

Order parameter for pairing systems

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We compare the static approximation to the Hubbard-Stratonovich representation of the partition function with the order parameter representation based on the Landau theory of phase transitions and we find that the two expressions for the partition function are very similar if we choose the expectation value of the potential for the order parameter of the system. This choice for the order parameter has certain advantages above choosing the BCS energy gap for the order parameter in that it generalizes naturally for momentum dependent pairing interactions and that it remains well defined in exact treatments. In a simple $i_{13/2}$ -model calculation different choices for the order parameter are compared with the exact grand canonical and with the static path integral results.

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I. INTRODUCTION

The mean-field BCS approach [1] for the pairing Hamiltonian

$$H = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma} - G \sum_{\mathbf{k}\mathbf{k}'} c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} \quad (1.1)$$

does not take into account quantum or statistical fluctuations. Quantum fluctuations arise because the mean-field wave functions are not the exact eigenfunctions of the pairing Hamiltonian and in particular they violate the particle number symmetry. Statistical or thermodynamic fluctuations arise in finite systems at finite temperature where states other than the most probable state of the system become accessible.

We consider two methods used to treat thermodynamic fluctuations of finite systems. The first is based on the functional integral representation of the partition function [2-6]. In this approach the saddle-point evaluation of the functional integral gives the standard BCS mean-field result, in the static approximation one includes thermodynamic fluctuations, and in higher order approaches one can include some of the quantum fluctuations [5,6].

The second method we look at is based on the Landau theory of phase transitions [7-10], where one uses a single order parameter to describe the macroscopic state of the system. Comparing the partition function in this formalism with the static approximation to the functional integral representation of the partition function, we notice that the expressions are very similar if we choose the expectation value of the pairing potential

$$\mathcal{G} = G \sum_{\mathbf{k}\mathbf{k}'} \langle c_{\mathbf{k}\uparrow}^{\dagger} c_{-\mathbf{k}\downarrow}^{\dagger} c_{-\mathbf{k}'\downarrow} c_{\mathbf{k}'\uparrow} \rangle \quad (1.2)$$

for the order parameter of the system.

This choice for the order parameter of a pairing sys-

tem has certain advantages above the traditional choice of the BCS energy gap Δ as the order parameter. First, the BCS energy gap is no longer a macroscopic parameter when one uses a momentum-dependent pairing interaction, because the gap becomes momentum dependent. Furthermore, the BCS energy gap is not defined in an exact calculation (it is only nonzero in the mean-field treatment). On the other hand \mathcal{G} remains a macroscopic parameter for momentum-dependent pairing potentials and is well defined in an exact calculation. In systems where the gap parameter is momentum dependent the expectation value of the potential has been previously used as a macroscopic parameter describing the system [11,12].

In the thermodynamic limit, both the BCS energy gap Δ and the expectation value of the pairing potential \mathcal{G} approach zero smoothly at the phase transition. Note that thermodynamic fluctuations vanish in this limit. For finite systems thermodynamic fluctuations give rise to a nonzero value of both parameters above the critical temperature. However, even if one excludes thermal fluctuations, \mathcal{G} remains finite above the critical temperature. This is not the case for the BCS energy gap.

Naturally, the above choice for the order parameter of a system is only valid for systems described by a pairing Hamiltonian. Should one include quadrupole-quadrupole terms or higher order terms, one would get a different expression for the order parameter and the system might have more than one order parameter. Furthermore, should one include higher order terms, it might not be possible to perform the Hubbard-Stratonovich transformation, making it very difficult to evaluate the partition function.

A simple numerical calculation with a small number of particles is performed in the $i_{13/2}$ model [9]. We compare the results with different choices of the order parameter with the exact grand canonical and the static path integral results in spite of the fact that the model Hamiltonian of the $i_{13/2}$ model may be viewed as an effective Hamiltonian to be used in mean-field calculations.

