Quark condensate at finite baryon density

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We discuss a recently derived, model-independent relation that expresses the value of a medium-modified quark condensate in terms of the vacuum value of the condensate and the value of the nucleon sigma term, σ_N. Our goal is to calculate the value of the quark condensate in nuclear matter, $\langle \mathrm{NM}|\bar{q}(0)q(0)|\mathrm{NM}\rangle$, using some standard many-body techniques. Here, we comment on the mean-field calculations of Cohen, Furnstahl, and Griegel and others. We then calculate the value of the nuclear matter quark condensate using linear response theory. In a sigma-dominance model, the linear response calculation relates the modification of the vacuum condensate to the matrix element of the operator $\bar{q}q$ taken between a state of the sigma meson and the vacuum. (That matrix element may be used to define a sigma decay constant, f_σ.) We also provide some additional insight into the relation between the dynamics of the quark condensate and the scalar fields of relativistic nuclear physics.

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I. INTRODUCTION

In recent years there has been a growing interest in the behavior of vacuum condensates in the presence of matter. While the most useful results are probably obtained using QCD sum rules, simple models, such as the Nambu–Jona-Lasinio (NJL) model [1] and the Gell-Mann–Levy sigma model [2], have been used to study the dynamics of the quark condensate [3]. (We will return to a discussion of the NJL model at a later point in this work.)

We consider a model-independent relation that has been discussed by a number of authors [3–5]. If we denote the quark condensate in matter as $\langle \mathrm{NM}|\bar{q}q|\mathrm{NM}\rangle$, the model-independent relation reads

$$\frac{\langle \mathrm{NM}|\bar{q}q|\mathrm{NM}\rangle-\langle 0|\bar{q}q|0\rangle}{\langle 0|\bar{q}q|0\rangle}=-\frac{\sigma_N}{f_\pi^2 m_\pi^2}\rho_B \tag{1.1}$$

and is valid for sufficiently small density, ρ_B. Here, $\langle 0|\bar{q}q|0\rangle$ is the vacuum value of the condensate, f_π is the pion decay constant, σ_N is the nucleon sigma term, and ρ_B is the baryon density. To the extent that hadronic masses are proportional to the strength of the condensate, we also have [3]

$$\frac{m^*}{m_{\mathrm{vac}}}=1-\frac{\sigma_N}{f_\pi^2 m_\pi^2}\rho_B+\cdots , \tag{1.2}$$

where m^* is the in-medium value of the quark mass.

A proof of Eq. (1.1) is given in Ref. [3]. A more elementary derivation, that is adequate for our purposes, is obtained by writing

$$\langle \mathrm{NM}|\bar{q}q|\mathrm{NM}\rangle=\langle 0|\bar{q}q|0\rangle+\langle N|\bar{q}q|N\rangle\rho_B , \tag{1.3}$$

where $\langle N|\bar{q}q|N\rangle$ is the nucleon matrix element of the quark scalar density. We now introduce the nucleon sigma term

$$\sigma_N=-2m_0\left\langle N\left|\frac{\bar{u}u+\bar{d}d}{2}\right|N\right\rangle , \tag{1.4}$$

where m_0 is the average current quark mass of the up and down quarks. Thus

$$\langle \mathrm{NM}|\bar{q}q|\mathrm{NM}\rangle=\langle 0|\bar{q}q|0\rangle+\frac{\sigma_N}{m_0}\rho_B+\cdots . \tag{1.5}$$

We make use of the Gell-Mann–Oakes-Renner relation,

$$f_\pi^2 m_\pi^2=-m_0\langle 0|\bar{q}q|0\rangle , \tag{1.6}$$

to write Eq. (1.5) as

$$\langle \mathrm{NM}|\bar{q}q|\mathrm{NM}\rangle=\langle 0|\bar{q}q|0\rangle\left[1-\frac{\sigma_N}{f_\pi^2 m_\pi^2}\rho_B\right] , \tag{1.7}$$

which is equivalent to Eq. (1.1).

