.

We consider neutrons scattered at bound atoms. Thus the extracted quantities, e.g., the neutron-electron scattering length, will not depend on the specific target nucleus. At low incident neutron energies the total neutron transmission cross sectrion $\sigma_{\rm tr}$ may be written as

$$\sigma_{\rm tr}(E) = \sigma_s(E) + \sigma_l(E) + \sigma_{LS}(E) + \sigma_{\rm abs}(E) + \sigma_{\rm nat}(E), \tag{1}$$

where E is the energy in the laboratory system. In this equation, $\sigma_s(E)$, essentially the elastic scattering cross section given by the nuclear s wave, is the quantity of interest. For neutron scattering from Pb and Bi in the energy range 1–150 eV, the absorption cross section $\sigma_{\rm abs}(E)$ and the cross section contributions from higher ($l > 0$) nuclear partial waves $\sigma_l(E)$ and from Schwinger scattering $\sigma_{LS}(E)$ are small corrections. The same is true for $\sigma_{\rm nat}(E)$ which accounts for solid or liquid state effects depending on the nature of the target. This latter term increases at lower energies $E < 5$ eV as then the wavelength of the neutron becomes of the order of the lattice spacings or the correlation length.

The cross section $\sigma_s(E)$ contains not only the elastic nuclear s-wave scattering but also electromagnetic contributions. The latter are relatively small compared to the nuclear part and are usually treated in the Born approximation. Thus, for a spinless nucleus, $\sigma_s(E)$ is given by

$$\sigma_s(E) = 4\pi(f_{\rm pot} + f_{\rm res} + f_{ne} + f_{\rm pol})^2, \tag{2}$$

where the first two terms, $f_{\rm pot}$ and $f_{\rm res}$, describe the nuclear s-wave potential and resonance scattering, respectively, while f_{ne} and $f_{\rm pol}$ are electromagnetic contributions. In the energy range up to 150 eV it is sufficient to describe $f_{\rm pot}$ by using a nuclear scattering length b_N, and the resonance scattering amplitude can be evaluated, using a standard form, from the resonance parameters of the target nucleus [6,11,21,22].

In this paper we focus on the determination of f_{ne} which stems from the electromagnetic interaction of the neutron with the charge distribution of the atom. This scattering amplitude is given by

$$f_{ne} = -Zb_{ne}[1 - \bar{F}_A(E)], \tag{3}$$

where Z is the charge of the target nucleus and $\bar{F}_A(E)$ is the angle averaged atomic form factor describing the electron charge distribution of the atom. The charge distribution of the nucleus also contributes to this interaction, but because we are dealing with low energies, the nuclear charge form factor can be taken as 1 in the whole energy range considered. It is useful to note that, for the scattering from a neutral atom, f_{ne} vanishes at $E = 0$.

The remaining scattering amplitude $f_{\rm pol}(E)$ takes into account that the scattering process is also affected by the interaction between the electric field of the nucleus and the electric dipole moment of the neutron induced by the strong electric fields at the surface of the nucleus [23]. This contribution can be written as

$$f_{\rm pol}(E) = b_P\ \bar{G}(E). \tag{4}$$

Here, $\bar{G}(E)$ is the angle averaged polarization form factor, which corresponds to the Fourier transform of the square of the electric field strength. The scattering length b_P is proportional to the electric polarizability of the neutron α_n and to Z^2.

Recently, the electric polarizability of the neutron α_n has been measured with high accuracy [23], and we use this value ($\alpha_n = 1.20\times10^{-3}$ fm^3) in our further consideration. However, for energies below 150 eV, the term (4) generates only a relatively small energy variation compared to that of the atomic form factor. Furthermore, a failure in the energy independent part of (3) and (4) can be compensated by varying the nuclear scattering length b_N, as it is poorly known and has to be determined also from the total cross section measurements.

The high precision experiments to determine b_{ne} are motivated by the direct relation of the neutron-electron scattering length to the charge distribution of the neutron. It is straightforward to show that b_{ne} is proportional to the mean square radius of the neutron $\langle r^2\rangle_n^{\rm ch}$, per

$$b_{ne} = \frac{1}{3}\frac{\alpha_f m_n c^2}{\hbar c}\langle r^2\rangle_n^{\rm ch}, \tag{5}$$

where m_n is the mass of the neutron. Furthermore, because the neutron is neutral and has no electric dipole moment, $\langle r^2\rangle_n^{\rm ch}$ then determines the electric charge form factor at zero momentum transfer q. But the total neutron-electron scattering length b_{ne} contains contributions due to the Foldy interaction which cannot be separated from the electrostatic one. Therefore $\langle r^2\rangle_n^{\rm ch}$ is related to the Sachs form factor $G_E^n(q^2)$,

$$\langle r^2\rangle_n^{\rm ch} = -\frac{1}{6}\frac{dG_E^n}{dq^2}\bigg|_{q=0}. \qquad (6)$$

