Pion absorption on ^{3}He. II. Antisymmetrization and angular decomposition of the Faddeev-based amplitude

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In order to develop a procedure for calculation of pion absorption on ^{3}He, we perform the decomposition of the absorption amplitude in three-nucleon partial waves, with full inclusion of antisymmetrization effects among the nucleons. Within this formalism, the matrix elements of the Δ-rescattering term, believed to be the dominant elementary absorption mechanism, are evaluated and embedded in the full (Faddeev-based) nuclear dynamics. This implies the treatment of the $3 \leftarrow 3$ and $2 \leftarrow 3$ processes in the final state. Since no attempt is made to include distortions due to the four-body (πNNN) dynamics, the trinucleon bound-state wave function is the only requirement in the initial state.

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I. INTRODUCTION

The advent of the meson factories at LAMPF, TRIUMF, and PSI has given us an important probe for high-precision studies in low- and intermediate-energy nuclear physics. The fact that the pion can be absorbed and emitted by nucleons and is the carrier of the longest-range part of the nucleon force presents theoretical investigations of pion reactions with both challenges and opportunities. A pion cannot be absorbed on a single, isolated nucleon, due to energy and momentun mismatch. Therefore in nuclei, single-pion absorption is strongly suppressed, and the dominant absorption process is on two nucleons. Thus, this mechanism has the potential to provide detailed information on short-range nucleon-nucleon (NN) correlations in nuclei.

The simplest system containing two nucleons is the deuteron. For this reason, there has been an extensive investigation, both experimental [1] and theoretical [2], on pion scattering and absorption by deuterons. Early work on this system was plagued by inconsistencies between different theories and the inability of most theoretical calculations to explain the data, especially data involving polarization observables. More recently, much of the theoretical controversy has subsided and a better understanding of the theory of these processes has emerged [3]. However, the original aim of learning more about short-range correlations has not been met [4]. This, in part, is due to the fact that, at least at energies available at the meson factories, pion absorption on two nucleons proceeds via the intermediate excitation of a Δ-resonance, followed by the $NN \leftarrow \Delta N$ transition. This mechanism masks the NN correlations [5, 6]. Instead, much of the focus now is on the reaction mechanism and the Δ resonance in nuclei.

The three-nucleon system is, potentially, a richer testing ground for studies of pion scattering and absorption. The experimental situation has recently been reviewed by Weyer [7]. This system is sufficiently more complicated than the deuteron, in that it has more than one bound state (^{3}H and ^{3}He), allows absorption on both isoscalar and isovector pairs, and supports absorption mechanisms where all three nucleons are involved. In the kinematical regions where the two-body absorption peaks are clearly discernable, much attention has been devoted to observables depending sensitively on the weaker s- and p-wave rescattering terms without intermediate isobar propagation, since these terms are believed to be more sensitive to the short-range NN correlations than the dominating (deuteronlike) two-nucleon absorption mechanism with Δ excitation. Such observables are, for example, the proton polarization from π^+ absorption on ^{3}He [5] and the isospin ratio $R = \sigma(\pi^+, pp)/\sigma(\pi^-, pn)$ [8–10].

Much of the interest in pion absorption, however, recently resulted from experimental indications that up to 30% of the total absorption cross section may be ascribed to mechanisms involving all three nucleons, with the available three-body phase space filled completely with a constant event density [11]. The existence of a significant three-nucleon absorption component has been established very recently also for ^{3}H in Ref. [12]. This fraction has been measured in other light nuclei as well (^{6}Li, ^{7}Li, and ^{9}Be) [13], with roughly the same result (namely, a value between $\frac{1}{4}$ and $\frac{1}{2}$ of the cross section may be ascribed to three-nucleon absorption mechanisms). At the present stage, the nature and magnitude of such a multinucleon absorption process, which exhibits a Δ resonancelike dependence upon the pion energy, still remain unexplained [14]. Although it is generally believed

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that the body of data indicates a new elementary pion-absorption mode on three nucleons, there is no unequivocal experimental signature for the absorption on three nucleons that would be distinguishable from the multistep effects involving initial- or final-state interactions. This could only be settled by Faddeev-theory-based calculations. Under this point of view, the π-^{3}He system is still sufficiently simple to be within reach of an "exact" calculation using modern few-body Faddeev-type techniques.

In the first paper of this series [15], hereafter denoted by I, a model for a Faddeev-based calculation of the π-^{3}He absorption process was introduced. Elementary pion-absorption amplitudes were embedded in the full three-nucleon dynamics, consisting of the initial-state breakup of the trinucleon in the presence of the pion, and the complete final-state interactions among the three nucleons. An exact description of the pion-nucleon correlations in the initial state, which would have required a unitary treatment for the coupled πNNN-NNN systems was not attempted, however. Such a treatment has been formally derived within the isobar separable model by Avishai and Mizutani [16]; the solution of the resulting equations, however, presents a formidable task, and consequently, we excluded such initial-state πN correlations in the Faddeev-based amplitudes of paper I.

The aim of this paper is still technical, in that we carry the work of I further towards a practical calculation scheme, by developing the detailed antisymmetrized and angular-momentum decomposed version of the Faddeev-based absorption amplitude (which was the final result of paper I). In particular, in Sec. II we treat the three nucleons according to the Pauli principle, deriving antisymmetrized Faddeev-based absorption amplitudes. The resulting amplitudes are then decomposed in three-nucleon partial waves (Sec. III), with the major emphasis given to the evaluation of the matrix elements of the (dominant) Δ-rescattering mechanism. Section IV summarizes the main results contained herein. Future work will analyze the intricacies of the numerical solution of the Faddeev equations for the $3 \leftarrow 3$ and $2 \leftarrow 3$ processes, and present results of calculations, both in a simplified model and for realistic ^{3}He wave functions and NN interactions.

II. ANTISYMMETRIZED ABSORPTION AMPLITUDES

In order to describe the absorption process on ^{3}He we treat the nucleons as a system of identical fermions (with use of the isospin formalism). We follow as closely as possible the classic treatment [17] for the standard three-identical-fermion problem. The absorption formalism for distinct particles has been developed in paper I.

The generalized Pauli principle dictates the total wave function to be antisymmetric under the simultaneous exchange of space (r), spin (σ), and isospin (τ) coordinates of any pair of nucleons. By labeling the nucleons as $(1,2,3)$, the particle-exchange operator for the nucleon pair $(2,3)$ is indicated as

$$P_{23} \equiv P_{23}^r P_{23}^\sigma P_{23}^\tau. \tag{2.1}$$

The two operators

$$P_2 \equiv P_{12}P_{23}, \quad P_3 \equiv P_{13}P_{23} \tag{2.2}$$

perform the cyclic permutations $(123) \rightarrow (231)$ and $(123) \rightarrow (312)$, respectively, and denoting with P_1 the identity operator $\mathbf{1}$ [i.e., $(123) \rightarrow (123)$], the idempotent symmetrizer is finally obtained:

$$\mathcal{S} = \frac{P_1 + P_2 + P_3}{3}. \tag{2.3}$$

An alternative form frequently employed for $\mathcal{S}$ is

$$\mathcal{S} = \frac{\mathbf{1} + P}{3}, \tag{2.4}$$

with

$$P = P_2 + P_3. \tag{2.5}$$

The three-body state $|\phi\rangle_{\mathcal{S}}$, fully antisymmetrized with respect to the permutations of *any* pair of particles, can be obtained by means of the operator $\mathcal{S}$,

$$|\phi\rangle_{\mathcal{S}} = \mathcal{S}|\phi\rangle_1, \tag{2.6}$$

where the three-body state $|\phi\rangle_1$ is assumed to be antisymmetrized with respect to one nucleon pair (23) only (however, any pair may be chosen).