In this work we wish to understand how the result given in Eq. (1.7) is obtained when we apply linear response theory to this problem. Note that the matrix element in Eq. (1.3) is the expectation value of $\bar{q}q$ taken between physical ("dressed") nucleons. In order to apply linear response theory, we need to identify a perturbation, so that the response to that perturbation may be calculated in lowest order. To that end, it is useful to introduce "valence quarks." These quarks have a constituent mass but are not dressed by their coupling to mesonic modes. This distinction may be made in a precise fashion by using the Nambu–Jona-Lasinio (NJL) model, for example. The "dressing" of the quarks of the NJL model has been extensively described by Weise and collaborators [6]. In a recent work, we have extended their discussion (that dealt mainly with the sigma meson) to include a description of the dressing of the NJL quarks by pions, the Goldstone bosons of the NJL model [7].

Now, consider the addition of nuclear matter to the vacuum. If the vacuum is not polarized, the contribution to the scalar density of the nucleons in nuclear matter is

$\langle N|\bar{q}(0)q(0)|N\rangle_{\rm val}\rho_B$, where the matrix element $\langle N|\bar{q}(0)q(0)|N\rangle_{\rm val}$ is calculated using the valence quarks only. That matrix element is approximately equal to 3. For example, we have calculated that matrix element using a (covariant) nontopological soliton model, with the result $\langle N|\bar{q}(0)q(0)|N\rangle_{\rm val}=3(1-2\alpha)$ [8]. Here, α is the fraction of the normalization integral for the (relativistic) quark wave functions that is due to the presence of lower components. Thus, if α is small, the matrix element is approximately equal to 3.

We define

$$\sigma_N^{\rm val}=m_0\langle N|\bar{q}(0)q(0)|N\rangle_{\rm val}\ , \qquad (1.8)$$

where m_0 is the average value of the up and down quark masses ($m_0\sim5$ MeV). We now apply linear response theory to write

$$\langle {\rm NM}|\bar{q}(0)q(0)|{\rm NM}\rangle=\langle 0|\bar{q}q|0\rangle+\delta\langle 0|\bar{q}q|0\rangle+\frac{\sigma_N^{\rm val}}{m_0}\ , \qquad (1.9)$$

where $\delta\langle 0|\bar{q}q|0\rangle$ represents the response of the vacuum state due to the presence of matter. If the model-independent result is to be reproduced, we should have

$$\delta\langle 0|\bar{q}q|0\rangle=\left[\frac{\sigma_N}{m_0}-\frac{\sigma_N^{\rm val}}{m_0}\right]\rho_B\ . \qquad (1.10)$$

Since $\sigma_N\simeq45\pm8$ MeV [9], we see that the polarization correction in Eq. (1.10) is about *twice* the last term in Eq. (1.9). Combining the polarization response with the perturbation, we have

$$\delta\langle 0|\bar{q}q|0\rangle+\frac{\sigma_N^{\rm val}}{m_0}\rho_B=\frac{\sigma_N}{m_0}\rho_B\ . \qquad (1.11)$$

The two terms on the left in Eq (1.11) are obtained simultaneously, if one uses a sigma-dominance model to calculate the change in the value of the quark condensate in the presence of matter. For example, we may consider the calculation of σ_N in the Nambu–Jona-Lasinio (NJL) model [5–7]. There, one sees that $\sigma_N=\kappa\sigma_N^{\rm val}$, where the *enhancement factor* κ is

$$\kappa=\frac{1}{1-G_SJ_{SS}(0)}\ . \qquad (1.12)$$

Here, G_S is the coupling constant of the NJL model and $J_{SS}(q^2)$ is a quark loop integral [6]. The relation to the sigma meson propagator may be seen by writing the approximate relation [7]

$$-\frac{g_{\sigma qq}^2}{q^2-m_\sigma^2}\simeq\frac{G_S}{1-G_SJ_{SS}(q^2)}\ . \qquad (1.13)$$

Here, m_σ is the sigma mass and $g_{\sigma qq}$ is the coupling constant for sigma-quark coupling. [Equation (1.13) may be made exact by making $g_{\sigma qq}$ and m_σ, q^2 dependent. Indeed, a momentum-space bosonization of the NJL model will yield a momentum-dependent coupling constant and sigma mass, $g_{\sigma qq}(q^2)$ and $m_\sigma(q^2)$ [10].] We note that κ has a value of about 3 [7].