Finally, it should be remarked that $\langle r^2\rangle_n^{\rm ch}$ differs from the usual definition of a mean square radius because of the zero charge on the neutron. Hence, the quantity $\langle r^2\rangle_n^{\rm ch}$ should be understood as $\frac{1}{e}\langle\rho_n^{\rm ch}r^2\rangle$ in this context.

III. COMPARISON OF ANALYSES

The determination of b_{ne} from neutron-atom total scattering cross sections requires the separation of a term which exhibits the characteristic energy dependence of the atomic form factor (3). Therefore it is advantageous to choose target nuclei with high Z that have no resonances in the interesting energy range from 0.1 eV up to 150 eV where the atomic form factor $\bar{F}_A(E)$ changes from almost 1 to 0.01. Fullfilling these requirements the most recent values of b_{ne} have been obtained from experiments on Bi and natural Pb by Koester *et al.* [5,6] and on Bi by Alexandrov *et al.* [11]. However, both evaluations of b_{ne} rely on data above 1 eV, where $\bar{F}_A(E)$ is already smaller than 0.15. Hence they are at the limit of which the energy dependence of the atomic form factor can be isolated. To gain additional information, both groups have included the corresponding coherent scattering length in their analyses. That has been determined separately with high accuracy [5].

In the following we compare the analyses of the total neutron-Bi cross section data made by both groups [5,6,11]. As has already been pointed out [14,18,19] the data sets used by these groups are almost equivalent. Therefore the discrepancy between their extracted b_{ne} values can only be attributed to the different procedures used.

To exhibit the differences between the analyses we must briefly consider neutron scattering from nuclei with spin. For a nucleus with spin quantum number I, the elastic channel for the scattering of a neutron can have the channel spin quantum numbers $J = I + \frac{1}{2}$ or $J = I - \frac{1}{2}$. Consequently, the scattering cross section $\sigma_s(E)$ becomes an incoherent sum over the different channel contributions,

$$\sigma_s(E) = g_+\ \sigma_s^+(E) + g_-\ \sigma_s^-(E)\ , \qquad (7)$$

where the factor g gives the spin statistical weight,

$$g = \frac{2J+1}{2(2I+1)} \qquad (8)$$

and the indices $+, -$ refer to the channel with $J = I + \frac{1}{2}$, $J = I - \frac{1}{2}$, respectively. Equivalently the cross section $\sigma_s(E)$ can also be written as

$$\sigma_s(E) = \sigma_{\rm coh}(E) + \sigma_{\rm inc}(E), \qquad (9)$$

here $\sigma_{\rm coh}(E)$ and $\sigma_{\rm inc}(E)$ are called the coherent and incoherent scattering cross sections, respectively.

Koester *et al.* [4–6] adopted the second decomposition of $\sigma_s(E)$ [Eq. (9)] either evaluating $\sigma_{\rm inc}(E)$ from the resonance parameters [22] or using the experimental values [4]. Estimating the contributions of high lying resonances, they corrected their measured transmission cross sections and determined b_{ne} from the resultant coherent cross sections.

On the other hand, Alexandrov *et al.* [11,12] used the first decomposition (7), using the values of $\sigma_s^\pm$ obtained from the well known resonance parameters [22]. They also included contributions from unknown, unmeasurable negative energy levels as well as high lying resonances by adding an energy independent correction term Σ, viz.,

$$\sigma_s^A(E) = g_+\sigma_s^+ + g_-\sigma_s^- + \Sigma = \sigma_{\rm coh}(E) + \sigma_{\rm inc}(E) + \Sigma, \qquad (10)$$

where Σ was determined by a fit to the data.

