Test of generalized seniority with pion double charge exchange on the nickel isotopes

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Recent experimental double charge exchange cross sections to the double analog state and the ground state are compared to predictions of the generalized seniority model.

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Recent (π^+,π^-) pion double charge exchange (DCX) experiments [1] on the nickel isotopes, with 292 MeV pions performed at the Clinton P. Anderson Meson Physics Facility (LAMPF), demonstrated that the differential cross section to the double isobaric analog state (DIAS) is consistent with the cross section predicted by generalized seniority [2,3]

$$\frac{d\sigma}{d\Omega}(\text{g.s.}; T=T_Z=N \to \text{DIAS}; T=N, T_Z=N-2)$$
$$= \left[\frac{60}{A_{\text{target}}}\right]^{4.04} N(2N-1)\left|\alpha+\frac{\beta}{(2N-1)}\right|^2, \quad (1)$$

which is identical to the cross section predicted in the seniority model [4,5]. In this expression N is the number of pairs of valence neutrons filling the ($n\ell_j = 1p_{3/2}, 0f_{5/2}, 1p_{1/2}$) single-particle orbitals outside of a doubly magic ^{56}Ni core. The complex amplitudes α and β are the long-range and short-range parts, respectively, of the DCX reaction on two valence neutrons [2]. The factor $\left[\frac{60}{A_{\text{target}}}\right]^{4.04}$ takes into account the pion distortion where A_{target} is the atomic mass of the target [1]. This cross section (1) is fitted to the experimental cross section [1] in Fig. 1 and the amplitudes α and β are extracted:

$$|\alpha| = 6.2\ [\text{nb/sr}]^{\frac{1}{2}}, \quad (2a)$$
$$|\beta| = 8.4\ [\text{nb/sr}]^{\frac{1}{2}}, \quad (2b)$$
$$\psi = 91^\circ, \quad (2c)$$

where ψ is the relative phase between α and β. Therefore the experimental cross section is consistent with generalized seniority, although it is not a stringent test, since there are only four cross sections and three real parameters.

For a target with $N=T=T_Z=1$ the DIAS is the ground state of the residual nucleus which also has $N=T=1$, but $T_Z=-1$. For $N>1$, however, the ground state of the residual nucleus will have isospin $T=N-2$. The authors of Ref. [1] predict the DCX cross section to the ground state of the neighboring zinc isotopes with $N>1$ using the cross section predicted by the seniority model [4,5] and are unable to reproduce these cross sections. Since the ground state of the residual zinc isotopes have neutrons and protons filling the same major shell, generalized seniority is not expected to be conserved and a fit is not expected.

However, in Ref. [2] it was shown that the cross section predicted by generalized seniority for transition to the ground state is not the same as that predicted by seniority. An additional long-range amplitude, γ, appears and a different N dependence is predicted:

$$\frac{d\sigma}{d\Omega}(\text{g.s.}; T=T_Z=N \to \text{g.s.}; T=T_Z=N-2)$$
$$= \left[\frac{60}{A_{\text{target}}}\right]^{4.04} K(N)\left|\gamma+\frac{\beta}{(2N-1)}\right|^2, \quad (3a)$$

where

$$K(N) = \eta_N^2(\bar\eta_N)^{-2}[N(N-1)]^2. \quad (3b)$$

In this expression $\eta_N(\bar\eta_N)^{-1}$ is the ratio of the normalization of the DIAS and the ground state of the neighboring zinc isotope, assuming it has good generalized seniority with isospin two units less than the target. This ratio can be calculated using the amplitudes c_j of the correlated $J=0$ pair of neutrons [2]:

$$S_1^\dagger = \frac{1}{2\sqrt{\Omega_e}}\sum_{j,m} c_j(-1)^{j-m}a_{jm}^\dagger a_{j-m}^\dagger, \quad (4)$$

where the effective occupancy is $\Omega_e = \sum c_j^2\,(j+\frac{1}{2})$, and

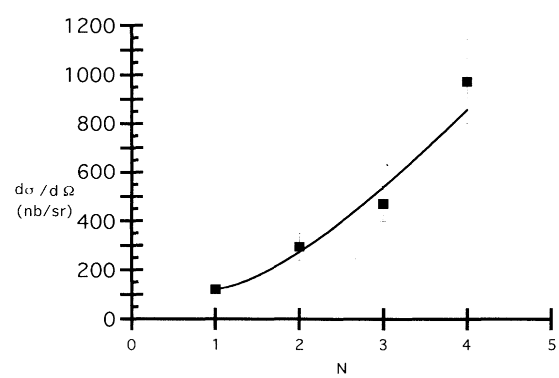

FIG. 1. The measured 292 MeV (π^+,π^-) Ni cross sections at 5° to the DIAS (black squares) vs the number of pairs of valence neutrons N [1]. The solid line is a fit using (1) with $|\alpha| = 6.2$ [nb/sr]$^{\frac{1}{2}}$, $|\beta| = 8.4$ [nb/sr]$^{\frac{1}{2}}$, and the relative phase $\psi = 91^\circ$.