We see that the enhancement factor is then $\kappa=(-g_{\sigma qq}/G_S)(-g_{\sigma qq}/m_\sigma^2)$, where the factor $(-g_{\sigma qq}/G_S)$ is what is required to obtain the condensate value from the value of the sigma field generated by the quark. That may be readily understood by noting that the bosonization of the NJL model proceeds by writing [10]

$$\bar{\sigma}(x)=-\frac{G_S}{g_{\sigma qq}}[\bar{u}(x)u(x)+\bar{d}(x)d(x)] \qquad (1.14)$$

and then setting $\bar{\sigma}=f_\pi+\sigma$, where f_π is the vacuum value of $\bar{\sigma}$.

From this discussion, we see that the appearance of σ_N in Eq. (1.11) tells us that the "quark sea" of the nucleon is quite important in this problem, since $\kappa\simeq3$. In the absence of the "sea," we would have $\kappa=1$. (We note that lattice simulations of QCD indicate that the sea and valence quarks make comparable contributions to σ_N [11].)

We may use Eq. (1.11) to make contact with relativistic nuclear physics, where large (Lorentz) scalar fields appear. In that case, we may express the sum of the second and the third term of the right-hand side of Eq. (1.9) in terms of the scalar field of the Walecka model [12] or of relativistic Brueckner-Hartree-Fock theory [13]. We have

$$\langle {\rm NM}|\bar{q}q|{\rm NM}\rangle=\langle 0|\bar{q}q|0\rangle-\frac{g_{\sigma qq}\sigma}{G_S} \qquad (1.15)$$

with $\sigma<0$. Note that $\langle 0|\bar{u}u|0\rangle=\langle 0|\bar{d}d|0\rangle\simeq(-250\ {\rm MeV})^3$, so that $\langle 0|\bar{q}q|0\rangle=2(-250\ {\rm MeV})^3=-0.031\ {\rm GeV}^3$. If we put $g_{\sigma qq}=2.58$, $G_S=7.91\ {\rm GeV}^{-2}$ [7], and note that $\sigma\simeq-36$ MeV in relativistic nuclear physics, we have $-g_{\sigma qq}\sigma/G_S\simeq0.012\ {\rm GeV}^3$. Thus the second term in Eq. (1.3) reduces the value of the vacuum condensate by about 38%. Note also that, with $\sigma_N=45$ MeV, $m_0=5$ MeV, and $\rho=0.17\ {\rm fm}^{-3}$, the second term in Eq. (1.5) is $\sigma_N\rho_B/m_0=0.012\ {\rm GeV}^3$, and also describes a 38% reduction in the value of the condensate. We see that the (Lorentz) scalar field of relativistic nuclear physics may be understood as providing a measure of the reduction of the value of the condensate from its vacuum value [14]. [See Eq. (1.3).]

II. MEAN-FIELD CALCULATIONS

Here we consider some mean-field calculations, made using the Nambu–Jona-Lasinio model, with the aim of obtaining the dependence of the condensate on the baryon density [3,5]. (The nature of these calculations will be clarified somewhat by the discussion of Sec. III, where we again consider linear response theory.)

A finite-density gap equation for the NJL model may be written in several forms depending upon the type of cutoff used [15]. For example, let

$$m^*=-G_S\langle {\rm NM}|\bar{q}q|{\rm NM}\rangle \qquad (2.1)$$

be the quark mass in the presence of nuclear matter. Then one may write [3]

$$m^* = N_c N_f G_S \left\{ \int_0^\Lambda \frac{d^4k_E}{(2\pi)^4} \frac{m^*}{[k_E^2+(m^*)^2]^{1/2}} - \frac{1}{2} \int_0^{k_F} \frac{d\mathbf{k}}{(2\pi)^3} \frac{m^*}{[\mathbf{k}^2+(m^*)^2]^{1/2}} \right\}, \quad (2.2)$$

where the first integral is evaluated in a Euclidean momentum space with a momentum cutoff Λ. The last term depends upon the choice of a Fermi momentum for the quarks, k_F, a quantity that is ultimately related to the density of nucleons. Equation (2.2) is clearly a *mean-field* result. The meaning of such an equation is probably more clearly seen if one uses the same cutoff procedure for both the integrals over the Dirac "sea" and the positive-energy states [15,16]. For example, Eq. (5.18) of Ref. [15] reads