II. THE FUNCTIONAL INTEGRAL REPRESENTATION OF THE PARTITION FUNCTION

The grand canonical partition function for a pairing system with constant attractive pairing potential G is given by

$$Z = \text{tr} [e^{-\beta K}] \quad (2.1)$$

with

$$K = H - \mu N = \sum_{\mathbf{k}\sigma} \epsilon_{\mathbf{k}} n_{\mathbf{k}\sigma} - GB^\dagger B, \quad (2.2)$$

where $n_{\mathbf{k}\sigma}$ counts the number of electrons in the state $|\mathbf{k}\sigma\rangle$:

$$n_{\mathbf{k}\sigma} = c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \quad (2.3)$$

and

$$B^\dagger = \sum_{\mathbf{k}} c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger. \quad (2.4)$$

$c_{\mathbf{k}\sigma}^\dagger$ and $c_{\mathbf{k}\sigma}$ are the fermion creation and annihilation operators and the single-particle energies $\epsilon_{\mathbf{k}} = e_{\mathbf{k}} - \mu$ are measured relative to the chemical potential μ . Defining

$$X = \frac{1}{2}(B + B^\dagger), \quad (2.5)$$

$$Y = \frac{i}{2}(B - B^\dagger),$$

we can write the Hamiltonian in a form suitable for the Hubbard-Stratonovich transformation [2,3]:

$$K = K_0 - G(X^2 + Y^2) \quad (2.6)$$

with

$$K_0 = \sum_{\mathbf{k}} \left[\epsilon_{\mathbf{k}} + \left(\epsilon_{\mathbf{k}} - \frac{G}{2} \right) (n_{\mathbf{k}\uparrow} + n_{-\mathbf{k}\downarrow} - 1) \right]. \quad (2.7)$$

The exact partition function is now given by the path integral

$$Z = \lim_{M \rightarrow \infty} \left(\frac{\beta G}{\pi M} \right)^M \int \prod_{n=1}^M d\phi_x(t_n) d\phi_y(t_n) \left\{ e^{-\frac{\beta G}{M} \sum_{n=1}^M \phi_x^2(t_n) + \phi_y^2(t_n)} \text{tr} \left[\tau e^{-\frac{\beta}{M} \sum_{n=1}^M \mathcal{K}_n} \right] \right\} \quad (2.8)$$

with

$$\mathcal{K}_n = K_0 - 2G[X\phi_x(t_n) + Y\phi_y(t_n)]. \quad (2.9)$$

Here τ is the imaginary time ordering operator. Note that \mathcal{K}_n is now only quadratic in the fermion operators, while K was quartic. In order to define the various approximation schemes one usually makes use of a Fourier transform of the fields and we assume that we have chosen an odd number of time slices M . Thus

$$\phi_x(t) = \sum_{p=-\frac{M-1}{2}}^{\frac{M-1}{2}} \eta_{xp} e^{-i\frac{2\pi p}{\beta} t} \quad (2.10)$$

with $n_{-xp}^* = n_{xp}$ and similarly for $\phi_y(t)$. In the case of the static-path approximation one approximates \mathcal{K}_n by the static Hamiltonian

$$\bar{\mathcal{K}} = K_0 - 2G[X\bar{\eta}_x + Y\bar{\eta}_y] \quad (2.11)$$

with $\bar{\eta}_x \equiv \eta_{x0}$ and $\bar{\eta}_y \equiv \eta_{y0}$. Further, converting the sum over the time slices to an integral, we obtain

$$\lim_{M \rightarrow \infty} \frac{\beta G}{M} \sum_{n=1}^M \phi_x^2(t_n) + \phi_y^2(t_n) = \beta G \left\{ \bar{\eta}_x^2 + \bar{\eta}_y^2 + 2 \sum_{p=1}^{\frac{M-1}{2}} [\eta_{xp}^2 + \eta_{yp}^2] \right\}. \quad (2.12)$$

The integrals over the η_{xp} and η_{yp} ($p \neq 0$) can now be readily performed. The static approximation to the partition function is then given by

$$\bar{Z} = \left(\frac{\beta G}{\pi} \right) \int d\bar{\eta}_x d\bar{\eta}_y \left\{ e^{-\beta G(\bar{\eta}_x^2 + \bar{\eta}_y^2)} \text{tr} \left[e^{-\beta \bar{\mathcal{K}}} \right] \right\}. \quad (2.13)$$

In order to evaluate the trace we define

$$\psi = G(\bar{\eta}_x - i\bar{\eta}_y) \quad (2.14)$$

so that

$$\bar{Z} = \frac{\beta}{\pi G} \int d\psi \left\{ \text{tr} \left[e^{-\beta h(\psi)} \right] \right\} \quad (2.15)$$

with

$$h = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} + \frac{|\psi|^2}{G} + \sum_{\mathbf{k}} \left[\left(\epsilon_{\mathbf{k}} - \frac{G}{2} \right) (n_{\mathbf{k}\uparrow} + n_{-\mathbf{k}\downarrow} - 1) - (\psi c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger + \psi^* c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}) \right]. \quad (2.16)$$