We have applied the procedure of Alexandrov *et al.* [11,12] to the neutron total cross section data from Pb and Bi that were used by Koester *et al.* [5,6]. Since we are interested in b_{ne} we consider the cross sections only at the relevant energies, $E = 1.26$, 5.19, 18.8, and 132 eV. The coherent scattering lengths as well as the corrections for absorption, Schwinger scattering, and solid state effects have been taken from [6], while the polarizability contributions has been evaluated for $\alpha_n = 1.20 \times 10^{-3}$ fm^3 according to the recent measurement of Schmiedmayer *et al.* [23]. However, the polarizability term will not influence the results as it can be compensated in this energy range by varying the nuclear scattering amplitude. Using the total scattering cross section at the four energies we have performed a fit by varying the neutron-electron scattering length b_{ne} in the range -0.6×10^{-3} to -2.0×10^{-3} fm and optimizing Σ at each b_{ne} value via the criterion function

$$\chi^2 = \sum_{i=1}^{4}\left(\frac{\sigma_{\rm tr}^A(E_i;b_{ne},\Sigma) - \sigma_{\rm tr}^{\rm expt}(E_i)}{\Delta\sigma_{\rm tr}^{\rm expt}(E_i)}\right)^2, \qquad (11)$$

where the index A indicates that the transmission cross section has been theoretically determined using (10).

The data for the two nuclei are fitted separately and in both cases it turns out that there exists a strong correlation between Σ and b_{ne}. The contour plots of χ^2 as functions of Σ and b_{ne} are shown in Figs. 1 and 2 and clearly they exhibit this correlation. Although the ensemble of data is rather small it is nevertheless obvious that Σ cannot be determined uniquely from the data sets of Koester *et al.* [5,6].

Nevertheless our fits clearly demonstrate that for both nuclei, b_{ne} values more negative than -1.33×10^{-3} fm are associated with negative Σ values. In particular the value of -1.55×10^{-3} fm as obtained by Alexandrov *et al.* [11,12] from Bi data requires in our calculation Σ to be -29 mb. Thus, we confirm that the values of the analyses of Alexandrov *et al.* are consistent. The physical meaning of this negative Σ value, however, requires a closer look of the relation (10) as one does not entertain negative cross sections.

The strong correlation between b_{ne} and the energy independent correction Σ indicates that the data are

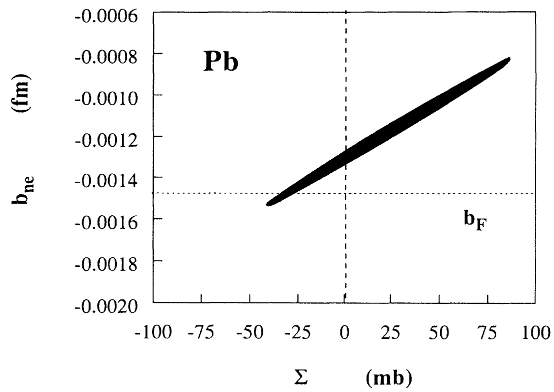

FIG. 1. Contour plot of the χ^2 values for Pb as a function of the neutron-electron scattering length b_{ne} and the correction term Σ. The black area indicates the range of b_{ne} and Σ which is in statistical agreement ($\chi^2 \leq 3$) with the experimental data [6]. For comparison the Foldy scattering length b_F (dotted line) and the assumption of Koester *et al.* [6] (dashed line) are indicated.

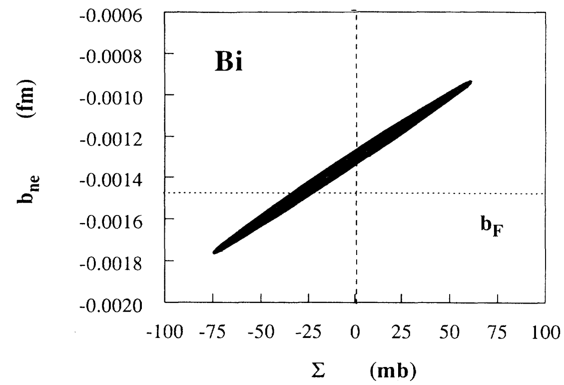

FIG. 2. Contour plot of the χ^2 values for Bi as a function of the neutron-electron scattering length b_{ne} and the correction term Σ. The black area indicates the range of b_{ne} and Σ which is in statistical agreement ($\chi^2 \leq 3$) with the experimental data [6]. For comparison the Foldy scattering length b_F (dotted line) and the assumption of Koester *et al.* [6] (dashed line) are indicated.