We may now consider the pion-$3N$ absorption amplitude

$$A^{\rm tot} = {}_{\mathcal{S}}\langle\psi^{(-)}|\mathcal{A}|\psi_{\rm BS}\rangle|\mathbf{P}_0^\pi\rangle, \tag{2.7}$$

where the initial three-nucleon bound-state wave function $|\psi_{\rm BS}\rangle$ and the final scattering state ${}_{\mathcal{S}}\langle\psi^{(-)}|$ have been antisymmetrized. Also, due to the idempotent character of the symmetrizer one may consider antisymmetrization only in one of the two nucleonic wave functions.

The last state has (ingoing) boundary conditions describing final channels with either three free nucleons or the deuteron-nucleon clusterization, when it occurs.

We will discuss the formalism assuming that the elementary absorption operator $\mathcal{A}$ is specified by the Δ-rescattering mechanism (Fig. 1). It is clear that this mechanism, although manifestly dominant, is not sufficient for an acceptable description of pion absorption around the Δ resonance [18–20]. However, the inclusion of other two-body or one-body elementary absorption mechanisms (or even three-body mechanisms) may be derived by similar procedures.

In the case of the two-body mechanism of Fig. 1, we note that there are six different contributions $\mathcal{A}(\beta \leftarrow \alpha)$ depending on which nucleon (α) undergoes the Δ ex-

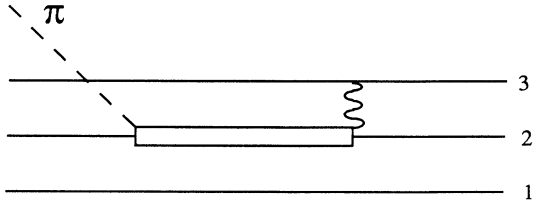

FIG. 1. Δ-rescattering mechanism for ^{3}He. The figure represents our conventions for the $\hat{\mathcal{A}}_1$ operator.

citation and which remaining nucleon (β) absorbs the rescattered meson.

If we define as $\hat{\mathcal{A}}_1$ the term $\mathcal{A}(3 \leftarrow 2)$, we can write all the remaining terms as

$$\mathcal{A}(1 \leftarrow 3) = P_2 \hat{\mathcal{A}}_1 P_3, \tag{2.8a}$$

$$\mathcal{A}(2 \leftarrow 1) = P_3 \hat{\mathcal{A}}_1 P_2, \tag{2.8b}$$

$$\mathcal{A}(2 \leftarrow 3) = P_{23} \hat{\mathcal{A}}_1 P_{23}, \tag{2.8c}$$

$$\mathcal{A}(3 \leftarrow 1) = P_{23} P_3 \hat{\mathcal{A}}_1 P_2 P_{23}, \tag{2.8d}$$

$$\mathcal{A}(1 \leftarrow 2) = P_{23} P_2 \hat{\mathcal{A}}_1 P_3 P_{23}. \tag{2.8e}$$

By virtue of the antisymmetry property of the initial and final nucleonic states, Eq. (2.7) can be expressed in terms of one elementary contribution only:

$$A^{\rm tot} = 6\ {}_{\mathcal{S}}\langle \psi^{(-)} | \hat{\mathcal{A}}_1 | \psi_{\rm BS} \rangle | \mathbf{P}_0^{\pi} \rangle. \tag{2.9}$$

The antisymmetrized scattering wave function describing three interacting particles in the final state ${}_{\mathcal{S}}\langle \psi^{(-)}|$ is expressed by means of the asymptotic three-nucleon plane-wave ${}_1\langle \phi_0|$ [where 1 denotes antisymmetrization with respect to the pair (23)]:

$${}_{\mathcal{S}}\langle \psi^{(-)}| = \frac{1}{3} \{ {}_1\langle \phi_0 | (\mathbf{1} + P) + {}_1\langle \phi_0 | U^{\mathcal{S}}_{3 \leftarrow 3} G_0 \}. \tag{2.10}$$

Here, $U^{\mathcal{S}}_{3 \leftarrow 3}$ is the symmetrized Faddeev-Alt-Grassberger-Sandhas (AGS) transition operator for the $3 \leftarrow 3$ process, which may be related to the symmetrized $2 \leftarrow 2$ transition operator $U^{\mathcal{S}}$ by the use of the expression

$$U^{\mathcal{S}}_{3 \leftarrow 3} = (\mathbf{1} + P)[T + T G_0 U^{\mathcal{S}} G_0 T](\mathbf{1} + P). \tag{2.11}$$

The operators T and G_0 represent the NN transition matrix and the free propagator, respectively, with detailed matrix elements given by Eqs. (3.35) and (3.36).

For the absorption process with two fragments ($d+N$) in the final state, the antisymmetrized final-state wave function may be written in terms of the channel state ${}_1\langle \phi_d|$, describing the free motion of the spectator (nucleon 1) plus the antisymmetrized (deuteron) bound state of the pair 23, by means of

$${}_{\mathcal{S}}\langle \psi_d^{(-)}| = \frac{1}{3}\ {}_1\langle \phi_d | U^{\mathcal{S}}_{2 \leftarrow 3} G_0, \tag{2.12}$$

where we have indicated with $U^{\mathcal{S}}_{2 \leftarrow 3}$ the antisymmetrized $(2 \leftarrow 3)$ AGS transition operator. This $(2 \leftarrow 3)$ operator is then related to the $(2 \leftarrow 2)$ one through

$$U^{\mathcal{S}}_{2 \leftarrow 3} = [G_0^{-1} + U^{\mathcal{S}} G_0 T](\mathbf{1} + P), \tag{2.13}$$

and the $(2 \leftarrow 2)$ operator is the solution of the symmetrized AGS equation

$$U^{\mathcal{S}} = G_0^{-1} P + P T G_0 U^{\mathcal{S}}. \tag{2.14}$$

It is convenient [21] to introduce the operator $\mathcal{T} \equiv U^{\mathcal{S}} G_0 T$, since we can then write the two final-state wave functions (2.10) and (2.12) as

$${}_{\mathcal{S}}\langle \psi^{(-)}| = {}_1\langle \phi_0 | \{ \mathbf{1} + (\mathbf{1} + P) T G_0 + (\mathbf{1} + P) T G_0 \mathcal{T} G_0 \} \mathcal{S}\ , \tag{2.15}$$

$${}_{\mathcal{S}}\langle \psi_d^{(-)}| = {}_1\langle \phi_d | \{ \mathbf{1} + \mathcal{T} G_0 \} \mathcal{S}, \tag{2.16}$$

respectively. $\mathcal{T}$ is the solution of the Faddeev-like equation

$$\mathcal{T} = PT + PTG_0\mathcal{T}. \tag{2.17}$$

The solution of this single three-body equation allows one to take into account the final-state three-body dynamics with both two and three particles in the asymptotic channel.