When performing the trace only terms with equal numbers of creation and annihilation operators will survive. We obtain

$$h_d = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} + \frac{|\psi|^2}{G} + \sum_{\mathbf{K}} R_{\mathbf{k}} (n_{\mathbf{k}\uparrow} + n_{-\mathbf{k}\downarrow} - 1) \quad (2.17)$$

with

$$R_{\mathbf{k}} = \sqrt{\left(\epsilon_{\mathbf{k}} - \frac{G}{2} \right)^2 + |\psi|^2}. \quad (2.18)$$

Note that the h_d is only a function of $|\psi|^2$. This allows us to write the partition function in the following form:

$$\bar{Z} = \frac{\beta}{G} \int_0^\infty d|\psi|^2 \left\{ e^{-\beta \Omega(|\psi|^2)} \right\} \quad (2.19)$$

with

$$\Omega(|\psi|^2) = \sum_{\mathbf{k}} \left[\epsilon_{\mathbf{k}} - R_{\mathbf{k}}(|\psi|^2) - \frac{2}{\beta} \ln \left(1 + e^{-\beta R_{\mathbf{k}}(|\psi|^2)} \right) \right] + \frac{|\psi|^2}{G}. \quad (2.20)$$

Note that the extremum of the integrand satisfies the BCS gap equation

$$1 = \frac{G}{2} \sum_{\mathbf{k}} \frac{1 - 2f_{\mathbf{k}}}{R_{\mathbf{k}}} \quad (2.21)$$

with

$$f_{\mathbf{k}} = \frac{1}{1 + e^{\beta R_{\mathbf{k}}}}. \quad (2.22)$$

If one replaces $|\psi|$ by the BCS energy gap Δ then Eq. (2.20) looks very similar to the BCS expression of the thermodynamic potential [8] and comparing (2.19) to the order parameter representation of the partition function

$$Z \propto \int_0^\infty e^{-\beta \Omega(\xi)} d\xi, \quad (2.23)$$

where ξ is the order parameter, one is led to identify $G = \frac{\Delta^2}{G}$ as the order parameter of the pairing system.

III. STATISTICAL FLUCTUATIONS IN THE ORDER PARAMETER

In the Landau theory of phase transitions one assumes that the macroscopic state of the system can be described by a single macroscopic parameter which is called the order parameter ξ . The pairing-normal phase transition is a second-order phase transition in which the order parameter approaches zero smoothly at the critical temperature. In the thermodynamic limit the specific heat has a discontinuity at the phase transition. Ignoring quantum fluctuations the isothermal probability distribution of the order parameter is given by [9,7]

$$P(\eta) \propto e^{-\beta \Omega(\xi)} = e^{-\frac{V}{k_B T} \Omega'(\xi)}, \quad (3.1)$$

where $\Omega' = \Omega/V$ is the thermodynamic potential per unit volume of the system. The partition function in this formalism is given by

$$Z = \frac{1}{\alpha} \int_0^\infty e^{-\beta \Omega(\xi)} d\xi, \quad (3.2)$$

where α is a normalization constant.

In the thermodynamic limit ($N \rightarrow \infty$, $V \rightarrow \infty$, $n = N/V$ finite) and also at zero temperature only the state which minimizes the thermodynamic potential is accessible to the system. Thermodynamic fluctuations can be ignored and neglecting quantum fluctuations the system is described well by the mean-field formalism. However, for finite systems at finite temperature the average of the order parameter calculated from [9]

$$\bar{\xi} = \frac{\int_0^\infty \xi e^{-\beta \Omega(\xi)} d\xi}{\int_0^\infty e^{-\beta \Omega(\xi)} d\xi} \quad (3.3)$$

need no longer be equal to the most probable value, i.e., the value which minimizes Ω .

Traditionally the order parameter for a pairing system governed by a Hamiltonian of the form (1.1) is chosen to be the gap parameter defined by $\Delta = G \langle B \rangle$. For a constant pairing potential the gap is not momentum dependent. Within the mean-field formalism this parameter remains finite below the critical temperature T_c and it approaches zero at T_c .