$$m^* = m_0 + 4(N_c N_f + \tfrac{1}{2}) \frac{G_S}{2} \left[\int_0^\Lambda \frac{d\mathbf{p}}{(2\pi)^3} \frac{m^*}{E_p} - \int_0^{k_F} \frac{d\mathbf{p}}{(2\pi)^3} \frac{m^*}{E_p} \Theta(|k_F|-|\mathbf{p}|) \right], \quad (2.3)$$

where m_0 is the current quark mass, and where the $\frac{1}{2}$ that accompanies $N_c N_f$ in Eq. (2.3) arises from a Fierz rearrangement of exchange matrix elements. Further, $E_p = [\mathbf{p}^2+(m^*)^2]^{1/2}$. If one considers a sum of discrete values of the momentum, one sees a pairwise cancellation between the (negative) contribution of the negative-energy states and the (positive) contributions of the positive-energy states to the value of the condensate.

It is also possible to assume that the positive-energy states are organized into "nucleons." Then, the new value of the condensate, or mass parameter, is obtained by solving a gap equation of the form

$$m^* = m_0 + 4(N_c N_f + \tfrac{1}{2}) G_S \left[\frac{1}{2} \int_0^\Lambda \frac{d\mathbf{p}}{(2\pi)^3} \frac{m^*}{E_p} \right] - G_S \rho_B \frac{\sigma_N^{\text{val}}}{m_0} \quad (2.4)$$

$$= m_0 + \frac{G_S}{2} [N_c N_f + \tfrac{1}{2}] \frac{m^*}{\pi^2} \{ \Lambda \sqrt{(m^*)^2+\Lambda^2} - (m^*)^2 \ln[\Lambda + \sqrt{(m^*)^2+\Lambda^2} \} - G_S \rho_B \frac{\sigma_N^{\text{val}}}{m_0} . \quad (2.5)$$

Alternatively, if the covariant cutoff is used, we have

$$m^* = m_0 + \frac{3m^*}{2\pi^2} G_S \left[\Lambda^2 - (m^*)^2 \ln \left[1 + \frac{\Lambda^2}{(m^*)^2} \right] \right] - \frac{\sigma_N^{\text{val}}}{m_0} G_S \rho_B . \quad (2.6)$$

The approach adopted in Ref. [3] was to adjust the parameters of the NJL model so that the model-independent result, Eq. (1.1), was reproduced at low density. The NJL model was then used to estimate corrections to the model-independent result at larger values of the density. (In general, this procedure has the unsatisfactory feature of taking the parameters m_0, Λ, and G_S away from the values that gave good fits to f_π and m_π, when using the NJL model.) In the next section, we discuss the modification of the condensate value using linear response theory. That calculation is somewhat less model dependent than the mean-field results [3,5] described here. For example, the mean-field result is based upon the description of the vacuum, or the vacuum plus quark matter, as an uncorrelated Fermi gas. In the linear response theory, one may, in principle, consider a vacuum state with a complex correlation structure.

III. LINEAR RESPONSE THEORY

In the last section we saw that it is possible to calculate the density dependence of the quark condensate using simple field-theoretic models. An alternate scheme is to use linear response theory to calculate the modification of the vacuum condensate in the presence of matter. The perturbation in this case is the *valence* quark scalar density of the quark matter added to the vacuum. Thus

$$H_{\text{pert}} = -\frac{\sigma_N^{\text{val}}}{m_0} G_S \rho_B [\bar{q}(x) q(x)] , \quad (3.1)$$

where G_S is the coupling constant that relates the value of the condensate to the constituent quark mass. The value of that parameter may be taken from the NJL model.

For example, consider the part of the Lagrangian of the NJL model that contains the mass term generated by the vacuum value of $\bar{q}(x)q(x)$. Supplemented by the perturbation given in Eq. (3.1), we have

$$\mathcal{L}(x) = \bar{q}(x) \left[i\gamma^\mu \partial_\mu - m_q + \frac{\sigma_N^{\text{val}}}{m_0} G_S \rho_B \right] q(x) + \cdots , \quad (3.2)$$

where $m_q = -G_S \langle 0|\bar{q}q|0\rangle$. Note that the last term of Eq. (3.2) represents about a 10% reduction of the mass parameter, m_q, at nuclear matter density. That is equivalent to a 10% reduction in the magnitude of the condensate. Since, in a theory of this type, we anticipate a 30–40 % reduction of the quark condensate at nuclear matter densities, we need to consider the linear response arising from the perturbation in Eq. (3.1) or (3.2). As we will see, the linear-response analysis will relate the modification of the condensate value to the decay constant of the sigma meson to be defined below.