Inclusion of the result (2.15) into (2.9) allows one to disentangle the various contributions to the absorption process with different levels of connectivity. Indeed, we can write

$$A^{\rm tot} = A^{\rm I} + A^{\rm II} + A^{\rm III} \tag{2.18}$$

with

$$A^{\rm I} = 2\ {}_1\langle \phi_0 | (\mathbf{1} + P) \hat{\mathcal{A}}_1 | \psi_{\rm BS} \rangle | \mathbf{P}_0^{\pi} \rangle, \tag{2.19}$$

$$A^{\rm II} = 2\ {}_1\langle \phi_0 | (\mathbf{1} + P) T (\mathbf{1} + P) G_0 \hat{\mathcal{A}}_1 | \psi_{\rm BS} \rangle | \mathbf{P}_0^{\pi} \rangle, \tag{2.20}$$

$$A^{\rm III} = 2\ {}_1\langle \phi_0 | (\mathbf{1} + P) T G_0 \mathcal{T} (\mathbf{1} + P) G_0 \hat{\mathcal{A}}_1 | \psi_{\rm BS} \rangle | \mathbf{P}_0^{\pi} \rangle. \tag{2.21}$$

The first term is the usual plane-wave approximation, while the terms $A^{\rm II}$ and $A^{\rm III}$ include, respectively, the disconnected and connected contributions of the final-state interaction. Much in a similar way, the absorption amplitude with the deuteron in the final channel splits into two contributions,

$$A_d^{\rm tot} = A_d^{\rm I} + A_d^{\rm II}, \tag{2.22}$$

where the first term is again a plane-wave-type approximation

$$A_d^{\rm I} = 2\ {}_1\langle \phi_d | (\mathbf{1} + P) \hat{\mathcal{A}}_1 | \psi_{\rm BS} \rangle | \mathbf{P}_0^{\pi} \rangle, \tag{2.23}$$

and the second term includes the interactions in the final channel

$$A_d^{\rm II} = 2\ {}_1\langle \phi_d | \mathcal{T} (\mathbf{1} + P) G_0 \hat{\mathcal{A}}_1 | \psi_{\rm BS} \rangle | \mathbf{P}_0^{\pi} \rangle. \tag{2.24}$$

III. PARTIAL-WAVE DECOMPOSITION

The symmetrized absorption amplitudes derived in the previous section are now expressed in detail in momentum space. In so doing, we follow standard three-body procedures to represent the various contributions to the absorption amplitude in terms of partial waves.

First, we specify the asymptotic three-body plane wave, previously denoted by ${}_1\langle \phi_0|$, as $\langle \mathbf{p}_1, \mathbf{q}_1, \hat{\alpha}_1|$, where the symbol $\hat{\alpha}_1$ is used as an abbreviation for the spin-isospin quantum numbers in the coupling scheme:

$$\hat{\alpha}_1 \equiv (\sigma_2 \sigma_3) s_1, (s_1 \sigma_1) S S^z; (\tau_2 \tau_3) t_1, (t_1 \tau_1) T T^z. \tag{3.1}$$

In other words, s_1 is the spin of the (23) subsystem, which

couples to the spin σ_1 of nucleon 1 to give the total spin of the system S and its magnetic component S^z. The identical scheme applies also to the isospin space. For the definition of the Jacobi momenta $\mathbf{p}_1$ and $\mathbf{q}_1$ we refer to Eqs. (A1) and (A2) of the Appendix. (From now on, we will skip the subscript "1" in these Jacobi momenta.)

Second, we introduce a complete set of plane-wave states in partial waves. In the jI-coupling scheme, we represent these states as

$$|p,q,\alpha\rangle_1 \equiv |p,q,(ls)j,(\lambda\sigma)I,(jI)JJ^z;TT^z\rangle_1, \quad (3.2)$$

where the angular-spin quantum numbers $(ls)j$ refer to the subsystem 23, $(\lambda\sigma)I$ refer to the spectator 1, and JJ^z indicate the total angular momentum of the system and its z component. Since we work with states which are supposed to be antisymmetrized in the (23) subsystem coordinates, this is achieved by imposing the condition $l+s+t=$ "odd" in the quantum numbers of such a subsystem.

The totally antisymmetrized bound-state wave function of the three-body system $|\psi_{\rm BS}\rangle$ is conveniently represented by means of only one Faddeev component (e.g, $|\psi_{\rm BS}\rangle_1$) via the relation

$$|\psi_{\rm BS}\rangle = (\mathbf{1}+P)|\psi_{\rm BS}\rangle_1, \quad (3.3)$$

and $|\psi_{\rm BS}\rangle_1$ satisfies a single Faddeev homogeneous-type equation (Ref. [17], pp. 153–157).

With the purpose of obtaining the desired decomposition in partial waves of the various absorption amplitudes (2.18)–(2.24), we have to introduce completeness relations in the three-nucleon space, namely,

$$\mathbf{I} = \sum_\alpha \int p^2 dp q^2 dq |p,q,\alpha\rangle\langle p,q,\alpha|, \quad (3.4)$$

and each operator or state which is relevant to the absorption amplitude has to be represented in terms of matrix elements between states of the type (3.2).

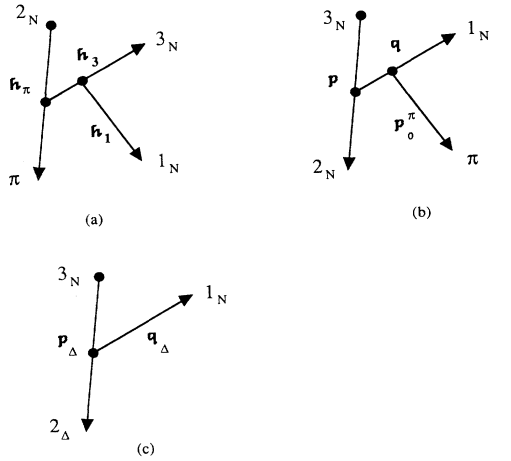

FIG. 2. Jacobi coordinates for the πNNN and ΔNN systems. (a) and (b) show the types of four-body Jacobi coordinates used in this work. (c) shows the coordinates used for the ΔNN system. The figure also displays the symbols used to indicate the corresponding momenta.

A. Matrix elements of the $\hat{\mathcal{A}}_1$ operator

The most interesting factor to be discussed is of course the matrix element of the operator $\hat{\mathcal{A}}_1$. As already mentioned, we focus our discussion on the case where $\hat{\mathcal{A}}_1$ represents the (dominant) Δ-rescattering term, but other elementary absorption mechanisms have eventually to be considered too. With the help of Fig. 2, it may be easier to visualize the kinematics involved when the Δ-rescattering mechanism is embedded in a three-nucleon space.

With a completely nonrelativistic treatment of the nucleon dynamics, we may write the elements of $\hat{\mathcal{A}}_1$ as

$$\langle p',q',\alpha'|\hat{\mathcal{A}}_1|p,q,\alpha\rangle|\mathbf{P}_0^\pi\rangle = \sum_{\alpha_\Delta}\int \langle p',q',\alpha'|V_{N\Delta}|p_\Delta,q_\Delta,\alpha_\Delta\rangle \frac{p_\Delta^2 dp_\Delta q_\Delta^2 dq_\Delta}{E-\frac{p_\Delta^2}{2\mu_\Delta}-\frac{q_\Delta^2}{2\nu_\Delta}} \langle p_\Delta,q_\Delta,\alpha_\Delta|F^+_{\pi N\Delta}|p,q,\alpha\rangle|\mathbf{P}_0^\pi\rangle. \quad (3.5)$$

In Eq. (3.5), the plane-wave states with the subscript "Δ" indicate the intermediate state with the isobar propagation. In particular, p_Δ and q_Δ are, respectively, the Jacobi momenta for the two-cluster partition $(2_\Delta 3_N)-(1_N)$ [Fig. 2(c)]. With $V_{N\Delta}$ we have denoted the $NN \leftarrow N\Delta$ transition potential, and with $F^+_{\pi N\Delta}$ the $\pi N\Delta$ vertex. The denominator in Eq. (3.5) refers to the intermediate propagation of the isobar, with

$$E = K + m_\pi + M - \mathcal{M}_\Delta, \quad (3.6)$$

K being the kinetic energy of the incoming π plus the binding energy of the target, and the last three terms are, respectively, the masses of the π, N, and Δ. In particular,

$$\mathcal{M}_\Delta = M_\Delta - \frac{i}{2}\Gamma_\Delta = 1232\ \text{MeV} - \frac{i}{2}\ \frac{2}{3}\frac{f^{*2}h_\pi^3}{4\pi m_\pi^2}\frac{M}{\sqrt{s}}, \quad (3.7)$$

where f^* is the $\pi N\Delta$ coupling constant, h_π is the relative πN momentum, and $\sqrt{s}$ is the total πN center-of-mass energy. Here, we have employed the expression for the Δ width given by the relativistically improved isobar model [22, 23]. This model provides a significant quantitative improvement with respect to the static Δ-isobar model, to which most of the earlier works referred [18–20]. The quantities μ_Δ and ν_Δ are the reduced masses for the systems Δ-N and N-(ΔN), respectively. As evidenced in Eq. (3.5), the matrix elements of $\hat{\mathcal{A}}_1$ involve momentum

integrations over three distinct factors.

