Pion double-charge-exchange operator

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We investigate the behavior of the operator responsible for sequential pion double charge exchange in nuclei in the plane wave limit with a view to studying its extension in space, especially the short-range part.

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I. INTRODUCTION

Pion double charge exchange (DCX) offers one of the most promising possibilities for studying correlations among nucleons in nuclei because it requires at least two interactions for the process to take place. A number of studies [1–3] have demonstrated that the reaction is indeed sensitive to nucleon-nucleon correlations. This sensitivity is a direct reflection of the structure of the DCX operator itself. Over the years several mechanisms which contribute to pion DCX have been investigated. These include pion charge exchange with virtual nuclear mesons [5], the delta rescattering mechanism [4], and the pion absorption mechanism [6]. In this paper we consider only the sequential operator, which is always present, and do not treat these other mechanisms which may also be important in certain cases. By the *sequential mechanism* we mean that the reaction proceeds through two successive single πN charge-exchange interactions. The DCX operator is (at least) a two-body operator in the nucleon coordinates and hence is moderately complicated. *It is the aim of this paper to seek a qualitative understanding of the sequential DCX operator.* It is *not* our purpose to provide state-of-the-art calculations of specific DCX cross sections suitable to be compared with experimental data. Such calculations have already been performed (and continue to be done), with varying degrees of success; see the references already cited. It is our hope that some of the results of this paper will influence future calculations in this area.

There are several reasons for studying the DCX (or more generally the double scattering) operator. These topics are explored in more depth in this paper.

(1) The range of the operator is not well known.

(2) DCX is very sensitive to the correlation stucture of the nuclear wave functions. Although implicit in many specific calculations, the explicit form of the DCX operator, including processes in which both nucleons flip their spins, has not, to our knowledge, been given. The calculation is normally done by performing the integrations over the nuclear wave functions first (it is indeed a good deal more efficient) and then doing the integration over the intermediate momentum of the neutral pion. The operators are particularly simple for forward DCX. They are given by Eqs. (22) and (23) for nonflip DCX and by Eqs. (27) and (28) for the double-flip DCX. We believe that these forms will be useful for those who want to see how sensitive DCX is to particular nuclear models.

(3) It has been observed in both distorted wave and plane wave calculations that for incident pion energies below 200 MeV the introduction of the double-spin-flip amplitude decreases the cross section. This reduction seems to be present independent of the distortion (and some other factors, such as nuclear size). Because of the resilience of the destructive-interference effect, it is natural to suspect that the cancellation is *inherent in the DCX operator itself.* Thus, in order to obtain some understanding of this cancellation a simple model can be examined. We will see in Sec. III that this cancellation should indeed be expected.

(4) The plane-wave DCX operator resembles the one-pion-exchange potential since the dominant (or only) coupling to the nucleon is p wave in nature. In both operators there is a δ function present in the relative spatial coordinate in the limit of zero-range form factors. The question regarding the removal from the DCX operator is no doubt similar to that of the case of the nucleon-nucleon potential. However, for pion double charge exchange the possibility of a test by the direct measurement of cross sections exists, giving an alternative to inferences from nucleon-nucleon scattering.

(5) By looking at the double-scattering operator in general we will see that, because of the low-lying Δ_{33} resonance, classical pion propagation in the nucleus is subject to possible very large corrections. This observation brings into question intranuclear cascade calculations frequently used to calculate inclusive reactions, but also suggests a possible technique for correcting them.

We concentrate here primarily on the reaction which takes place on two active neutrons external to an inert core. If their angular momentum is nonzero, they may be coupled to other external neutrons (as in the case of the calcium isotopes) to form a spin-zero ground state.

From somewhat indirect arguments it has been inferred [7, 8] that the principal internucleon spacing being sampled is the order of 1 fm. This range is determined by the following three elements.

(1) The intrinsic coordinate dependence of the DCX operator, $\tilde{F}$, itself: This operator, which is the focus of this paper, is an intricate function of the coordinates of the active nucleons.

(2) The nuclear wave function or two-body density matrix: The correlations, even in a simple shell model, strongly affect the number of close neutron pairs, and so greatly influence the DCX cross section. The DCX cross section is obtained, in the impulse approximation, by integrating $\tilde{F}$ over the nuclear wave function. Our aim is to provide a convenient form for $\tilde{F}$ which the reader may then evaluate their own nuclear wave functions.

(3) The pion-nuclear optical potential: The pion wave function is distorted by the nuclear medium. For example, the absorptive nature of the potential tends to suppress DCX for configurations in which the nucleons are widely spaced; conversely, a pion-nucleus resonance in the optical potential might enhance DCX from widely spaced pairs.

A completely clean description of $\tilde{F}$ independent of the nuclear wave function and the optical distortions is not possible. In order to make some progress in this direction we invoke two approximations. First, we adopt the closure approximation on the nuclear intermediate wave function so that $\tilde{F}$ becomes a local function of the nucleon coordinates. This approximation could be improved by applying binding corrections to the active nucleons. Second, within the sequential mechanism, we will ignore the optical distortion of the intermediate π^0 meson. This is, unfortunately, only a rough approximation, although it may be partially softened by a choice of the effective energy of π^0. Although our results are strictly valid only within the plane-wave impulse approximation (PWIA), optical distortions of the incident π^+ and outgoing π^- may in principle be included by (a) replacing the exponential factor in front of Eq. (3) by the overlap of the incident and outgoing pionic (distorted) wave functions and (b) treating the initial and final pion momenta, $\mathbf{k}$ and $\mathbf{k}'$, as derivative operators on the incoming and outgoing pion distorted waves, respectively. Consequently, certain aspects of our results hold in the distorted-wave case. The advantage of our approach is that the operator F (which is $\tilde{F}$ with the external pion waves removed) is then a function of the *relative* coordinates of the two neutrons rather than of the coordinates of both neutrons. Consequently, we will be able to express the results in terms of a single central function and its derivatives.