Now, we have for the response to the perturbation [17]

$$\delta\langle 0|\bar{q}q|0\rangle = -i\left[\frac{\sigma_N}{m_0}G_S\rho_B\right] \times\int_{-\infty}^{0} dt'\int d\mathbf{x}'\langle 0|[\bar{q}(x')q(x'),\bar{q}(0)q(0)]|0\rangle\ . \tag{3.3}$$

Equation (3.3) may be evaluated by using the relation

$$\bar{q}(x)q(x)=e^{i(Ht-\mathbf{P}\cdot\mathbf{x})}\bar{q}(0)q(0)e^{-i(Ht-\mathbf{P}\cdot\mathbf{x})}\ , \tag{3.4}$$

after inserting a complete set of states between the operators $\bar{q}(x)q(x)$ and $\bar{q}(0)q(0)$. Since a calculation of σ_N that uses the sigma-dominance model [6,7] appears to be satisfactory, we will assume the intermediate state of a zero-momentum σ meson is most important. Justification of this procedure may be found in our recent work [18]. There we inserted physical two-pion states between the operators $\bar{q}(x)q(x)$ and $\bar{q}(0)q(0)$ and calculated the resulting matrix elements to one-loop order, making use of a generalized NJL model. We calculated the imaginary part of the correlator that arises from the coupling to the two-pion continuum and then obtained the real part by means of a dispersion relation. It was then found that for spacelike q^2 ($q^2\leq 0$), an expression based upon sigma dominance provided an excellent representation of the (scalar-isoscalar) correlator. In our model, there is no *physical*, low-mass sigma to be found for timelike q^2, a result in accord with the absence of such a meson in the data tables.

Once we are convinced that the sigma-dominance model is useful, we proceed by denoting a state of the sigma meson as $|\sigma,\mathbf{p}=0)$, and use the normalization condition

$$(\sigma,\mathbf{p}'|\sigma,\mathbf{p})=(2\pi)^3 2\omega(\mathbf{p})\delta^{(3)}(\mathbf{p}-\mathbf{p}')\ . \tag{3.5}$$

We then define

$$(\sigma,\mathbf{p}=0|\bar{q}(0)q(0)|0)=F^2\ . \tag{3.6}$$

We have

$$\delta\langle 0|\bar{q}(0)q(0)|0\rangle=(2)\frac{F^4}{2m_\sigma^2}\left[\frac{\sigma_N^{\rm val}}{m_0}G_S\rho_B\right]\ , \tag{3.7}$$

where the factor of 2 on the right-hand side of Eq. (3.7) arises because each term of the commutator in Eq. (3.2) contributes equally.

Now, upon using Eq. (3.7), we have

$$\langle {\rm NM}|\bar{q}q|{\rm NM}\rangle=\langle 0|\bar{q}q|0\rangle+G_S\left[\frac{\sigma_N^{\rm val}}{m_0}\rho_B\right]\frac{F^4}{m_\sigma^2} +\frac{\sigma_N^{\rm val}}{m_0}\rho_B\ , \tag{3.8}$$

where, as before, the last term in Eq. (3.8) is the contribution of the valence quarks to the scalar density. Equation (3.8) may be written as

$$\langle {\rm NM}|\bar{q}q|{\rm NM}\rangle=\langle 0|\bar{q}q|0\rangle+\frac{\sigma_N^{\rm val}}{m_0}\rho_B\left[1+\frac{G_SF^4}{m_\sigma^2}\right]\ . \tag{3.9}$$

TABLE I. Results of the calculation of the factor $(1+G_SF^4/m_\sigma^2)$ are shown. Parameters of the NJL model determined in Ref. [7] are given, with Λ being the cutoff in Euclidean momentum space. Models A and B refer to Table I of Ref. [7], while parameters for model C are in Table II of that reference. For model D we put $m_q\simeq m_N/3$ and have used $m_\sigma=550$ MeV, which is a value often used in the one-boson-exchange model of the nucleon-nucleon interaction. (Note that $F^2=m_\sigma f_\sigma$.)