It must be observed that, from previous analyses of the πd system (see Ref. [3], Chap. 6), the use of the Δ-rescattering mechanism in the form presently discussed [Eq. (3.5)] is not completely satisfactory. The limitation of applicability arises for two reasons: (i) The $NN \leftarrow \Delta N$ transition is described in first-order perturbation theory. To overcome this, the transition potential $V_{N\Delta}$ should be replaced by a true $NN \leftarrow \Delta N$ transition matrix $T_{N\Delta}$, taking into account the ΔN rescatterings to all orders. In this case, however, the $NN \leftarrow NN$ and $NN \leftarrow \Delta N$ transitions have to be handled consistently through isobar-coupled equations. Clearly, this has implications also for the Faddeev-based treatment of the final-state dynamics. (ii) The model is not unitary: it does not couple the absorption channel to all the available πNNN-NNN channels. Moreover, the use of first-order perturbation in the $NN \leftarrow \Delta N$ transition, and of a complex Δ mass in Eq. (3.7), breaks unitarity.

The above limitations could be completely overcome only through a consistent four-body treatment of the πNNN-NNN dynamics. As already observed in the Introduction, the computational problems brought about by a fully consistent theory are presently unsurmountable. With reference to this point, an approach recently proposed by Ueda [24] is worthy of mention in that it preserves unitarity up to the three-body level, and allows for a nonperturbative treatment of the ΔN-NN dynamics. The relativistic four-body problem is described within the multibody, three-, and two-body cluster coupling approximation, through an explicit treatment of the couplings among the πdN, $NN\Delta$, NNN, and dN channels. Each subsystem interacts via two-body separable interactions; the three-body channels πdN, $NN\Delta$, and NNN are coupled by separable $d\pi$-$N\Delta$-NN transition matrices, while the channels πdN, NNN, and dN are connected through the rearrangement processes $N \leftarrow \pi N$ and $d \leftarrow NN$. This model, which requires the fitting of various adjustable parameters, represents a computationally workable approach to the non-perturbative aspects of the πNNN-NNN dynamics. Numerical results for the $\pi^0\,{}^3\mathrm{He} \leftarrow pd$ reaction give indications that in certain partial waves the inclusion of higher-order processes is important.

1. Matrix elements of the $\pi N\Delta$ vertex

We proceed by calculating first its matrix elements between three-dimensional plane waves. In so doing, the interpretation discussed in Ref. [25] is here assumed. Namely, the vertex depends intrinsically upon the momentum of relative pion-nucleon motion, h_π. This is somehow different from the field-theoretic description, where the absorbing nucleon is static, and the pion momentum intervenes in the coupling vertex. The two descriptions are coincident only in the pion-nucleon c.m. system. The coupling vertex between full plane waves is

$$\langle \mathbf{p}_\Delta, \mathbf{q}_\Delta, \hat{\alpha}_\Delta | F^+_{\pi N\Delta} | \mathbf{p}, \mathbf{q}, \hat{\alpha} \rangle | \mathbf{P}^\pi_0 \rangle = \sum_{\hat{\beta}} \int \langle \mathbf{p}_\Delta, \mathbf{q}_\Delta, \hat{\alpha}_\Delta | F^+_{\pi N\Delta} | \mathbf{h}_3, \mathbf{h}_1, \mathbf{h}_\pi, \hat{\beta} \rangle \langle \mathbf{h}_3, \mathbf{h}_1, \mathbf{h}_\pi, \hat{\beta} | \mathbf{p}, \mathbf{q}, \hat{\alpha} \rangle | \mathbf{P}^\pi_0 \rangle d\mathbf{h}_3 d\mathbf{h}_1 d\mathbf{h}_\pi, \tag{3.8}$$

where we have inserted a completeness relation for the πNNN space with the Jacobi scheme shown in Fig. 2(a). The last matrix element requests the transformation between two sets of four-body variables. In the Appendix, we introduce and discuss an approximated kinematical transformation which yields

$$\langle \mathbf{h}_3, \mathbf{h}_1, \mathbf{h}_\pi, \hat{\beta} | \mathbf{p}, \mathbf{q}, \hat{\alpha} \rangle | \mathbf{P}^\pi_0 \rangle = \delta \left(\mathbf{h}_\pi - \frac{3M + m_\pi}{3M + 3m_\pi} \mathbf{P}^\pi_0 \right) \delta \left(\mathbf{h}_3 + \mathbf{p} + \frac{3M + m_\pi}{6M + 3m_\pi} \mathbf{P}^\pi_0 \right) \delta \left(\mathbf{h}_1 - \mathbf{q} + \frac{1}{3} \mathbf{P}^\pi_0 \right) \delta_{\hat{\beta}\hat{\alpha}}. \tag{3.9}$$

In Eq. (3.8) the matrix element of the coupling vertex is

$$\langle \mathbf{p}_\Delta, \mathbf{q}_\Delta, \hat{\alpha}_\Delta | F^+_{\pi N\Delta} | \mathbf{h}_3, \mathbf{h}_1, \mathbf{h}_\pi, \hat{\beta} \rangle = \delta(\mathbf{p}_\Delta + \mathbf{h}_3) \delta(\mathbf{q}_\Delta - \mathbf{h}_1) \mathcal{F}_{\hat{\alpha}_\Delta \hat{\beta}}(\mathbf{h}_\pi). \tag{3.10}$$

With the detailed expression of $\mathcal{F}_{\hat{\alpha}_\Delta \hat{\beta}}(\mathbf{h}_\pi)$ given in the Appendix, the embedding of the coupling vertex in the $3N$ space yields

$$\langle \mathbf{p}_\Delta, \mathbf{q}_\Delta, \hat{\alpha}_\Delta | F^+_{\pi N\Delta} | \mathbf{p}, \mathbf{q}, \hat{\alpha} \rangle | \mathbf{P}^\pi_0 \rangle = \mathcal{F}_{\hat{\alpha}_\Delta \hat{\alpha}}(\mathbf{h}_\pi) \delta \left(\mathbf{p}_\Delta - \mathbf{p} - \frac{3M + m_\pi}{6M + 3m_\pi} \mathbf{P}^\pi_0 \right) \delta \left(\mathbf{q}_\Delta - \mathbf{q} + \frac{1}{3} \mathbf{P}^\pi_0 \right). \tag{3.11}$$

We now derive the coupling vertex between angular-decomposed three-body plane waves. For simplicity, instead of working with states in the jI-coupling scheme, we consider here angular-spin *uncoupled* partial waves, and the details of the transformation to the jI scheme are found in the Appendix.