In this way one can explictly examine certain features of the problem, such as the vanishing of the "long-range term" for the two nucleons aligned along the z axis at the same energy that the 0° single charge exchange cross section has a deep minimum or that the "double-spin-flip" part of the amplitude is of "near-zone" character, i.e., gives no contribution to the amplitude at long range. Because the farther the intermediate π^0 travels, the more its propagation is affected by the nuclear medium, we expect that the model is more reliable for DCX from close neutron pairs. The longer-range results might be also expected to be relevant, at least qualitatively, at low pion energies, for which the nucleus is relatively transparent.

Although the emphasis in this paper is on the DCX process, the same general formulas will also apply to any double-scattering process of a spin-0 particle on a pair of nucleons for which the s- and p-wave elementary amplitudes dominate.

II. GENERAL FORMULATION

The sequential (double-scattering) operator for the pion double-charge-exchange amplitude in the plane wave limit and assuming closure over the intermediate nuclear states is

$$\tilde{F}(\mathbf{k},\mathbf{k}';\mathbf{r_1},\mathbf{r_2}) = \frac{1}{2\pi^2}\int d\mathbf{q}\frac{e^{-i\mathbf{k}'\cdot\mathbf{r_2}}f(\mathbf{q},\mathbf{k}')e^{i\mathbf{q}\cdot(\mathbf{r_2}-\mathbf{r_1})}f(\mathbf{k},\mathbf{q})e^{i\mathbf{k}\cdot\mathbf{r_1}}}{q^2-\kappa^2-i\epsilon} \tag{1}$$

$$= \frac{e^{-i(\mathbf{k}'\cdot\mathbf{r_2}-\mathbf{k}\cdot\mathbf{r_1})}}{2\pi^2}\int d\mathbf{q}\frac{f(\mathbf{q},\mathbf{k}')e^{i\mathbf{q}\cdot\mathbf{r}}f(\mathbf{k},\mathbf{q})}{q^2-\kappa^2-i\epsilon} \tag{2}$$

$$= e^{-i(\mathbf{k}'\cdot\mathbf{r_2}-\mathbf{k}\cdot\mathbf{r_1})}F(\mathbf{k},\mathbf{k}',\mathbf{r}). \tag{3}$$

The DCX operator is a function of the coordinates $\mathbf{r_1},\mathbf{r_2}$ and spin variables $\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2$ (implicit in the two single charge exchange operators f) of the two nucleons. $\tilde{F}$ is the lowest-order two-nucleon DCX operator, and since it changes two neutrons (protons) into two protons (neutrons), it represents the isotensor part of the general two-nucleon operator. Here $\mathbf{r}$ ($\equiv \mathbf{r_2}-\mathbf{r_1}$) is the radial vector between the two nucleons. It is precisely the dependence of F on this variable which is of interest. The quantity f, the pion-nucleon charge-exchange (off-shell) amplitude, is an operator in the nucleon spin space. We shall use the form

$$f(\mathbf{q},\mathbf{q}',E) = \lambda_0(E)v(q)v(q') + \lambda_1(E)v(q)v(q')\mathbf{q}\cdot\mathbf{q}' + i\lambda_f(E)v(q)v(q')\boldsymbol{\sigma}\cdot\mathbf{q}\times\mathbf{q}', \tag{4}$$

where the λ_0, λ_1, and λ_f are taken from the phase shifts [9] (see Table I) and $v(q)$ is the off-shell form factor for the pion-nucleon interaction. While many of the results which follow are independent of the particular choice of $v(q)$, a form must be chosen when calculations are made. In these cases we will use

TABLE I. Table of λ's calculated from the phase shifts of Ref. [9].

$T_{\rm lab}$ (MeV)	$P_{\rm c.m.}$ (MeV/c)	λ_0 (fm)		λ_1 (fm^3)		λ_f (fm^3)	
		Real	Imag.	Real	Imag.	Real	Imag.
20	66.3	-0.187	-0.006	0.615	0.013	0.282	0.007
40	95.3	-0.185	-0.007	0.631	0.041	0.300	0.021
60	118.6	-0.184	-0.007	0.651	0.089	0.321	0.045
80	138.9	-0.183	-0.005	0.669	0.162	0.340	0.081
100	157.3	-0.181	-0.003	0.674	0.270	0.353	0.134
120	174.4	-0.180	0.000	0.640	0.415	0.345	0.206
140	190.5	-0.179	0.003	0.527	0.575	0.298	0.285
160	205.7	-0.177	0.006	0.321	0.680	0.203	0.337
180	220.3	-0.176	0.010	0.087	0.665	0.094	0.329
200	234.3	-0.175	0.014	-0.083	0.555	0.016	0.275
220	247.8	-0.173	0.018	-0.168	0.424	-0.020	0.210
240	260.9	-0.172	0.021	-0.195	0.312	-0.029	0.155
260	273.6	-0.170	0.025	-0.195	0.229	-0.024	0.116
280	285.9	-0.169	0.028	-0.184	0.169	-0.015	0.088
300	297.9	-0.167	0.031	-0.169	0.125	-0.005	0.070

$$v(q) = \frac{\alpha^2 + k^2}{\alpha^2 + q^2}, \tag{5}$$

which has a single parameter α to describe the range of the pion-nucleon interaction. For the figures we use $\alpha =$ 800 MeV/c. k is the on-shell momentum corresponding to the center-of-mass energy E. It is a simple matter to use independent ranges for each pion-nucleon partial wave, but for simplicity we have used a common range.

Since we have used the closure approximation to sum over all intermediate nuclear states, we have introduced the intermediate momentum κ, usually taken to be the on-shell momentum of the intermediate π^0. It could also include an effective excitation of the nucleus, chosen to improve the closure approximation. The reaction amplitude is given by the matrix element of this operator taken between the initial and final nuclear wave functions.