Model	A	B	C	D
Λ (GeV)	1.00	1.05	1.00	1.00
G_S (GeV^{-2})	7.91	6.97	7.91	7.91
m_q (GeV)	0.260	0.245	0.302	0.310
m_σ (GeV)	0.519	0.489	0.603	0.550
F^2 (GeV^2)	0.240	0.265	0.237	0.263
f_σ (GeV)	0.462	0.542	0.393	0.478
$(1+G_SF^4/m_\sigma^2)$	2.69	3.04	2.64	2.81
κ	3.12	3.57	2.50	

To obtain the model-independent result, we should have

$$\sigma_N=\sigma_N^{\rm val}\left[1+\frac{G_SF^4}{m_\sigma^2}\right]\ , \tag{3.10}$$

so that, upon using the Gell-Mann–Oakes-Renner relation, we regain Eq. (1.7). We see that, for consistency, we should have $(1+G_SF^4/m_\sigma^2)=\kappa$. If that were the case, the last term in Eq. (3.9) would equal $\sigma_N\rho_B/m_0$. (We recall that κ was approximately equal to 3.)

In Table I we present the results of a calculation of $(1+G_SF^4/m_\sigma^2)$ made using the NJL model. We make use of the parameter sets determined in Ref. [7]. (Some details of the calculation are given in the Appendix.) From Table I, we see that $(1+G_SF^4/m_\sigma^2)$ is reasonably close to κ for the various parameter sets considered. Rather than working with F^2, it may be useful to define a sigma decay constant, f_σ, such that $F^2=m_\sigma f_\sigma$. In that case, Eq. (3.10) reads $\sigma_N=\sigma_N^{\rm val}(1+G_Sf_\sigma^2)$, etc. Values of f_σ are also given in Table I.

Finally, we once again note that the (Lorentz) scalar fields of relativistic nuclear physics [12,13] may be related to the change in the value of the quark condensate in the presence of matter, $\sigma_N\rho_B/m_0$. We have

$$\sigma=-\frac{G_S}{g_{\sigma qq}}\left[\frac{\sigma_N}{m_0}\rho_B\right] \tag{3.11}$$

as may be inferred from the use of Eqs. (1.14) and (1.15). With $G_S=7.91$ GeV^{-2}, $\sigma_N/m_0\simeq 9$, and $g_{\sigma qq}=2.58$, we have $\sigma\simeq -36$ MeV at nuclear matter density, where $\rho_{\rm NM}=0.17$ fm$^{-3}=(109$ MeV$)^3$.

IV. DISCUSSION AND CONCLUSIONS

In this work we have clarified the meaning of Eq. (1.1) and provided an alternate derivation to that given in Ref. [3], for example. We have seen that Eq. (1.3) organizes the scalar density of the valence quarks and the induced vacuum polarization effects into a single term. By separating these contributions as in Eq. (1.9), we can apply standard many-body techniques to the system as a whole. This procedure clarifies the nature of the mean-field calculations of the modification of the vacuum condensate value in the presence of matter.

For relatively small perturbations, the contribution of vacuum polarization to the scalar density may be calculated using linear response theory. Here we assumed that the excitation of the sigma meson of the NJL model saturated the response and we saw a reasonably consistent picture emerged for the various methods of calculation. We also saw that the effects under consideration could be evaluated by calculating the vacuum polarization induced by a single nucleon using a sigma dominance model. (In that case, σ_N described the *sum* of the valence quark contribution and the vacuum polarization induced by a single nucleon.) However, for the term linear in ρ_B, we saw we could calculate the vacuum polarization induced by all the nucleons using linear response theory. The linear response theory describes the response to a perturbation based upon the fully correlated ground state of the Hamiltonian, while the mean-field analysis describes the vacuum, or the vacuum plus quark matter, as an uncorrelated Fermi gas. Because of that, one may argue that the application of linear response theory is somewhat less model dependent than the mean-field analysis described in Sec. II.

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APPENDIX

In this appendix we describe the calculation of the constant, F^2, defined in Eq. (3.6). We use the wave function of the sigma meson obtained in the NJL model [7].