For angular-spin uncoupled states, the $\pi N\Delta$ coupling vertex is written as follows:

$$\begin{aligned}
&\langle p_\Delta, q_\Delta, (l_\Delta, \lambda_\Delta) L_\Delta, L^z_\Delta, \hat{\alpha}_\Delta | F^+_{\pi N\Delta} | p, q, (l, \lambda) L, L^z, \hat{\alpha} \rangle | \mathbf{P}^\pi_0 \rangle \\
&\quad \equiv \mathcal{F}_{\hat{\alpha}_\Delta \hat{\alpha}}(\mathbf{h}_\pi) \mathbf{I}(\mathbf{P}^\pi_0, (l_\Delta, \lambda_\Delta) L_\Delta, L^z_\Delta; (l, \lambda) L, L^z) \\
&\quad = \mathcal{F}_{\hat{\alpha}_\Delta \hat{\alpha}} \left(\frac{3M + m_\pi}{3M + 3m_\pi} \mathbf{P}^\pi_0 \right) \int d\hat{\mathbf{P}}_\Delta d\hat{\mathbf{Q}}_\Delta d\hat{\mathbf{P}} d\hat{\mathbf{Q}} \mathcal{Y}^{*L_\Delta L^z_\Delta}_{l_\Delta \lambda_\Delta}(\hat{\mathbf{P}}_\Delta, \hat{\mathbf{Q}}_\Delta) \mathcal{Y}^{LL^z}_{l\lambda}(\hat{\mathbf{P}}, \hat{\mathbf{Q}}) \\
&\qquad \times \delta \left(\mathbf{p}_\Delta - \mathbf{p} - \frac{3M + m_\pi}{6M + 3m_\pi} \mathbf{P}^\pi_0 \right) \delta \left(\mathbf{q}_\Delta - \mathbf{q} + \frac{1}{3} \mathbf{P}^\pi_0 \right).
\end{aligned} \tag{3.12}$$

In Eq. (3.12) we have denoted by **I** the integral over the $4(\times 2)$ angular variables, with the symbol $\mathcal{Y}$ representing the bipolar spherical harmonics [17, 26]. Once $\mathcal{Y}$ is represented in terms of the regular spherical harmonics, the eightfold integral factorizes into two angular integrals involving only **p**-type or **q**-type momenta, respectively,

$$\begin{aligned}\mathbf{I}(\mathbf{P}_0^\pi,(l_\Delta,\lambda_\Delta)L_\Delta,L_\Delta^z;(l,\lambda)L,L^z)=\sum_{mn}\sum_{m_\Delta n_\Delta} & C(l\lambda L,mnL^z)C(l_\Delta\lambda_\Delta L_\Delta,m_\Delta n_\Delta L_\Delta^z)\\ &\times\int d\hat{\mathbf{P}}_\Delta d\hat{\mathbf{P}}\delta\left(\mathbf{p}_\Delta-\mathbf{p}-\frac{3M+m_\pi}{6M+3m_\pi}\mathbf{P}_0^\pi\right)Y^*_{l_\Delta m_\Delta}(\hat{\mathbf{P}}_\Delta)Y_{lm}(\hat{\mathbf{P}})\\ &\times\int d\hat{\mathbf{Q}}_\Delta d\hat{\mathbf{Q}}\delta\left(\mathbf{q}_\Delta-\mathbf{q}+\frac{1}{3}\mathbf{P}_0^\pi\right)Y^*_{\lambda_\Delta n_\Delta}(\hat{\mathbf{Q}}_\Delta)Y_{\lambda n}(\hat{\mathbf{Q}}).\end{aligned}\tag{3.13}$$

We stress the importance of the approximated kinematical transformations, Eqs. (A8)–(A10). It is because of such an approximation, indeed, that (a) the factorization shown in Eq. (3.12) occurs, where the first factor $\mathcal{F}$ depends only upon the initial momentum of the pion, and (b), the remaining eightfold integral **I** can be calculated through Eq. (3.13), where it is expressed as a product of two separate and similar fourfold angular integrals which can be performed analytically. To this end, we choose the azimuthal axis parallel to $\mathbf{P}_0^\pi$, and we separate the spherical harmonics in

$$Y_{lm}(\theta,\phi)=\Theta_{lm}(\cos\theta)\frac{e^{im\phi}}{\sqrt{2\pi}},\tag{3.14}$$

where the function $\Theta_{lm}(\cos\theta)$ is proportional to the associated Legendre function.

The result is

$$\begin{aligned}\mathbf{I}(\mathbf{P}_0^\pi,(l_\Delta,\lambda_\Delta)L_\Delta,L_\Delta^z;(l,\lambda)L,L^z)=\sum_{mn}\sum_{m_\Delta n_\Delta} & C(l\lambda L,mnL^z)C(l_\Delta\lambda_\Delta L_\Delta,m_\Delta n_\Delta L_\Delta^z)\\ &\times\Upsilon\left(p_\Delta,p,\frac{3M+m_\pi}{6M+3m_\pi}P_0^\pi\right)\Theta_{l_\Delta m_\Delta}(\cos\hat{P}_\Delta)\Theta_{lm}(\cos\hat{P})\delta_{m_\Delta m}\\ &\times\Upsilon\left(q_\Delta,q,\frac{1}{3}P_0^\pi\right)\Theta_{\lambda_\Delta n_\Delta}(\cos\hat{Q}_\Delta)\Theta_{\lambda n}(\cos\hat{Q})\delta_{n_\Delta n},\end{aligned}\tag{3.15}$$

where the angular variables can be expressed through Pythagora's generalized theorem *("law of cosines")*

$$\cos\hat{P}_\Delta=\left[\left(\frac{3M+m_\pi}{6M+3m_\pi}P_0^\pi\right)^2+p_\Delta^2-p^2\right]\left[\frac{6M+3m_\pi}{2(3M+m_\pi)P_0^\pi p_\Delta}\right],\tag{3.16}$$

$$\cos\hat{P}=\left[p_\Delta^2-\left(\frac{3M+m_\pi}{6M+3m_\pi}P_0^\pi\right)^2-p^2\right]\left[\frac{6M+3m_\pi}{2(3M+m_\pi)P_0^\pi p}\right],\tag{3.17}$$

$$\cos\hat{Q}_\Delta=\left[q^2-q_\Delta^2-\left(\frac{1}{3}P_0^\pi\right)^2\right]\left[\frac{3}{2P_0^\pi q_\Delta}\right],\tag{3.18}$$

$$\cos\hat{Q}=\left[q^2-q_\Delta^2+\left(\frac{1}{3}P_0^\pi\right)^2\right]\left[\frac{3}{2P_0^\pi q}\right].\tag{3.19}$$

In Eq. (3.15), the *triangular* function Υ has been introduced. It may be defined as

$$\Upsilon(a,b,c)\equiv\frac{H(a-|b-c|)H(b+c-a)}{abc},\tag{3.20}$$

with $H(x)$ being the Heaviside function.

A very important case to be discussed separately is the evaluation of the integrals displayed in Eq. (3.13) in the limit $p\ll P_0^\pi$ ($q\ll P_0^\pi$). This corresponds to neglecting the pair (spectator) momentum with respect to the incoming pion momentum when dealing with the intermediate isobar propagation. Then, the first integral in Eq. (3.13) becomes

$$\begin{aligned}&\int d\hat{\mathbf{P}}_\Delta d\hat{\mathbf{P}}\delta\left(\mathbf{p}_\Delta-\mathbf{p}-\frac{3M+m_\pi}{6M+3m_\pi}\mathbf{P}_0^\pi\right)Y^*_{l_\Delta m_\Delta}(\hat{\mathbf{P}}_\Delta)Y_{lm}(\hat{\mathbf{P}})\\ &\qquad=\sqrt{2}\Theta_{l_\Delta 0}(\cos\hat{P}_\Delta=1)\frac{\delta\left(p_\Delta-\frac{3M+m_\pi}{6M+3m_\pi}P_0^\pi\right)}{p_\Delta^2}\delta_{l0}\delta_{m0}\delta_{m_\Delta 0},\end{aligned}\tag{3.21}$$

and the second integral is

$$\int d\hat{\mathbf{Q}}_\Delta d\hat{\mathbf{Q}}\delta(\mathbf{q}_\Delta - \mathbf{q} + \tfrac{1}{3}\mathbf{P}_0^\pi) Y^*_{\lambda_\Delta n_\Delta}(\hat{\mathbf{Q}}_\Delta) Y_{\lambda n}(\hat{\mathbf{Q}}) = \sqrt{2}\Theta_{\lambda_\Delta 0}(\cos\hat{Q}_\Delta = -1)\frac{\delta(q_\Delta - \frac{1}{3}P_0^\pi)}{q_\Delta^2}\delta_{\lambda 0}\delta_{n0}\delta_{n_\Delta 0}. \tag{3.22}$$

This, together with the discussion in the Appendix for the transformation to the jI-coupling scheme, completes the evaluation of the matrix elements of the $\pi N\Delta$-coupling vertex.