In general, all cross terms from the two occurrences of the SCX operators and the three terms in Eq. (4) interfere, but for a double-analog transition between shell-model wave functions made up of orbitals all with the same parity, the spin-dependent and spin-independent terms do not interfere, i.e., there are non-spin-flip (NSF) and double-spin-flip (DSF) terms only. We shall examine the structure of the operator for this case. Thus we have two terms:

$$F_{\rm NSF}(\mathbf{k},\mathbf{k}',\mathbf{r}) = \frac{1}{2\pi^2}\int d\mathbf{q}\frac{[\lambda_0 + \lambda_1\mathbf{q}\cdot\mathbf{k}'][\lambda_0 + \lambda_1\mathbf{k}\cdot\mathbf{q}]v^2(q)e^{i\mathbf{q}\cdot\mathbf{r}}}{q^2 - \kappa^2 - i\epsilon} \tag{6}$$

and

$$F_{\rm DSF}(\mathbf{k},\mathbf{k}',\mathbf{r}) = -\frac{\lambda_f^2}{2\pi^2}\int d\mathbf{q}\frac{[\boldsymbol{\sigma_1}\cdot\mathbf{k}\times\mathbf{q}]\ [\boldsymbol{\sigma_2}\cdot\mathbf{q}\times\mathbf{k}']\ v^2(q)e^{i\mathbf{q}\cdot\mathbf{r}}}{q^2 - \kappa^2 - i\epsilon}. \tag{7}$$

Replacing the vector $\mathbf{q}$ in the amplitudes by $-i\boldsymbol{\nabla}$ we can write the operators in terms of the function $g(r)$ (considered by Glauber [10]) and its derivatives:

$$F_{\rm NSF}(\mathbf{k},\mathbf{k}',\mathbf{r}) = (\lambda_0 - i\lambda_1\mathbf{k}\cdot\boldsymbol{\nabla})(\lambda_0 - i\lambda_1\mathbf{k}'\cdot\boldsymbol{\nabla})g(r), \tag{8}$$

$$F_{\rm DSF}(\mathbf{k},\mathbf{k}',\mathbf{r}) = \lambda_f^2(\boldsymbol{\sigma}_1\cdot\mathbf{k}\times\boldsymbol{\nabla})\ (\boldsymbol{\sigma_2}\cdot\boldsymbol{\nabla}\times\mathbf{k}')\ g(r), \tag{9}$$

where

$$g(r) = \frac{1}{2\pi^2}\int d\mathbf{q}\frac{v^2(q)e^{i\mathbf{q}\cdot\mathbf{r}}}{q^2 - \kappa^2 - i\epsilon}. \tag{10}$$

For large r, $-i\nabla g(r) = \kappa\hat{\mathbf{r}}g(r)$ so that, for $\mathbf{r}\|\mathbf{k}$ or $\mathbf{r}\|\mathbf{k}'$ and $|\mathbf{k}| = |\mathbf{k}'| = \kappa$ the amplitude contains a factor $\lambda_0 + \lambda_1 k^2$ and hence is small at 50 MeV where the single charge exchange is nearly zero. For short ranges the reaction still proceeds even for $\mathbf{r}$ aligned along the momenta.

For the model case [Eq. (5)]

$$g(r) = \frac{e^{i\kappa r} - e^{-\alpha r}}{r} - \frac{(k^2+\alpha^2)(k'^2+\alpha^2)}{2\alpha(\kappa^2+\alpha^2)}e^{-\alpha r}. \tag{11}$$

In the case $|\mathbf{k}| = |\mathbf{k}'| = \kappa$ we have, for small r,

$$g(r) \to C_0 + \tfrac{1}{2}C_2r^2 + \tfrac{1}{3}C_3r^3 + \cdots, \tag{12}$$

where

$$C_0 = \frac{(\alpha + ik)^2}{2\alpha},\ C_2 = -(2ik^3 + \alpha^3 + 3k^2\alpha)/6,$$
$$C_3 = (\alpha^2 + k^2)^2/8. \tag{13}$$

Note that for a function $v(q)$ with a stronger suppression for large q (such as a Gaussian) all odd coefficients would be zero.

For $|\mathbf{k}| = |\mathbf{k}'| = \kappa$ the needed derivatives are given by

$$g'(r) = \frac{ike^{ikr} + \alpha e^{-\alpha r}}{r} - \frac{e^{ikr} - e^{-\alpha r}}{r^2} + \frac{1}{2}(\alpha^2 + k^2)e^{-\alpha r}, \tag{14}$$

$$g''(r) = -\frac{k^2 e^{ikr} + \alpha^2 e^{-\alpha r}}{r} - 2\frac{ike^{ikr} + \alpha e^{-\alpha r}}{r^2} + 2\frac{e^{ikr} - e^{-\alpha r}}{r^3} - \frac{1}{2}\alpha(\alpha^2 + k^2)e^{-\alpha r}. \tag{15}$$

We note the appearance of three types of terms: (1) those of short range ($e^{-\alpha r}$), (2) those of intermediate range ("near zone") (e^{ikr}/r^2 and e^{ikr}/r^3), and (3) those of long range ("radiation zone") (e^{ikr}/r). Only the short-range terms are model dependent, the others are independent of the explicit form of $v(q)$. This last class of terms is due entirely to the p-wave nature of the pion-nucleon interaction.

Two useful combinations of these functions are

$$g^+(r) \equiv g''(r) + \frac{2}{r}g'(r) = \nabla^2 g(r) = -k^2\frac{(e^{ikr} - e^{-\alpha r})}{r} - \frac{\alpha(\alpha^2 + k^2)}{2}e^{-\alpha r} \tag{16}$$

and

$$g^-(r) \equiv g''(r) - \frac{1}{r}g'(r). \tag{17}$$

Note that $g^+(r)$ contains a function which limits to a δ function, $\frac{\alpha^3}{8\pi}e^{-\alpha r}$, so that

$$\lim_{\alpha\to\infty} g^+(r) = -k^2\frac{e^{ikr}}{r} - \frac{\delta(r)}{r^2}$$

or

$$\lim_{v(q)\to 1}(q^2 - k^2)g(r) = \frac{\delta(r)}{r^2}. \tag{18}$$

This last expression can also be obtained directly from Eq. (10). It is often argued that the δ function should not appear in the analogous case of the one-pion-exchange contribution to the nucleon-nucleon potential. The extraction of this term in the same manner as is done in the nucleon-nucleon problem [11] leads to the result that $g^+(r)$ becomes simply $-k^2 g(r)$. While it can be argued that the δ function is as inappropriate here as in the nucleon-nucleon case (all DCX calculations to date have included it to our knowledge), we will restrict ourselves to simply considering the two possibilities, i.e., with and without its inclusion.

