Description of alpha clustering including continuum configurations

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Alpha clustering is analyzed within a shell-model-like Berggren representation which includes the bound and outgoing single-particle resonant states of a realistic Woods-Saxon potential. It is found that the clustering of the alpha particle in ^{212}Po, leading to α decay, can be described even for distances which are well beyond the surface of the daughter nucleus.

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The calculation of α-decay widths, which has been one of the current challenges of nuclear theory for many years, requires a good description of the preformed α particle in the mother nucleus. Since the α particles being emitted are those formed either at the surface of the nucleus or beyond it, a reasonable microscopic description of the clustering requires the use of a realistic finite single-particle potential, including its continuum states.

The importance of configuration mixing in the shell model wave function of the alpha-like particle was realized already in the earliest microscopic calculations of alpha decay [1–3]. Nevertheless an attempt to mock up the continuum part (through a large number of harmonic oscillator single-particle states) was not intended until comparatively recently [4]. Further evidence of the role played by highly excited single-particle states, this time bound by a realistic Woods-Saxon potential, to induce the clustering of pairs of nucleons was obtained later [5]. This led to the introduction of pairing giant resonances [6].

Parallel to this development, the study of particle-hole giant resonances had reached a critical point. Their description by means of harmonic oscillator representations had been tremendously successful to account for properties such as energies and sum rules. However, there are other quantities, such as escape widths, which require a proper treatment of the continuum. This was evident when the first experimental data on partial decay width became available and important disagreements between theory and experiment were found [7, 8].

To deal with the difficult task of treating the continuum properly an alternative procedure was presented in Ref. [9]. In this procedure one uses a single particle basis which is composed of bound states and single-particle resonances (Gamow states) of a realistic local potential. This expansion, i.e., the use of a completeness relation in which bound, resonant, and scattering states are involved, was proposed by Berggren [10]. A truncation of the Berggren expansion by neglecting the contribution of the scattering states was used to calculate the escape widths of the multipole giant resonances in ^{208}Pb (Ref. [9]). A very similar approach in which the single particle Green function was expanded on the same basis gave a remarkably good agreement with the exactly calculated quantities in an analytically solvable model [11, 12].

Encouraged by these results we will in this paper treat the alpha clustering of ^{212}Po (leading to α decay) by using the truncated Berggren expansion as a representation of the shell model. We approximate the ^{212}Po wave function restricting it to a product of two-neutron and two-proton states:

$$|^{212}\mathrm{Po(g.s.)}\rangle = |^{210}\mathrm{Pb(g.s.)}\rangle \otimes |^{210}\mathrm{Po(g.s.)}\rangle. \quad (1)$$

It has been claimed that one of the possible causes for the persistent failure of the earlier calculations to give a good account of the experimental decay width may be a deficient treatment of the continuum, since in those calculations only bound representations (harmonic oscillator or other) were used [4, 13]. We choose first a Woods-Saxon potential in which the calculated bound state energies fit the available single-particle experimental data reasonably well [9]. Then we calculate the basis in this potential including not only the bound but also the resonant states. This basis is the same as the one used in Ref. [9] where the regularization procedure for normalizing Gamow states is explained in detail. It is worthwhile to point out that inside the nucleus the bound single-particle wave functions are very similar to those obtained with a harmonic oscillator potential or to the Sturmian solutions where the depth of the Woods-Saxon potential is adjusted each time

so that all single-particle energies coincide with a fixed value (e.g., half of the pair binding energy [5]). In fact, inside the nucleus, even resonant wave functions are similar to those carrying the same set of quantum numbers obtained with an infinite well potential. But beyond the surface region notable differences between these states start to be seen. With the basis presented in Ref. [9] we calculated the two-particle wave function using a surface delta residual interaction. In that reference it was discussed that the single-particle solutions corresponding to the wide resonances increase exponentially immediately from the nuclear surface and at the same time oscillate very rapidly. Therefore their contributions to the two-particle matrix elements are small and they are not very important in describing two-particle states (a detailed discussion of the criteria to use the basis is also given in Ref. [9]).