2. *Treatment of the intermediate Δ propagation*

In particular, we focus here on the inclusion of relativistic kinematical corrections for the pion. Returning to expression (3.5), by virtue of the two triangular functions in Eq. (3.15), the integration intervals in the $\mathbf{p}$ and $\mathbf{q}$ variables are, respectively $[|p - \frac{3M+m_\pi}{6M+3m_\pi}P_0^\pi|, p + \frac{3M+m_\pi}{6M+3m_\pi}P_0^\pi]$ and $[|q - \frac{1}{3}P_0^\pi|, q + \frac{1}{3}P_0^\pi]$. By means of a suitable change of variables, in the exact expression (3.5) the singularities due to the triangular functions can be integrated over, so as to make numerical evaluation feasible. However, the integration is totally eliminated in the limits $p \ll P_0^\pi$ and $q \ll P_0^\pi$, because of the presence of the δ's in Eqs. (3.21) and (3.22). For simplicity, we discuss this situation only. In such a case, instead of Eq. (3.5), we have

$$\langle p', q', \alpha'|\hat{\mathcal{A}}_1|p, q, \alpha\rangle|\mathbf{P}_0^\pi\rangle = \sum_{\alpha_\Delta}\langle p', q', \alpha'|V_{N\Delta}|\tilde{p}_\Delta, \tilde{q}_\Delta, \alpha_\Delta\rangle \frac{\tilde{p}_\Delta^2\tilde{q}_\Delta^2}{E - \frac{\tilde{p}_\Delta^2}{2\mu_\Delta} - \frac{\tilde{q}_\Delta^2}{2\nu_\Delta}}\langle\tilde{p}_\Delta, \tilde{q}_\Delta, \alpha_\Delta|F^+_{\pi N\Delta}|p, q, \alpha\rangle|\mathbf{P}_0^\pi\rangle\ , \tag{3.23}$$

with

$$\tilde{p}_\Delta = \frac{3M + m_\pi}{6M + 3m_\pi}P_0^\pi, \tag{3.24}$$

$$\tilde{q}_\Delta = \frac{1}{3}P_0^\pi. \tag{3.25}$$

We note that, because of the one-body character of the coupling vertex, it follows that

$$\tilde{p}_\Delta^2 = h_3^2, \quad \tilde{q}_\Delta^2 = h_1^2. \tag{3.26}$$

We may investigate to which extent the denominator in Eq. (3.23) can be approximated in the following way:

$$E - \frac{\tilde{p}_\Delta^2}{2\mu_\Delta} - \frac{\tilde{q}_\Delta^2}{2\nu_\Delta} \simeq E - \frac{h_3^2}{2\mu_3} - \frac{h_1^2}{2\nu_1}, \tag{3.27}$$

the only difference being in the expression of the reduced masses. Here, μ_Δ (ν_Δ) is the reduced mass of the system Δ-N [(ΔN)-N], while μ_3 (ν_1) is the reduced mass of the system (πN)-N [(πNN)-N].

If we take the bare pion's rest mass value in the expressions for the reduced masses, μ_3 underestimates μ_Δ by $\simeq 5\%$, and ν_1 underestimates ν_Δ by $\simeq 2\%$. However, a remarkably better agreement is obtained around the isobar resonance when minimal relativistic corrections are introduced for the reduced masses μ_3, ν_1, such as the replacement of the pion rest mass m_π with $\omega_\pi = \sqrt{h_\pi^2 + m_\pi^2}$. This correction is obtained when performing a nonrelativistic reduction of the three-body relativistic kinematics [27].

When $\mathbf{p}, \mathbf{q} \simeq 0$, the following identity holds:

$$\frac{P_0^{\pi\,2}}{2\mu_\pi} = \frac{h_\pi^2}{2\nu_\pi} + \frac{h_3^2}{2\mu_3} + \frac{h_1^2}{2\nu_1}, \tag{3.28}$$

μ_π and ν_π being, respectively, the reduced masses of the systems π-(NNN) and π-N. This identity is exactly verified also with the approximate transformation discussed in the Appendix [Eqs. (A8) and (A9)].

By virtue of Eqs. (3.6), (3.27), and (3.28), we can write the denominator in Eq. (3.23) as follows:

$$E - \frac{\tilde{p}_\Delta^2}{2\mu_\Delta} - \frac{\tilde{q}_\Delta^2}{2\nu_\Delta} = m_\pi + M + \frac{h_\pi^2}{2\nu_\pi} - B - \mathcal{M}_\Delta, \tag{3.29}$$

where we have indicated with $-B$ the trinucleon binding energy.

On the right-hand side of Eq. (3.29), the first three terms correspond to the fully nonrelativistic reduction of the invariant mass $\sqrt{s}$ of the πN pair, evaluated in its c.m. system. Therefore, the relativistic denominator for the intermediate Δ propagation is (neglecting the binding energy)

$$\sqrt{s} - \mathcal{M}_\Delta, \tag{3.30}$$

with

$$\sqrt{s} = \sqrt{m_\pi^2 + h_\pi^2} + \sqrt{M^2 + h_\pi^2}. \tag{3.31}$$

In the context of pionic disintegration of deuterons, this relativistic form (3.30) has been already used by several authors [18–20].

3. *Treatment of the NN ← ΔN transition potential*

The treatment of the $NN \leftarrow \Delta N$ transition is the highly nontrivial problem in the $\hat{\mathcal{A}}_1$ mechanism. Within the pionic disintegration of deuterons, this question has been carefully analyzed by Tanabe and Ohta [28]. Indeed, various approaches with different levels of sophistication have been developed for the description of the $NN \leftarrow \Delta N$ inelasticities (perturbative methods, coupled-channel methods, unitary three-body methods; for their references see [3]).

Of course, the embedding of the $NN \leftarrow \Delta N$ transition in a three-nucleon framework makes these treatments a much more difficult task and, therefore, we consider here only the perturbative-type approach because

of its greater simplicity. Following this approach, the $NN \leftarrow \Delta N$ process is described by a transition potential $V_{N\Delta}$ originated by the exchange of one or two pions. Because of isospin conservation, the exchanged boson has to be of isovector nature; thus, the two-pion exchange must have the spin-isospin structure of a ρ meson. Therefore, we have

$$V_{N\Delta} = V^{\pi}_{N\Delta} + V^{\rho}_{N\Delta}. \tag{3.32}$$

We recall that, following our definition of $\hat{\mathcal{A}}_1$, the isobar excitation may occur only in particle 2, with deexcitation through the exchange of a meson with particle 3. The matrix elements of the $NN \leftarrow \Delta N$ transition potential, when embedded in a three-baryon space, are

$$\langle p', q', \alpha' | V_{N\Delta} | p_\Delta, q_\Delta, \alpha_\Delta \rangle = \frac{\delta(q' - q_\Delta)}{q'^2} \delta_{\lambda' \lambda_\Delta} \delta_{I' I_\Delta} \delta_{J' J_\Delta} \delta_{J'^z J^z_\Delta} \delta_{T' T_\Delta} \delta_{T'^z T^z_\Delta} \langle p', l' s' j', t' | V_{N\Delta} | p_\Delta, l_\Delta s_\Delta j_\Delta, t_\Delta \rangle \ . \tag{3.33}$$

In the meson-exchange framework, the detailed matrix elements for the static transition potential have been given in the literature a long time ago [29, 30]. In connection with the "perturbative" Δ-rescattering mechanisms, this transition potential has been discussed also by other authors [18–20, 22, 31].