However, for the case where the pairing potential is momentum dependent the gap parameter also becomes momentum dependent and it is thus no longer a single parameter defining the macroscopic state of the sys-

tem. Furthermore, in an exact calculation the expectation value of the gap parameter is identically zero for all temperatures. These considerations have led us to look for alternative choices for the order parameter.

Comparing the Hubbard-Stratonovich representation of the partition function (2.19) with the order parameter representation (3.2) we were led to considering the expectation value of the pairing potential for the order parameter. In the thermodynamic limit for the case of a constant pairing potential this is given by $\mathcal{G} = \frac{\Delta^2}{G}$.

Let us therefore look at the behavior of the expectation value of the pairing potential \mathcal{G} . In the thermodynamic limit this parameter has a similar behavior to that of the gap parameter Δ ; i.e., it remains finite below T_c and approaches zero at T_c . But unlike the gap parameter \mathcal{G} remains a single macroscopic parameter for a momentum-dependant pairing interaction. Furthermore, it is well defined in an exact treatment. It is interesting to note that in finite systems, unlike the gap parameter, \mathcal{G} remains finite above the critical temperature, even when one does not include statistical fluctuations.

Now, in order to calculate the partition function in the order-parameter representation (3.2) let us transform the Hamiltonian (1.1) into quasiparticle space

$$\begin{aligned} a_{\mathbf{k}\uparrow} &= u_{\mathbf{k}} c_{\mathbf{k}\uparrow} - v_{\mathbf{k}} c_{-\mathbf{k}\downarrow}^\dagger, \\ a_{-\mathbf{k}\downarrow} &= u_{\mathbf{k}} c_{-\mathbf{k}\downarrow} + v_{\mathbf{k}} c_{\mathbf{k}\uparrow}^\dagger. \end{aligned} \quad (3.4)$$

Here $a_{\mathbf{k}\sigma}^\dagger$ and $a_{\mathbf{k}\sigma}$ are the quasiparticle creation and annihilation operators and we have chosen the arbitrary phase factor acquired in the transformation such that $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are real. We require further that the quasi-particles obey Fermi statistics. This is equivalent to requiring $u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2 = 1$ and we can replace the two parameters $u_{\mathbf{k}}$, $v_{\mathbf{k}}$ by a single transformation parameter $x_{\mathbf{k}}$ defined by

$$\left. \begin{array}{l} u_{\mathbf{k}}^2 \\ v_{\mathbf{k}}^2 \end{array} \right\} = \frac{1}{2}(1 \pm x_{\mathbf{k}}). \quad (3.5)$$

Taking the expectation value of the transformed Hamiltonian we obtain

$$\mathcal{E} = \langle E \rangle = 2 \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \rho_{\mathbf{k}} - G \sum_{\mathbf{k}\mathbf{k}'} \tau_{\mathbf{k}}^* \tau_{\mathbf{k}'} - G \sum_{\mathbf{k}} \rho_{\mathbf{k}}^2, \quad (3.6)$$

where $\rho_{\mathbf{k}}$ is the single-particle density

$$\rho_{\mathbf{k}} = \langle c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} \rangle = 1 - x_{\mathbf{k}}(1 - 2f_{\mathbf{k}}) \quad (3.7)$$

with $f_{\mathbf{k}} \equiv \langle a_{\mathbf{k}\uparrow}^\dagger a_{\mathbf{k}\uparrow} \rangle = \langle a_{-\mathbf{k}\downarrow}^\dagger a_{-\mathbf{k}\downarrow} \rangle$ and $\tau_{\mathbf{k}}$ is the pairing tensor defined by

$$\tau_{\mathbf{k}} = \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle = \frac{1}{2} \sqrt{1 - x_{\mathbf{k}}^2} (1 - 2f_{\mathbf{k}}). \quad (3.8)$$

The third term in Eq. (3.6) tends to zero in the thermodynamic limit and in this case the expectation value of the pairing potential is simply given by $\mathcal{G} = \frac{\Delta^2}{G}$. On the other hand, in finite systems the third term in (3.6) does contribute and \mathcal{G} remains finite above the phase transition.