primarily sensitive to the difference between the total cross section and the coherent cross section at the energy $E = 0$ as given by the coherent scattering length,

$$\Delta(E) = \sigma_s^A(E) - \sigma_{\rm coh}(E = 0), \quad (12)$$

and are much less sensitive to the energy dependence of the atomic form factor $\bar{F}_A(E)$. Using (10) we may write $\Delta(E)$ as

$$\Delta(E) = \sigma_{\rm coh}(E) - \sigma_{\rm coh}(E = 0) + \sigma_{\rm inc}(E) + \Sigma \ , \quad (13)$$

which simplifies the interpretation of Σ to two possibilities. The first is that Σ accounts for contributions of high lying resonances which are not explicitly taken into account. In this case Σ contains their contributions to $\sigma_{\rm inc}$ and $\sigma_{\rm abs}(E)$ but not to $\sigma_{\rm coh}(E) - \sigma_{\rm coh}(E = 0)$ because this difference depends at least linearly on E. This interpretation implies that

$$\sigma_{\rm inc}(E) + \sigma_{\rm abs}(E) + \Sigma \geq 0 \ . \quad (14)$$

Since the sum $\sigma_{\rm inc}(E) + \sigma_{\rm abs}(E)$ evaluated from known resonances is of the order of 13 mb for Bi and 26 mb for Pb the physical Σ values are restricted by (14), thus allowing only b_{ne} values greater than b_F (see Figs. 1 and 2). The second possibility is that Σ accounts for low lying, extremely narrow resonances or negative energy levels which show up in the coherent scattering length but not at the higher energies. This possibility cannot be completely excluded but it seems unlikely that it happens for both nuclei Bi and Pb with a negative contribution exceeding all other correction terms.

IV. CONCLUSIONS

We have discussed the determination of the neutron-electron scattering length b_{ne} from total neutron-Bi cross section measurements with regard to the discrepancy between the values of b_{ne} extracted by the analyses of Koester *et al.* [4–6] and of Alexandrov *et al.* [9–11]. It has been already pointed out previously [14,18] that the discrepancy is caused by a different treatment of the resonance contributions in these analyses. In particular Alexandrov *et al.* [9–11] include an energy independent term Σ which should account for unknown negative energy levels as well as high lying resonances.

We have reanalyzed the data of Koester *et al.* [5,6] using the procedure of Alexandrov *et al.* [7], finding that the extracted value of b_{ne} is strongly correlated with that of Σ. Furthermore, the data do not determine the Σ value with accuracy, thus allowing a wide variation in the deduced values of b_{ne}. This variation encompasses the extracted values of both groups. The interval of possible b_{ne} values is immediately restricted if we assume that Σ accounts for high lying resonances. In this case the value of b_{ne} should be less negative than b_F because otherwise for Pb and Bi, unphysical values for the corrections are needed. However, if the term Σ corrects for unknown negative energy levels, not showing up in the total cross sections at the energies $E = 1$ eV and above but by modifying the coherent scattering length, there is no direct argument to exclude certain of these Σ values. Nevertheless, considering the analyses of Bi and Pb data together, this latter interpretation is as unlikely as both systems having such strongly contributing negative energy levels.

Summarizing our arguments, it seems well justified that the neutron-electron scattering length b_{ne} is less negative than b_F, and since one expects only relatively small contributions from high lying resonances, the value extracted by Koester *et al.* [6] appears to be the most reliable one at present.

The favored value of b_{ne} which is less negative than the Foldy scattering length is frequently criticized to be in contradiction with standard models of the nucleon. This criticism is based on the fact that a neutron charge distribution with a negative charged skin is expected from the anomalous magnetic moment. Indeed we obtain from our favored value of b_{ne} that, via (5), a negative mean square charge radius is obtained. However, in many analyses, it is the so-called intrinsic charge radius of the neutron,

$$\langle r^2 \rangle_n^{\rm intr} = -\frac{1}{6}\frac{dF_1}{dq^2}\bigg|_{q=0}, \tag{15}$$

that is considered. This radius is associated with the slope of the Dirac form factor and depends on the difference between Foldy scattering length and b_{ne}. It is positive in our case. However, this sign does not contradict our understanding of the neutron because the distinction between Foldy term and intrinsic charge distribution depends on the model applied and is therefore not observable. This becomes obvious in the standard quark model where no Foldy term appears [24]. Therefore the criticism on the sign of the charge radius is not justified.

The discrepancy between the diffraction experiments on tungsten and the analyses of total neutron-atom cross section with regard to the extracted b_{ne} values is still an open question. However, we have cleared up the situation for total neutron-atom cross section measurements. The remaining uncertainty concerning the negative energy levels can be circumvented by a high precision measurement of the energy dependence of the total neutron-atom cross sections allowing us to separate the term proportional to the atomic form factor. Such a measurement is now under way at ORELA [25].

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