In the spirit of the perturbative approach, the distortions in the final state are calculated using conventional nucleon-nucleon interactions (extrapolated up to the Δ resonance region) and nucleonic degrees of freedom only. These distortions are fully described by Eqs. (2.15) and (2.16). With respect to this point, we note that a possible double counting problem may arise if we consider the higher-order process consisting of the $NN \leftarrow \Delta N$ transition via the pion term, plus a further one-pion exchange between the same two nucleons in the final state. In such a case, the whole process actually involves the exchange of two pions, and has therefore to be subtracted from $V^{\rho}_{\Delta N}$. In the context of the reaction $pp \leftarrow \pi^+ d$, this problem has been discussed by Maxwell *et al.* [19], where it has been shown that the cross section may change by 20% at the resonance peak, depending on the possible different ways of subtraction.

From another point of view, the $NN \leftarrow \Delta N$ transition is not well determined because it requires the highly off-shell extrapolation of the $\pi N \Delta$ vertex [2]. This implies the introduction of cutoff form factors which in the perturbative approach are generally adjusted to reproduce the (pion-deuteron) absorption cross sections. Of course, when the same approach is applied to pion-^{3}He, the cutoffs must be consistent with the ones obtained from the deuteron problem.

B. Other matrix elements

The other matrix elements one needs to evaluate for the calculation of the amplitudes (2.18)–(2.24) are in principle conventional three-body constituents. These involve matrix elements of the following operators: **1**, G_0, T, P, and [through Eq. (2.17)] $\mathcal{T}$.

The detailed expressions for these quantities can be found in many references (e.g, Refs. [17, 32]). In particular,

$$\langle p, q, \alpha | p', q', \alpha' \rangle = \delta_{\alpha\alpha'} \frac{\delta(q - q')}{q^2} \frac{\delta(p - p')}{p^2}, \tag{3.34}$$

$$\langle p, q, \alpha | G_0(E) | p', q', \alpha' \rangle = \delta_{\alpha\alpha'} \frac{\delta(q - q')}{q^2} \frac{\delta(p - p')}{p^2} \frac{1}{E - \dfrac{p^2}{2\mu} - \dfrac{q^2}{2\nu}}, \tag{3.35}$$

and

$$\langle p, q, \alpha | T(E) | p', q', \alpha' \rangle = \frac{\delta(q - q')}{q^2} \delta_{\lambda\lambda'} \delta_{II'} \delta_{JJ'} \delta_{J^z J'^z} \delta_{TT'} \delta_{T^z T'^z} \left\langle p, lsj, t \middle| T\left(E - \frac{q^2}{2\nu}\right) \middle| p', l's'j', t' \right\rangle. \tag{3.36}$$

The matrix elements of the permutation operator P are, by far, more complicated compared to the previous ones, since they contain all the intricacies inherent to the recouplings of linear momenta corresponding to three particles. For brevity, their expressions will not be given here but we refer instead to the specialized literature [32].

In the final state, the matrix element between three-dimensional and angular decomposed $3N$ plane waves is also required. For the jI-coupling scheme, we have

$$\langle \mathbf{p}_f, \mathbf{q}_f, \hat{\alpha}_f | p, q, \alpha \rangle = \delta_{t_f t} \delta_{T_f T} \delta_{T_f^z T^z} \delta_{s_f s} \frac{\delta(p_f - p)}{p^2} \frac{\delta(q_f - q)}{q^2} \times \sum_L \hat{j} \hat{I} \hat{L} \hat{S}_f \begin{Bmatrix} l & s_f & j \\ \lambda & \frac{1}{2} & I \\ L & S_f & J \end{Bmatrix} \left(\sum_{L^z} C(LS_f J, L^z S_f^z J^z) \, \mathcal{Y}^{LL^z}_{l\lambda}(\hat{\mathbf{p}}_f, \hat{\mathbf{q}}_f) \right), \tag{3.37}$$

with $\hat{j} \equiv \sqrt{2j+1}$.

IV. SUMMARY

This work has carried the theory of paper I forward to include full antisymmetrization and angular-momentum decomposition. The absorption amplitude, Eqs. (2.1) and (2.2) of I [Eq. (2.7) here], is fully antisymmetrized in the nucleon variables in Sec. II, leading to the results given in Eqs. (2.18)–(2.24). Angular-momentum decomposition of the various components of this amplitude is carried out in Sec. III, with the essential results given by Eqs. (3.5), (3.12), (3.15), (A11), and (A13). The angular-momentum reduction is a rather hybrid procedure, in that it is possible, and indeed convenient, to treat the momentum vector of the incoming pion as a vector variable. That is, the angular-momentum reduction is only done on the nucleon variables. However, the full complexity of the three-body system, antisymmetrized in the nucleonic degrees of freedom, is completely taken into account. The resulting system of equations are in a form suitable for numerical computations. What is needed is selection of suitable three-body wave functions and the parameters in the pion absorption vertex [Eq. (A12)], and the transition potential $V_{N\Delta}$ [Eq. (3.33)]. Then the problem can be solved with a choice of efficient algorithms for computation.

Since it is anticipated that the dominant pion-absorption mechanism in the energy region around the delta resonance is the two-body delta-rescattering term (Fig. 1), this work has taken only that term into account. However, it is straightfoward to include other absorption mechanisms such as those represented by Figs. 1(b) (the Koltun-Reitan term, Ref. [33]) and 1(c) (one-body absorption without intermediate delta propagation) of I. Because of the antisymmetrization, any of the three nucleons can be involved in the absorption mechanism. True three-body absorption which cannot be described by a two-body mechanism preceded (followed) by (purely nucleonic) ISI (FSI) is not discussed in the present paper, although it can be accomodated within this formalism *via* the proper redefinition of the operator $\hat{\mathcal{A}}_1$. With respect to this point, an accurate calculation of FSI (ISI) preceded (followed) by two- or one-body absorption mechanisms would be extremely important because it could shed some light on the extent to which true three-body absorption is indeed needed to explain multinucleon absorption processes. As mentioned in the Introduction, there is experimental evidence that as much as 30% of the total absorption cross section is attributable to mechanisms involving all three nucleons, and it is of fundamental importance to assess which fraction has to be ascribed to *true* three-body absorption.

Much of the previous theoretical work in pion absorption on light nuclei does not go beyond the spectator model. In this approximation the pioneering work by Maxwell and Cheung [31] has been carried further on with more sophisticated and modern treatments of the $NN \leftarrow \Delta N$ inelasticities, which nowadays can be described more precisely, e.g., with the coupled-channel model by Niskanen [34]. It is worth mentioning, here, a particularly sophisticated version of the spectator model, which resorts to the unitary treatment of the πNN-$N\Delta$ system in the framework of the three-body Argonne-Hannover model [9]. The spectator model, however, restricts the dynamical description of the absorption process to two nucleons only, and consequently limits drastically the kinematical regions for the ejected nucleons. On the contrary, there is a wealth of experimental data entirely covering the available phase space, which has not yet been tackled theoretically. This requires theoretical work with Faddeev-based treatments of the three-nucleon dynamics [35], along the lines described in the present paper.