To calculate the partition function as a function of \mathcal{G} we minimize the free energy for each value of \mathcal{G} with respect to the transformation parameters $x_{\mathbf{k}}$ subject to the constraints that the expectation value of the pairing potential is given by

$$\mathcal{G} = G \left[\sum_{\mathbf{k}\mathbf{k}'} \tau_{\mathbf{k}}^* \tau_{\mathbf{k}'} + \sum_{\mathbf{k}} \rho_{\mathbf{k}}^2 \right] \quad (3.9)$$

and that the number of particles is given by \mathcal{N}

$$\mathcal{N} = \langle N \rangle = 2 \sum_{\mathbf{k}} \rho_{\mathbf{k}}. \quad (3.10)$$

Let

$$\begin{aligned} F' &= F - \mu \langle N \rangle - \lambda \mathcal{G} \\ &= \langle E \rangle - T \langle S \rangle - \mu \langle N \rangle - \lambda \mathcal{G}. \end{aligned} \quad (3.11)$$

Here λ and the chemical potential μ are the Lagrange multipliers for the constraints (3.9) and (3.10), respectively. The entropy is given by

$$\langle S \rangle = -2k_B \sum_{\mathbf{k}} [f_{\mathbf{k}} \ln f_{\mathbf{k}} + (1 - f_{\mathbf{k}}) \ln(1 - f_{\mathbf{k}})]. \quad (3.12)$$

We want to minimize F' with respect to the transformation parameters $x_{\mathbf{k}}$. But

$$\frac{dF'}{dx_{\mathbf{k}}} = \frac{\partial F'}{\partial x_{\mathbf{k}}} + \frac{\partial F'}{\partial f_{\mathbf{k}}} \frac{df_{\mathbf{k}}}{dx_{\mathbf{k}}} \quad (3.13)$$

has a minimum (and we assume in what follows that it is a global minimum) where both

$$\frac{\partial F'}{\partial x_{\mathbf{k}}} = 0 \quad (3.14)$$

and

$$\frac{\partial F'}{\partial f_{\mathbf{k}}} = 0. \quad (3.15)$$

From Eq. (3.14) we obtain an expression for the transformation parameters

$$x_{\mathbf{k}} = \frac{\epsilon'_{\mathbf{k}}}{\sqrt{\epsilon_{\mathbf{k}}'^2 + \Delta'^2}} \quad (3.16)$$

with

$$\epsilon'_{\mathbf{k}} = \epsilon_{\mathbf{k}} - G(1 + \lambda)\rho_{\mathbf{k}} \quad (3.17)$$

and

$$\Delta' = G(1 + \lambda) \sum_{\mathbf{k}} \tau_{\mathbf{k}} \quad (3.18)$$

and from (3.15) we obtain the quasiparticle occupation probabilities

$$f_{\mathbf{k}} = \frac{1}{1 + \exp(\beta \sqrt{\epsilon_{\mathbf{k}}'^2 + \Delta'^2})}. \quad (3.19)$$

We now calculate the free energy as a function of \mathcal{G} by

solving the coupled set of equations (3.9), (3.10), and (3.18) simultaneously for Δ' , μ , and λ for each value of \mathcal{G} .

IV. A SIMPLE MODEL CALCULATION

In order to compare the different methods we did the calculation in the $i_{13/2}$ model [9] where it is possible to do an exact grand canonical calculation. Similar to Goodman, we chose the number of nucleons as $\mathcal{N}=6$ and the pairing potential such that the zero-temperature BCS energy gap is given by $\Delta(T=0) = 1.0$ MeV. The single-particle energies in this model are given by

$$e_m = \kappa \frac{3m^2 - j(j+1)}{j(j+1)} \quad (4.1)$$

with $j = \frac{13}{2}$, $\kappa = 2$, and $m - \frac{1}{2}$ an even integer between $-j$ and j .

In Fig. 1 we plot the expectation value of the pairing potential. If one ignores the second term in (3.9) the expectation value of the pairing potential \mathcal{G} in the mean-field BCS goes to zero above the phase transition. However, if one includes this term \mathcal{G} remains finite for finite systems, even when one does not take into account thermodynamic fluctuations. Note that using \mathcal{G} for the order parameter one approximates the magnitude of the exact grand canonical results much better than when one uses the BCS energy gap Δ for the order parameter. The remaining discrepancy in the shape is mainly due to the neglect of quantum fluctuations.