To analyze the clustering of the pairs of particles we follow the procedure of Ref. [5]. That is, we locate both particles at a given distance from the center of the nucleus and vary the angle θ between the two particles. Clustering occurs if the two-particle wave function is strongly peaked around θ=0 (see Refs. [5, 6]). In our case we found that considerable clustering is seen even at a distance of 11 fm from the nucleus center but there are also non-vanishing contributions for $\theta \neq 0$. In Fig. 1 we show this fact for the case of the ^{210}Pb ground state.

We now consider the α-particle formation amplitude

$$I_L(R) = \int d\xi_d d\xi_\alpha d\hat{R}\, \phi_d(\xi_d)\phi_\alpha(\xi_\alpha) Y_0^{*L}(\hat{R})\phi_m(\xi_d, \xi_\alpha, R), \tag{2}$$

where the indices d and m label the degrees of freedom of the daughter and mother nuclei, respectively. Taking into account that the function corresponding to the mother nucleus has been written as a product of two-neutron and two-proton wave functions [Eq. (1)], the amount of pair clustering (Fig. 1) should reflect in $I_L(R)$. As we have assumed that the nucleus ^{208}Pb is an inert core, the formation amplitude reduces to the overlap between the four-nucleon wave function and the internal wave function of the alpha particle. The four-nucleon wave function is expressed as an expansion in terms of four-nucleon configurations:

$$\Psi_4(\mathbf{n}_1, \mathbf{n}_2, \mathbf{p}_1, \mathbf{p}_2) = \mathcal{N}\left[\left[\phi^{j_1}(\mathbf{n}_1)\phi^{j_1}(\mathbf{n}_2)\right]^0 \left[\phi^{j_2}(\mathbf{p}_1)\phi^{j_2}(\mathbf{p}_2)\right]^0\right]_0^0 .$$

For the internal α-particle wave function we adopt a standard Gaussian form, i.e.,

$$\Phi(\mathbf{n}_1, \mathbf{n}_2, \mathbf{p}_1, \mathbf{p}_2) = (8\lambda/\pi)^{9/4} e^{-4\lambda(\rho_n^2+\rho_p^2+\sigma^2)}$$

with

$$\rho_n = \frac{\mathbf{n}_1 - \mathbf{n}_2}{\sqrt{2}}\ ,\ \rho_p = \frac{\mathbf{p}_1 - \mathbf{p}_2}{\sqrt{2}}\ ,$$
$$\sigma = \frac{(\mathbf{n}_1 + \mathbf{n}_2) - (\mathbf{p}_1 + \mathbf{p}_2)}{2}.$$

For the evaluation of Eq. (2) we must perform a 9-dimensional integration over the above three vector coordinates with the constraint that $\mathbf{R} = \frac{1}{4}(\mathbf{n}_1+\mathbf{n}_2+\mathbf{p}_1+\mathbf{p}_2)$. We note that a simpler integral could be possible if one makes an *a priori* assumption about two-nucleon clusterization (i.e., $\theta \equiv 0$ in Fig. 1), in which case no integration over the angular part is needed. This procedure will surely render larger α widths but they will correspond to the unreal neglect of the $\theta \neq 0$ contributions (Fig. 1). We therefore perform the full 9-dimensional integration by resorting to the method developed in Ref. [14]. This procedure is an extension to alpha-like structures of the Bayman-Kallio [15] method for two nucleons. It makes it possible to project out in an exact way the relative angular momentum, spin, and isospin-zero part of a wave function of four nucleons bound in a general single particle potential. For our purposes we have generalized the method of Ref. [14] in order to be able to include complex functions (resonant states).

A large value of I_L requires a large overlap between the mother wave function and the product of the α particle and daughter nucleus wave functions. That is, the more the clustering features are pronounced in the initial state, the larger is the value of the formation amplitude. Therefore, the formation amplitude as a function of R gives an indication of the clustering of the four nucleons that constitute the α particle. This can be seen by gradually increasing the number of configurations used to describe the mother nucleus because the influence of the continuum upon clustering proceeds through high-lying configurations [13]. This indeed happens in our case, as is seen in Fig. 2 where the amplitude is plotted as a function of the α center of mass radius. Actually the impressive increase of I_L as a function of the number of four-nucleon configurations seen in Fig. 2 is a feature which was also present in calculations where bound representations were used [4, 17]. The reason for that increase is that all configuration mixing components contribute with the same phase to the formation amplitude. One might then expect to obtain an unlimited value of the for-