We define the vertex for sigma coupling to a quark and antiquark to be $\Gamma = iN/\sqrt{n_c}$, where $n_c = 3$ is the number of colors, and N is to be found by normalizing the wave function appropriately. We have

$$F^2 = (-1)i^3 N\sqrt{n_c} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{\not k - m_q}\frac{1}{\not p + \not k - m_q}\right], \tag{A1}$$

where p^μ is the momentum of the sigma meson. [In the rest frame $p^\mu = (m_\sigma, \mathbf{0})$.] We evaluate the integral for $m_\sigma < 2m_q$ by passing to a Euclidean momentum space, with a momentum cutoff Λ. The result is

$$F^2 = \frac{N\sqrt{n_c}}{2\pi^2}\int_0^1 dx \left\{\left[\Lambda^2 - a^2 \ln\left[\frac{\Lambda^2+a^2}{a^2}\right]\right] - 2a^2\left[\ln\left[\frac{\Lambda^2+a^2}{a^2}\right] - \frac{\Lambda^2}{\Lambda^2+a^2}\right]\right\}, \tag{A2}$$

with $a^2 = m_q^2 - x(1-x)p^2 = m_q^2 - x(1-x)m_\sigma^2$. Typical values for Λ, m_q, and m_σ are found in Table I.

We now have to provide a value for the factor N. That may be done by calculating a "form factor" at zero momentum transfer, $F^\mu(0)$. We define

$$2\omega(\mathbf{p})F^\mu(0) = 2(iN)^2(-1)i^3 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[S(k)\gamma^\mu S(k) S(-p+k)], \tag{A3}$$

where the factor of 2 arises from an isospin trace.

With reference to Eq. (A3), we can see that, if the Dirac matrix γ^μ were inserted in *both* quark lines, when calculating the form factor, we would have $F^\mu(0) = 0$. That is a statement that the sigma meson carries zero baryon number. However, one may still normalize the sigma meson state, even if there is no conserved quantum number, such as the charge, to determine the normalization. The procedure to adopt in this case has been discussed by several authors [19], and it may be seen that our normalization is equivalent to that defined by those authors in the case of a $q\bar{q}$ system.

We have

$$2\omega(\mathbf{p})F^\mu(0) = 2N^2 i \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\frac{[(\not k + m_q)\gamma^\mu(\not k + m_q)(-\not p + \not k + m_q)]}{(k^2 - m_q^2)^2[(-p+k)^2 - m_q^2]} \tag{A4}$$

$$= 2N^2 i \int \frac{d^4k}{(2\pi)^4} \frac{4p^\mu(k^2 - m_q^2) - 4k^\mu[2(k\cdot p) - (k^2 + 3m_q^2)]}{(k^2 - m_q^2)^2[(-p+k)^2 - m_q^2]} \tag{A5}$$

$$= N^2 I(p^2) p^\mu, \tag{A6}$$

where Eq. (A6) serves to define the quantity $I(p^2)$. Recalling the normalization of our states, given in Eq. (3.5), we have $F^0(0) = 1$, or

$$N = \frac{2}{I(m_q^2)}. \tag{A7}$$

The integral of Eq. (A5) is again evaluated by transforming to a Euclidean momentum space, with cutoff Λ. We define

$$I(p^2) = J_1(p^2) + J_2(p^2),$$

and find

$$J_1(p^2) = -8i \int_0^1 dx\, I_2(A) , \tag{A8}$$

with $A = m_q^2 - x(1-x)p^2$ and

$$I_2(A) = \frac{i}{16\pi^2} \left[\ln \left[\frac{\Lambda^2 + A}{A} \right] - \frac{\Lambda^2}{\Lambda^2 + A} \right] . \tag{A9}$$

Also,

$$J_2(p^2) = -16i \int_0^1 x\, dx \{ (1 - \tfrac{3}{2}x)[I_2(A) + AI_3(A)] + (1-x)[3m^2 - (1-x^2)p^2] I_3(A) \} , \tag{A10}$$

with

$$I_3(A) = -\frac{i}{32\pi^2} \left[\frac{1}{A} - \frac{3\Lambda^2 + A}{(\Lambda^2 + A)^2} \right] . \tag{A11}$$

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