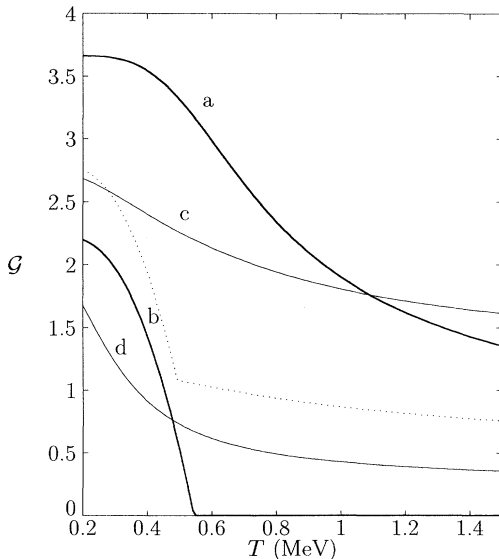


FIG. 1. The expectation value of the potential in the exact grand canonical treatment (thick solid line labeled *a*), in the standard mean-field BCS approach (thick solid line labeled *b*), in the mean-field BCS approach including the third term in Eq. (3.6) which vanishes in the thermodynamic limit (dotted line), and in the Landau treatments using the expectation value of the potential (thin solid line marked *c*), and the BCS energy gap (thin solid line marked *d*) for the order parameter.

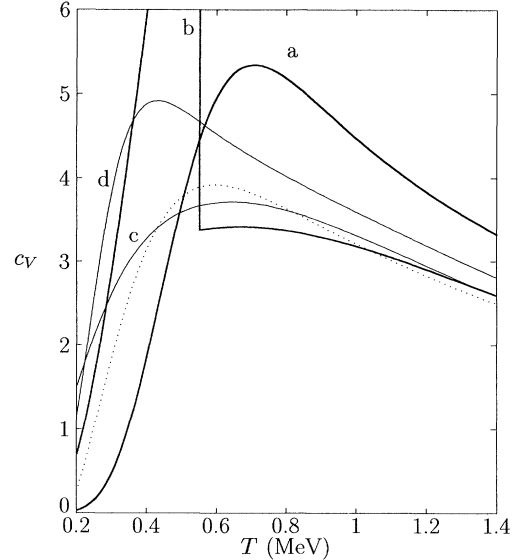


FIG. 2. The specific heat as a function of temperature. The thick solid line labeled *a* gives the exact grand canonical result, the thick solid line labeled *b* is the standard mean-field BCS result, the dotted line gives the result from the static path integral approach, the thin solid line labeled *c* is the result obtained from the Landau theory of phase transitions using the expectation value of the potential for the order parameter, and the thin solid line labeled *d* gives the result obtained from the Landau theory using the BCS gap for the order parameter.

In Fig. 2 we plot the specific heat as a function of temperature. The standard mean-field BCS treatment has a discontinuity at the phase transition. The static path approximation and the Landau treatment using \mathcal{G} for the order parameter give very similar results, especially close to the phase transition. They yield virtually the same critical temperature which is close to that in the exact grand canonical treatment. When one uses the BCS energy gap for the order parameter one reproduces the height of the peak in the specific heat reasonably well, but the critical temperature is far too low.

V. CONCLUSIONS

By comparing the static path approximation to the Hubbard-Stratonovich representation of the partition function with the form of the partition function based on Landau theory of phase transitions, we identify the expectation value of the pairing potential as a good choice for the order parameter. In both treatments one takes into account thermodynamic fluctuations, but not quantum fluctuations. We note that even in a mean-field treatment of finite systems where thermodynamic fluctuations are not taken into account, the expectation value of the pairing potential remains finite above the phase transition. It is, however, not clear how the results generalize for a more complicated effective interaction.

We find that using \mathcal{G} for the order parameter we obtain a value for the thermodynamic average of the expectation

value of the pairing potential which is closer to the exact grand canonical result than the value obtained if one uses Δ for the order parameter. Furthermore, the behavior of the specific heat is very similar to that obtained in a static path integral approach and the exact transition temperature is very well reproduced.

Finally we note that using the gap parameter Δ for the order parameter results in certain conceptual difficulties which are resolved if one uses the expectation value of the pairing potential for the order parameter. First, Δ is always exactly zero (on both sides of the phase transition) in an exact canonical calculation since it is defined by $\Delta = G \sum_{\mathbf{k}} \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$. Furthermore, if one uses a momentum-dependent pairing potential, the BCS en-

ergy gap Δ also becomes momentum dependent and is thus no longer a suitable parameter for describing the macroscopic state of the system. On the other hand \mathcal{G} is well defined and nonzero (at least below the phase transition) in an exact canonical calculation. Furthermore, it remains a single macroscopic parameter for momentum-dependent pairing potentials.

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