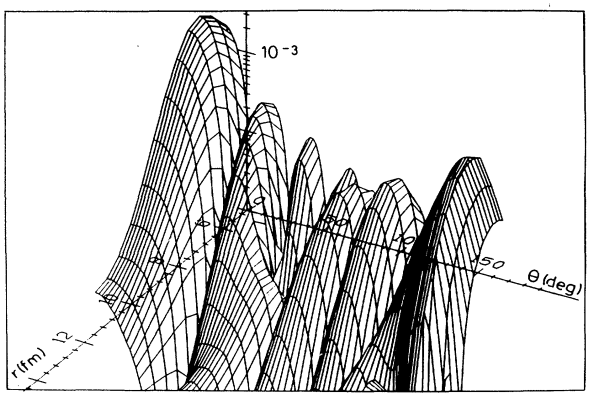

FIG. 1. Square of the two-neutron wave function, when $|\mathbf{n}_1| = |\mathbf{n}_2| = r$, plotted as a function of the relative angle θ and the radius r. The values of the radius, common to the two neutrons, are plotted starting from 6 fm.

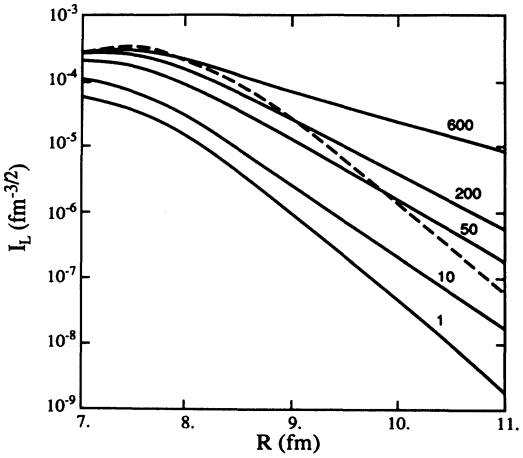

FIG. 2. The formation amplitude $I_L(R)$ as a function of the distance R. The solid lines correspond to the Berggren representation. The different curves are labeled by the number of configurations included in the calculation. The dashed line indicates the result of a calculation performed with a bound representation.

mation amplitude if enough configurations are included. This point was discussed in Ref. [17], where it was shown that the amplitude increases up to a certain limit, but after that a "saturation" is reached. This was to be expected, since the shell model should in principle be able to describe the clustering of the four nucleons, provided a sufficiently large configuration space is used. Since the clustering is better described, the value of the formation amplitude will increase.

The effect of using the Berggren representation can be assessed by comparing with the calculation performed with the same method of Ref. [14] but using a realistic *bound* representation (dashed line in Fig. 2). As seen in this figure, in that case the formation amplitude (2) is practically the same as the one obtained with the Berggren representation (solid lines) up to around 8.5 fm, but falls off much more rapidly beyond this radius. The remarkable feature of Fig. 2 is that the Berggren value of I_L does not diminish drastically as R increases. But, as was the case with the real (bound) representations mentioned above, one may wonder whether our calculated formation amplitude would diverge as more configurations are included. That this will not be the case is suggested by the fact that on the nuclear surface the Berggren representation reaches the value of the real representations, but no more. This will also be expected because within the Berggren representation one can, eventually, describe processes that occur far from the nuclear surface (i.e., high in the continuum [18]) by including even wider single particle terms. The convergence is not evident in Fig. 2 because the results there shown were obtained by a numerical procedure that takes one two-proton configuration at a time, beginning by the most bound, and multiplies it by a number of two-neutron states (taken also in increasing order of energy). Therefore, the clustering shown for a given number of configurations (i.e., 200) includes a few low energy p-p configurations and a large number (bound and continuum) n-n configurations and, hence, the rather considerable difference for large radii when including 200 or 600 configurations is mainly due to the addition of the continuum p-p configurations that have a significative amplitude beyond the nuclear radius.

Figure 2 shows that one has succeeded in maintaining the alpha particle clustering up to large distances outside the nuclear surface. This was just the purpose of using the Berggren representation in the description of alpha decay. However, in the region around the nuclear surface (where the alpha-decay process takes place [4, 7]) the maximum value of I_L is about the same in both representations. As a result, the disagreement between the calculated and experimental alpha decay width persists indicating that other process, as the coupling to other channels (polarization), or a proper treatment of the proton-neutron interaction [16], may be important.

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