While each of $g''(r)$ and $g'(r)/r$ contain a δ function, $g^-(r)$ is exactly the combination for which the δ functions cancel, in fact, $g^-(r)$ satisfies the condition

$$g^-(0) = 0. \tag{19}$$

Thus for large α, we expect that $g^+(r)$ is large and $g^-(r)$ is small for small r.

A. Spin-independent contribution

Evaluating Eq. (8)

$$F_{\mathrm{NSF}}(\mathbf{k},\mathbf{k}',\mathbf{r}) = \lambda_0^2 g(r) - i\lambda_0\lambda_1(\mathbf{k}\cdot\hat{\mathbf{r}} + \mathbf{k}'\cdot\hat{\mathbf{r}})g'(r) - \lambda_1^2\left[\tfrac{1}{3}\mathbf{k}\cdot\mathbf{k}'\ g^+(r) + (\mathbf{k}\cdot\hat{\mathbf{r}}\ \ \mathbf{k}'\cdot\hat{\mathbf{r}}\ - \tfrac{1}{3}\mathbf{k}\cdot\mathbf{k}')g^-(r)\right]. \tag{20}$$

The last term has the same form as the "tensor operator" $S_{12}(\mathbf{r}) \equiv \sigma_1\cdot\hat{\mathbf{r}}\ \sigma_2\cdot\hat{\mathbf{r}} - \frac{1}{3}\sigma_1\cdot\sigma_2$, so familiar in the nucleon-nucleon problem. In fact this last term has the same form as the one-pion-exchange potential [with replacement by the corresponding $g^+(r)$ and $g^-(r)$] except the spin operators have been replaced by momenta. The radiation (large r) limit of this expression is

$$(\lambda_0 + \lambda_1\mathbf{k}\cdot\boldsymbol{\kappa})(\lambda_0 + \lambda_1\boldsymbol{\kappa}\cdot\mathbf{k}')\frac{e^{i\kappa r}}{r}, \tag{21}$$

where we have set $\boldsymbol{\kappa} = \hat{\mathbf{r}}\kappa$.

The classical expression for the cross section for double charge exchange from two nucleons is simply the cross section for a single charge exchange on the first particle $[\sigma(\mathbf{k},\boldsymbol{\kappa})]$ multiplied by the probability that another charge exchange occurs on the second nucleon $[\sigma(\boldsymbol{\kappa},\mathbf{k}')/r^2]$. This product is equal to the absolute square of F_{NSF} in the radiation limit, given in Eq. (21). Since it is the classical expression which is used in modeling nuclear reactions with intranuclear cascades, it is of interest to compare the two expressions over the range of r of importance in the nucleus. Figure 1 shows the quantum and classical cross sections for $\mathbf{k} = \mathbf{k}'$, i.e., 0° and $|\mathbf{k}| = \kappa$ as a function of the angle between the two nucleons (equal to the two single-charge-exchange scattering angles). It is seen that the agreement between the two is very poor in the near-zone region. This potentially significant quantum correction is due largely to the properties of the pion in the following sense: the near-zone corrections arise from the p-wave nature of the interaction; if λ_1 were zero the expression would contain only the function $g(r)$, which has no near-zone components. For small κ, the near-zone corrections are particularly important, so it is precisely the pion-nucleon interaction, with its pre-

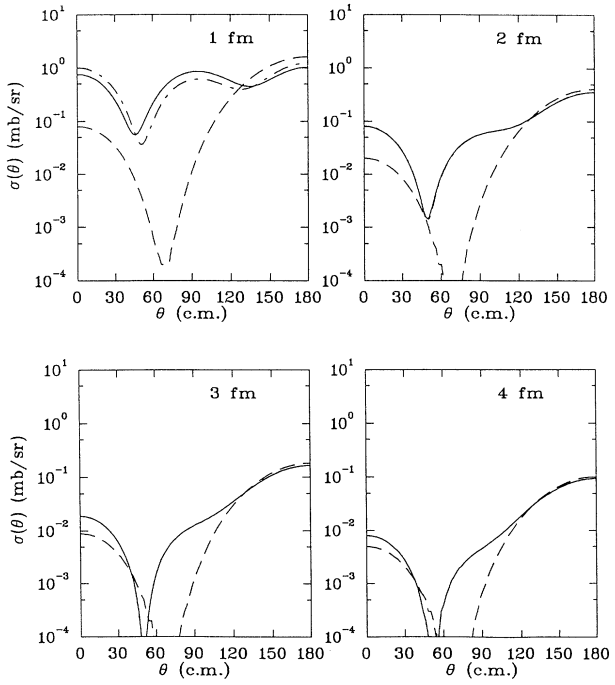

FIG. 1. Comparison of the classical and quantum-mechanical double-scattering cross sections at 100 MeV. The solid line represents the quantum-mechanical cross section with the δ function included and the dash-dotted curve the same quantity without it. The dashed line gives the classical expression.

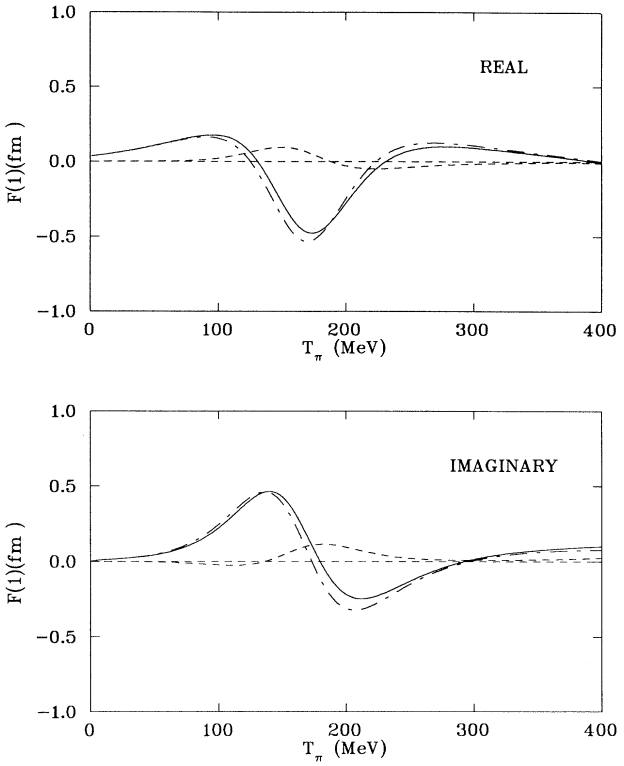

FIG. 2. Non-spin-flip and double-spin-flip DCX operators at $x = 0$ (90°) as a function of energy at an internucleon spacing of 1 fm. The solid line corresponds to the total, the dashed line to the DSF operator, and the dash-dotted line to the NSF operator.

cocious p-wave strength due to the 3-3 resonance, which gives the strongest violation of the classical limit.