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APPENDIX

1. Kinematical transformations

The calculation of the matrix elements of $\hat{\mathcal{A}}_1$ requires the transformation between two sets of four-body kinematical variables. Denoting by $\mathbf{K}_1^L$, $\mathbf{K}_2^L$, $\mathbf{K}_3^L$, and $\mathbf{K}_\pi^L$ the momenta of the four particles in the Laboratory system, the two sets of (nonrelativistic) Jacobi momenta are

$$\mathbf{p} = \tfrac{1}{2}(\mathbf{K}_2^L - \mathbf{K}_3^L) \ , \tag{A1}$$

$$\mathbf{q} = \tfrac{1}{3}\left(2\mathbf{K}_1^L - (\mathbf{K}_2^L + \mathbf{K}_3^L)\right) \ , \tag{A2}$$

$$\mathbf{P}_0^\pi = \frac{3M\mathbf{K}_\pi^L - m_\pi(\mathbf{K}_1^L + \mathbf{K}_2^L + \mathbf{K}_3^L)}{3M + m_\pi}, \tag{A3}$$

and

$$\mathbf{h}_\pi = \frac{1}{M + m_\pi}(M\mathbf{K}_\pi^L - m_\pi\mathbf{K}_2^L) \ , \tag{A4}$$

$$\mathbf{h}_3 = \frac{1}{2M + m_\pi}\left((M + m_\pi)\mathbf{K}_3^L - M(\mathbf{K}_\pi^L + \mathbf{K}_2^L)\right) \ , \tag{A5}$$

$$\mathbf{h}_1 = \frac{(2M + m_\pi)\mathbf{K}_1^L - M(\mathbf{K}_\pi^L + \mathbf{K}_2^L + \mathbf{K}_3^L)}{3M + m_\pi}. \tag{A6}$$

Since in the laboratory system it is $\mathbf{K}_1^L + \mathbf{K}_2^L + \mathbf{K}_3^L = 0$, the pion momentum in the c.m system is simply

$$\mathbf{P}_0^\pi = \frac{3M\mathbf{K}_\pi^L}{3M + m_\pi}. \tag{A7}$$

If in Eqs. (A4) and (A6) we neglect terms of the type $(m_\pi/M)\mathbf{K}^L$ and higher orders in (m_π/M), where the

momenta $\mathbf{K}^L$ refer *to the nucleon coordinates only*, we obtain the following transformation:

$$\mathbf{h}_\pi \simeq \frac{3M+m_\pi}{3M+3m_\pi}\mathbf{P}_0^\pi, \tag{A8}$$

$$\mathbf{h}_3 \simeq -\mathbf{p} - \frac{3M+m_\pi}{6M+3m_\pi}\mathbf{P}_0^\pi, \tag{A9}$$

$$\mathbf{h}_1 \simeq \mathbf{q} - \frac{1}{3}\mathbf{P}_0^\pi. \tag{A10}$$

We stress the fact that this transformation does not involve approximations on the pion momentum $\mathbf{K}_\pi^L$.

2. Expression for $\mathcal{F}_{\hat\alpha\hat\alpha'}(\mathbf{h}_\pi)$

The detailed expression for $\mathcal{F}_{\hat\alpha\hat\alpha'}(\mathbf{h}_\pi)$, where $\hat\alpha$, $\hat\alpha'$ denote, respectively, the spin-isospin quantum number for the $(1_N2_\Delta3_N)$ and $(1_N2_N3_N)$ systems, are given as follows:

$$\begin{aligned}\mathcal{F}_{\hat\alpha\hat\alpha'}(\mathbf{h}_\pi) &= \sum_{\sigma_1^z\sigma_3^z}\sum_{\sigma_2^z\sigma'^z_2}\sum_{s^zs'^z}\sum_{\tau_1^z\tau_3^z}\sum_{\tau_2^z\tau'^z_2}\sum_{t^zt'^z} C(s\tfrac12 S,s^z\sigma_1^zS^z)C(\tfrac32\tfrac12 s,\sigma_2^z\sigma_3^zs^z)C(s'\tfrac12 S',s'^z\sigma_1^zS'^z)\\
&\times C(\tfrac12\tfrac12 s',\sigma'^z_2\sigma_3^zs'^z)C(t\tfrac12 T,t^z\tau_1^zT^z)C(\tfrac32\tfrac12 t,\tau_2^z\tau_3^zt^z)C(t'\tfrac12 T',t'^z\tau_1^zT'^z)C(\tfrac12\tfrac12 t',\tau'^z_2\tau_3^zt'^z)\\
&\times C(1\tfrac12\tfrac32,t_\pi^z\tau'^z_2\tau_2^z)\left[\sum_m C(1\tfrac12\tfrac32,m\sigma'^z_2\sigma_2^z)[Y_{1m}(\hat{\mathbf{h}}_\pi)]^*\right] f(h_\pi)\\
&= (-)^{3+s+s'+S+t+t'+T}4\hat s\hat s'\hat S'\hat t\hat t'\hat T'\begin{Bmatrix}1 & s' & s\\ \frac12 & S & S'\end{Bmatrix}\begin{Bmatrix}1 & \frac12 & \frac32\\ \frac12 & s & s'\end{Bmatrix}\begin{Bmatrix}1 & t' & t\\ \frac12 & T & T'\end{Bmatrix}\begin{Bmatrix}1 & \frac12 & \frac32\\ \frac12 & t & t'\end{Bmatrix}\\
&\times\left[\sum_m C(1S'S,mS'^zS^z)[Y_{1m}(\hat{\mathbf{h}}_\pi)]^*\right] C(1T'T,t_\pi^zT'^zT^z)f(h_\pi)\ .\end{aligned} \tag{A11}$$

The functional dependence of $f(h_\pi)$ is

$$f(h_\pi) = \frac{i}{(2\pi)^{\frac32}}\sqrt{\frac{4\pi}{3}}\frac{f^*}{m_\pi}\frac{h_\pi}{\sqrt{2\omega_\pi(h_\pi)}}, \tag{A12}$$

where the standard choice for the $\pi N\Delta$ coupling constant f^* is derived from the Chew-Low model, $f^* = 2f$ (f is the πNN empirical coupling constant). Alternative choices including relativistic descriptions have been discussed in the literature [19].

3. Matrix elements of the coupling vertex in the jI-coupling scheme

The $\pi N\Delta$ vertex in the jI-coupling scheme is derived from the matrix elements for the *uncoupled* scheme, Eq. (3.12). The result is

$$\begin{aligned}\langle p_\Delta,q_\Delta,\alpha_\Delta|F^+_{\pi N\Delta}|p,q,\alpha\rangle|\mathbf{P}_0^\pi\rangle &= \sum_{LS}\sum_{L_\Delta S_\Delta}\hat j\hat I\hat L\hat S\hat j_\Delta\hat I_\Delta\hat L_\Delta\hat S_\Delta\begin{Bmatrix}l & s & j\\ \lambda & \frac12 & I\\ L & S & J\end{Bmatrix}\begin{Bmatrix}l_\Delta & s_\Delta & j_\Delta\\ \lambda_\Delta & \frac12 & I_\Delta\\ L_\Delta & S_\Delta & J_\Delta\end{Bmatrix}\\
&\times\left(\sum_{L^zS^z}\sum_{L_\Delta^zS_\Delta^z}C(LSJ,L^zS^zJ^z)\,C(L_\Delta S_\Delta J_\Delta,L_\Delta^zS_\Delta^zJ_\Delta^z)\right.\\
&\left.\times\mathbf{I}(\mathbf{P}_0^\pi,(l_\Delta,\lambda_\Delta)L_\Delta,L_\Delta^z;(l,\lambda)L,L^z)\mathcal{F}_{\hat\alpha_\Delta\hat\alpha}(\mathbf{h}_\pi)\right).\end{aligned} \tag{A13}$$

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