For 0° DCX to the double analog state ($\mathbf{k}' = \mathbf{k}$) and $\kappa = k$ we find

$$F_{\rm NSF}(\mathbf{k},\mathbf{k},\mathbf{r}) = \sum_{L=0,1,2} g_L(r) P_L(x), \tag{22}$$

where $x = \hat{\mathbf{k}} \cdot \hat{\mathbf{r}}$ and

$$g_0(r) = \lambda_0^2 g(r) - \frac{\lambda_1^2 k^2}{3} g^+(r), \quad g_1(r) = -2i\lambda_0\lambda_1 k g'(r),$$
$$g_2(r) = -\tfrac{2}{3}\lambda_1^2 k^2 g^-(r). \tag{23}$$

The dash-dotted line in Fig. 2 shows the non-spin-flip DCX operator at $x = 0$ (90 °) as a function of energy at $r = 1$ fm. We have extended the plots into the resonance region even though the plane-wave impulse approximation is a very poor approximation in an absolute sense, to illustrate the comparison between spin-flip and nonflip components. This configuration, in which the nucleons are oriented perpendicular to the beam axis, provides the most important contribution to DCX for pions of energy of approximately 50 MeV; at this energy the single-charge-exchange cross section is nearly zero in the forward direction.

B. Double-spin-flip term

For the NSF part of the operator the spin structure was irrelevant but for the DSF it is necessary to assume something about the singlet-triplet decomposition of the two-nucleon wave function. The Appendix develops the equations necessary to treat the general case, but here we assume the pure singlet case for which the operator is [see Eq. (48)]

$$\mathbf{e}_1 \cdot \mathbf{e}_2 = -(\mathbf{k} \times \mathbf{q}) \cdot (\mathbf{k}' \times \mathbf{q}) = -\mathbf{k} \cdot \mathbf{k}' q^2 + (\mathbf{k} \cdot \mathbf{q})(\mathbf{k}' \cdot \mathbf{q}), \tag{24}$$

which in coordinate space is

$$\mathbf{k} \cdot \mathbf{k}' \nabla^2 - (\mathbf{k} \cdot \boldsymbol{\nabla})(\mathbf{k}' \cdot \boldsymbol{\nabla}). \tag{25}$$

The singlet projection of the DSF operator becomes

$$F^0_{\rm DSF}(\mathbf{k},\mathbf{k}',\mathbf{r}) = \lambda_f^2 [\tfrac{2}{3}\mathbf{k} \cdot \mathbf{k}'\, g^+(r) + (\mathbf{k} \cdot \hat{\mathbf{r}}\ \ \mathbf{k}' \cdot \hat{\mathbf{r}} - \tfrac{1}{3}\mathbf{k} \cdot \mathbf{k}') g^-(r)]. \tag{26}$$

At $\mathbf{k} = \mathbf{k}'$,

$$F^0_{\rm DSF}(\mathbf{k},\mathbf{k},\mathbf{r}) = \sum_{L=0,2} s_L(r) P_L(x), \tag{27}$$

where

$$s_0(r) = \tfrac{2}{3}\lambda_f^2 k^2 g^+(r), \quad s_2(r) = -\tfrac{2}{3}\lambda_f^2 k^2 g^-(r). \tag{28}$$

The dashed line in Fig. 2 shows the DSF operator at $x = 0$ and $r = 1$ fm as a function of energy.

It is curious that even if $x = 1$ (both nucleons lined up in the beam direction) the double-flip term is not zero. At first sight this seems paradoxical since each scatter would then be in the forward direction where each single-spin-flip amplitude vanishes. The contribution comes completely from the off-shell and near-zone regions of $F^0_{\rm DSF}$. Its contribution to the projected amplitude from x near 1 (i.e., the nucleons lined up in the beam direction) is comparable to the nonflip amplitude at 50 MeV. The cancellation is exact (as it must be) in the far zone, since for $x = 1$ $F_{\rm DSF}$ is proportional to $g^+(r) - g^-(r) = 3g'(r)/r$ which falls as $1/r^2$.

III. PROJECTED OPERATOR

As pointed out in the Introduction, the DCX amplitude is obtained by integrating $\tilde{\mathbf{F}}(\mathbf{k}, \mathbf{k}', \mathbf{r}_1, \mathbf{r}_2)$ over the wave function of the two active neutrons. Although $\tilde{F}$ is a function of both vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, for forward scattering ($\mathbf{k} = \mathbf{k}'$) it depends only on the relative coordinate, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ as can be seen from Eq. (3). The most important component of the wave function [12] is that in which the neutron pair is in a relative s state (and hence in a spin-singlet state). For this case, the angular integration over $\mathbf{r}$ is easily performed and leads to Q(r), a function only of the distance between the neutrons. In this section we will examine this function in detail:

$$Q(r) \equiv \frac{1}{2}\int_{-1}^{1} dx e^{-i\mathbf{k}\cdot\mathbf{r}} \{[g_0(r) + s_0(r)]\, P_0(x) + g_1(r) P_1(x) + [g_2(r) + s_2(r)]\, P_2(x)\} \tag{29}$$

$$= j_0(kr)\,[g_0(r) + s_0(r)] - i j_1(kr) g_1(r) - j_2(kr)\,[g_2(r) + s_2(r)] \tag{30}$$

$$= j_0(kr)\lambda_0^2 g(r) - \frac{k^2}{3} j_0(kr)(\lambda_1^2 - 2\lambda_f^2) g^+(r) - 2\lambda_0\lambda_1 k g'(r) j_1(kr) + \frac{2k^2}{3} j_2(kr)(\lambda_1^2 + \lambda_f^2) g^-(r). \tag{31}$$

The short-range part of this operator contains a δ function in the limit of large α:

$$\lim_{\alpha\to\infty} Q(r) = -\frac{k^2}{3}(\lambda_1^2 - 2\lambda_f^2)\delta(r)/r^2 + \cdots. \tag{32}$$

Here the explicit cancellation between the NSF and DSF terms is evident. For Δ_{33} dominance $\lambda_f = \frac{1}{2}\lambda_1$ so that the short-range part of the operator is decreased by a factor of 2 by the inclusion of double spin flip. Figure 3 shows the absolute value of the projected operator with and without the inclusion of the double spin flip. It is seen that when the DSF operator is important below 300 MeV it diminishes the sensitivity of DCX to the short-range part of the nucleon-nucleon wave function.

It is interesting to consider the large r limit of Eq. (31),

$$\lim_{r\to\infty} Q(r) = \frac{-i}{2kr^2}\left[(\lambda_0 - \lambda_1 k^2)^2 e^{2ikr} - (\lambda_0 + \lambda_1 k^2)^2\right]. \tag{33}$$

This limit is most useful at low energies, for which the pion mean free path within the nucleus is large. The double-spin-flip contributions do not appear, they have canceled exactly. Also noteworthy is that the coefficient of the nonoscillating part of the expression is proportional to the square of the forward single-charge-exchange operator, which is very small near 50 MeV. What remains is an operator proportional to $\frac{e^{2ikr}}{r^2}$. The expectation value of this residual operator will tend toward zero as k becomes large. Thus we see again that the long-range part of the DCX amplitude is small near 50 MeV to the extent that the form factor constrains each individual single charge exchange to take place in the forward direction. Figure 4 shows the behavior as a function of energy of the two coefficients in Eq. (33) as well as that of the δ function.

Of course Eq. (33) represents only the large r limit, and we must look at the complete expression to observe the rest of the dependence. To this end we write the operator as the sum of two terms, one representing the part that oscillates at long range and another which goes as $1/r^2$ for large r:

$$Q = Q^+ + Q^-. \tag{34}$$

This can be done in a simple (nonunique) fashion by writing the Bessel functions in Eq. (31) in terms of spherical Hankel functions:

$$Q^\pm(r) = \frac{h_0^\pm(kr)[g_0(r) + s_0(r)] - i h_1^\pm(kr) g_1(r) - h_2^\pm(kr)[g_2(r) + s_2(r)]}{2}. \tag{35}$$

Figure 5 shows the behavior of $r^2Q^{\pm}(r)$ the projected operator as a function of r at 200 MeV. The oscillatory behavior of Q^+ and the total operator is apparent while Q^- is nearly constant.

To reveal the difference in the "range" of the oscillating part and the "$1/r^2$" part, the total operator has been integrated over a Gaussian sample function with a radial extent R by multiplying by $\psi^2(R,r)$, where

$$\psi(R,r) = Ne^{-\frac{3r^2}{4R^2}}, \qquad N^2 = \frac{3}{R^3}\sqrt{\frac{6}{\pi}}, \tag{36}$$

and $R^2 = \langle r^2 \rangle$:

$$\bar{Q}^{\pm}(R) \equiv \int_{r_{\min}}^{\infty} r^2 dr \psi^2(R,r) Q^{\pm}(r). \tag{37}$$

Figure 6 shows the absolute integrated operator for $r_{\min} = 1$ fm at 200 and 50 MeV as a function of R. At 200 MeV the sum approaches $\bar{Q}(R)$ for large r so the oscillating part dies out. At 50 MeV the same thing must happen eventually but because of the smaller value of k the cancellation between Q^+ and Q^- dominates over the nuclear volume.

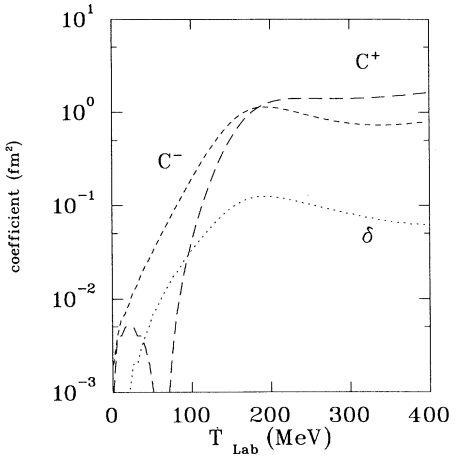

FIG. 4. Behavior as a function of energy of the two coefficients in Eq. (33) as well as that of the δ function. $C^{\pm} \equiv |\lambda_0 \pm \lambda_1 k^2|^2$.

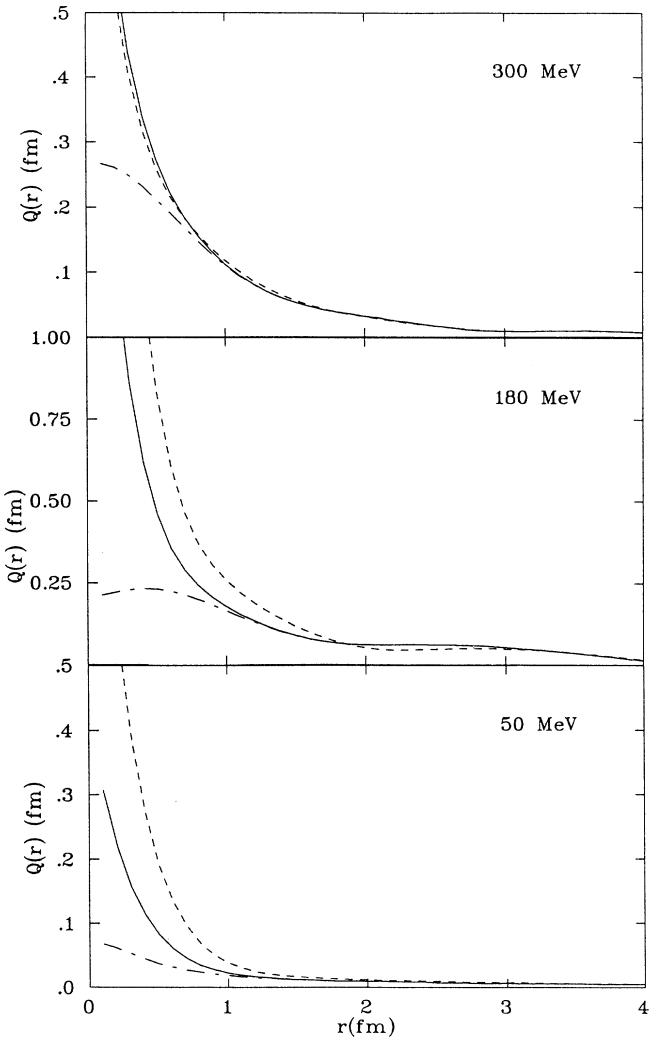

FIG. 3. Absolute value of the projected operator with (solid curve) and without (dotted curve) the inclusion of the double-spin-flip amplitude. The dash-dotted curve is the absolute value of the sum without the δ function.

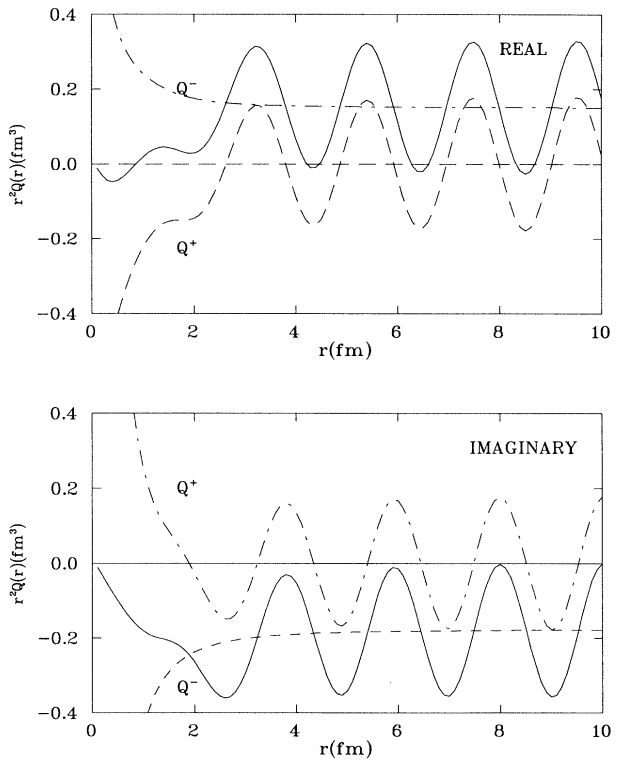

FIG. 5. Behavior of the real and the imaginary parts of $r^2Q^{\pm}(r)$ as a function of r at 200 MeV. The oscillatory behavior of Q^+ and the total operator is apparent while Q^- is nearly constant.

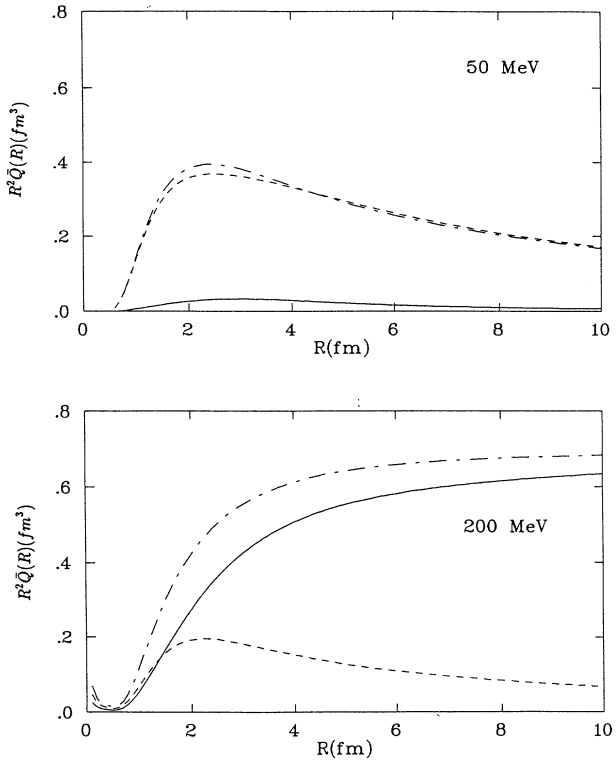

FIG. 6. Absolute value of the integrated operator $\bar{Q}(R)$ from 1 fm at the energies (a) 200 MeV and (b) 50 MeV as a function of R. The dotted curve gives $\bar{Q}^{+}(R)$, the dash-dotted curve is $\bar{Q}^{-}(R)$, and the solid curve is the sum of the two.

IV. CONCLUSIONS

In this paper we have presented explicit forms for the DCX operator [Eqs. (20) and (26) and Appendix for the general spin state of the struck nucleon pair]. These expressions are suitable for integration over model nuclear wave functions. The operators corresponding to forward scattering are particularly simple [Eqs. (22), (23), (26), and (27)]. We have also examined several qualitative features of the operator and have tracked down the origin of the strong cancellation between the nonflip and spin-flip amplitudes which is so prominent in previous calculations. Further, the question was raised as to whether the portion of the operator which is proportional to $\delta(\mathbf{r})$ (in the $\alpha \to \infty$ limit) should be removed as is sometimes an issue in the one-pion-exchange contribution to NN scattering. This may have dramatic effects on the flip-nonflip cancellation just mentioned. Lastly, we have examined the somewhat intricate $r = |\mathbf{r}_1 - \mathbf{r}_2|$ dependence of the DCX operator.

The DCX operator has three basic behaviors corresponding to three "ranges" in the internucleon spacing.

The first behavior corresponds to a classical range in that it falls off with a rapid decrease after a characteristic distance (with the monopole function used for illustration it has an exponential fall off with range $1/\alpha$). The form of the operator in this zone depends on the inclusion (or not) of the δ function.

After this initial short-range phase of the operator there is a "near-zone" range which is of varying importance depending on the energy of the reaction. In a typical case this range lies within the nuclear volume.

The specification of the next two behavioral characteristics is not as simple. Beyond the near-zone region the function can be separated into one part which behaves as e^{2ikr}/r^2 and another which goes as $1/r^2$. In the mean (i.e., after integrating over a wave function) the oscillating function will decrease more rapidly than the other with the increasing size of the system. In this sense it has a "shorter range," the characteristic size of which is determined by $\frac{1}{2k}$.

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APPENDIX: SPIN REDUCTION FOR THE DOUBLE-SPIN-FLIP TERM

We define the basic tensor operators by

$$t^{+1} = -(t_x + it_y)/\sqrt{2}, \quad t^{-1} = (t_x - it_y)/\sqrt{2}, \quad t^0 = t_z, \tag{A1}$$

and a tensor operator made up of a bilinear combination of two of these elementary quantities $\mathbf{t}_1$ and $\mathbf{t}_2$ by

$$T_L^M = \sum_{\mu,\mu'} C^{1,1,L}_{\mu,\mu',M} t_1^{\mu} t_2^{\mu'}. \tag{A2}$$

In this notation the dot product is

$$\mathbf{t}_1 \cdot \mathbf{t}_2 = \sum_{\mu} t_1^{\mu} t_2^{-\mu} (-1)^{\mu} = -\sqrt{3} T_0^0, \tag{A3}$$

and the inversion is given by

$$t_1^{\mu} t_2^{\mu'} = \sum_{L,M} C^{1,1,L}_{\mu,\mu',M} T_L^M. \tag{A4}$$

To express the operator

$$\mathcal{O} = (\boldsymbol{\sigma}_1 \cdot \mathbf{k} \times \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q} \times \mathbf{k}') \tag{A5}$$

in tensor form we put $\mathbf{e}_1 = \mathbf{k} \times \mathbf{q}$, $\mathbf{e}_2 = \mathbf{q} \times \mathbf{k}'$, and write

$$\mathcal{O} = \sum_{\mu} \boldsymbol{\sigma}_1^{-\mu} e_1^{\mu} (-1)^{\mu} \times \sum_{\mu'} \boldsymbol{\sigma}_2^{-\mu'} e_2^{\mu'} (-1)^{\mu'}$$

$$= \sum_{\mu,\mu'} \sigma_1^{-\mu} \sigma_2^{-\mu'} e_1^{\mu} e_2^{\mu'} (-1)^{\mu+\mu'} \tag{A6}$$

$$= \sum_{L,M} (-1)^{L+M} \Sigma_L^{-M} E_L^M, \tag{A7}$$

where Σ and E are defined as bilinear tensor operators in

terms of the Pauli matrices $(\boldsymbol{\sigma_1}, \boldsymbol{\sigma_2})$ and the pair $(\mathbf{e_1}, \mathbf{e_2})$.

The operators E_L^M are

$$E_0^0 = -\frac{1}{\sqrt{3}}\mathbf{e}_1 \cdot \mathbf{e}_2, \tag{A8}$$

$$E_1^1 = \tfrac{1}{2}[e_1^z(e_2^x + ie_2^y) - (e_1^x + ie_1^y)e_2^z] = \frac{-i}{2}[(\mathbf{e_1} \times \mathbf{e_2})^x + i(\mathbf{e_1} \times \mathbf{e_2})^y], \tag{A9}$$

$$E_1^0 = \frac{i}{\sqrt{2}}(e_1^x e_2^y - e_1^y e_2^x) = \frac{i}{\sqrt{2}}(\mathbf{e_1} \times \mathbf{e_2})^z, \quad E_1^{-1} = E_1^{1*}, \tag{A10}$$

$$E_2^2 = \tfrac{1}{2}(e_1^x + ie_1^y)(e_2^x + ie_2^y), \tag{A11}$$

$$E_2^1 = -\tfrac{1}{2}[(e_1^x + ie_1^y)e_2^z + e_1^z(e_2^x + ie_2^y)], \tag{A12}$$

$$E_2^0 = \sqrt{\frac{3}{2}}e_1^z e_2^z - \frac{1}{\sqrt{6}}\mathbf{e_1} \cdot \mathbf{e_2}, \quad E_2^{-1} = -E_2^{1*}, \quad E_2^{-2} = E_2^{2*}. \tag{A13}$$

Since for a double-analog transition the intitial and final states are the same, only the symmetric combinations will enter.

$$E_2^1 + E_2^{-1} = i(e_1^y e_2^z + e_1^z e_2^y), \quad E_2^2 + E_2^{-2} = e_1^x e_2^x - e_1^y e_2^y. \tag{A14}$$

For $\hat{\mathbf{k}}=\hat{\mathbf{k}}'=\hat{\mathbf{z}}$, since $e_1^z = e_2^z = 0$ and $\mathbf{e_1} = -\mathbf{e_2}$,

$$\begin{aligned} E_0^0 &= \mathbf{e_1} \cdot \mathbf{e_1}/\sqrt{3}, \\ E_1^M &= 0, \\ E_2^2 + E_2^{-2} &= (e_1^x)^2 - (e_1^y)^2, \\ E_2^1 = E_2^{-1} &= 0, \\ E_2^0 &= -\tfrac{1}{\sqrt{6}}\mathbf{e_1} \cdot \mathbf{e_2} \\ &= \tfrac{1}{\sqrt{6}}\mathbf{e_1} \cdot \mathbf{e_1}. \end{aligned} \tag{A15}$$

We can evaluate the spin operator in the singlet-triplet basis using the Wigner-Eckart theorem:

$$\langle S\Sigma \mid \Sigma_L^M \mid S'\Sigma' \rangle = C_{M,\Sigma',\Sigma}^{L,S',S} \langle S \parallel \Sigma_L \parallel S' \rangle. \tag{A16}$$

The reduced matrix elements are

$$\langle 0 \parallel \Sigma_0 \parallel 0 \rangle = \sqrt{3}, \quad \langle 1 \parallel \Sigma_0 \parallel 1 \rangle = -\frac{1}{\sqrt{3}}, \tag{A17}$$

$$\langle 1 \parallel \Sigma_2 \parallel 1 \rangle = 2\sqrt{\frac{5}{3}}, \quad \langle 0 \parallel \Sigma_1 \parallel 1 \rangle = \sqrt{6}, \quad \langle 1 \parallel \Sigma_1 \parallel 1 \rangle = 0. \tag{A18}$$

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