TABLE I. The factor $K(N)$ for the shell model and for the strong pairing limit.

N	$K(N)$	
	Shell model	Strong pairing
2	1.9	1.7
3	3.6	3.6
4	6.3	7.4
5	10.0	14.4
6	19.5	35.5

$a^{\dagger}_{jm}$ creates a valence neutron with single-nucleon angular momentum j and projection m. It is only in the strong pairing limit, $c_j = 1$, that the generalized seniority limit becomes equivalent to the seniority limit and $\gamma = 0$ in this limit. Also in this limit,

$$K(N) = \frac{(2N-1)N(N-1)(2\Omega+1)}{(\Omega - N + 1)}, \quad c_j = 1, \quad (5)$$

where the total occupancy is $\Omega = \sum_j \left(j + \frac{1}{2}\right)$.

We used the c_j's determined from a shell model calculation [6,7] of the nickel isotopes; $c_{\frac{3}{2}} = 0.275$, $c_{\frac{5}{2}} = 0.158$, and $c_{\frac{1}{2}} = 0.157$. In Table I we tabulate $K(N)$ calculated for these values [2] and for the strong pairing limit, $c_j = 1$. We see that the K's grow much slower with respect to N compared to the strong pairing limit. In Fig. 2 the experimental ground-state cross sections are compared to those calculated in the strong pairing limit: $\gamma = 0$ and $K(N)$ given in column 2 of Table I (dotted line). Since this limit is the same as the seniority model, we see there is no agreement, as stated previously. In Fig. 2 we also compare the experimental cross section with generalized seniority (solid line): Eq. (3) and $K(N)$ given in column 1 of Table I. We extract γ,

$$|\gamma| = 1.3\,[\text{nb/sr}]^{\frac{1}{2}}, \quad (6a)$$

$$\Psi = 170^\circ. \quad (6b)$$

However, the calculated cross sections are at the extremes

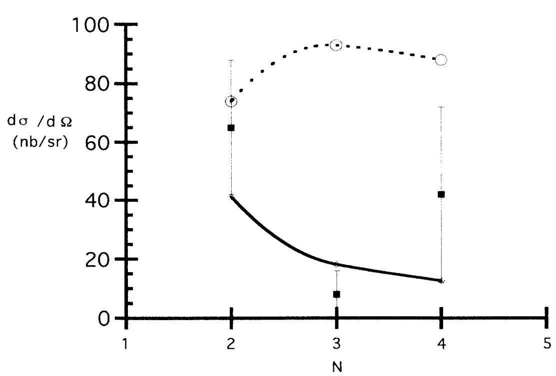

FIG. 2. The measured 292 MeV (π^+, π^-) Ni cross sections at 5° to the ground state (black squares) vs the number of pairs of valence neutrons N [1]. The solid line is a fit using (3) with $|\gamma| = 1.3$ [nb/sr]$^{\frac{1}{2}}$, $|\beta| = 8.4$ [nb/sr]$^{\frac{1}{2}}$, and the relative phase $\Psi = 170°$. The dashed line is a prediction with $|\gamma| = 0$.

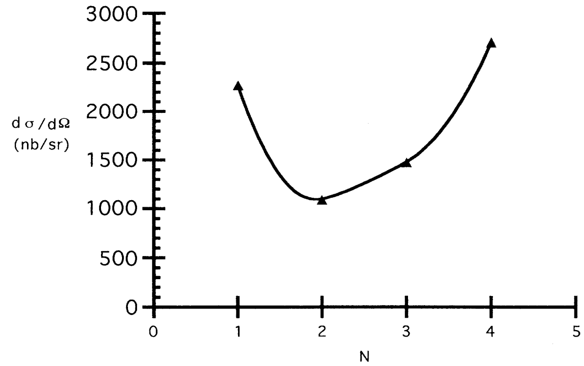

FIG. 3. The predicted 35 MeV (π^+, π^-) Ni cross sections at 25° to the DIAS (black diamonds) vs the number of pairs of valence neutrons N using (1) with $|\alpha| = 13.5$ [nb/sr]$^{\frac{1}{2}}$, $|\beta| = 56.0$ [nb/sr]$^{\frac{1}{2}}$, and the relative phase $\psi = 134°$.

of the error bars which are quite large. Clearly more accurate experimental cross sections need to be measured to determine whether the cross section predicted by generalized seniority can reproduce the experimental cross sections.

Experience on the calcium isotopes [4,5] has taught us that lower pion energies produce larger DCX cross sections to both the DIAS and the ground state. In Figs. 3 and 4 we plot the predicted cross sections to the DIAS and ground state for the nickel isotopes using α and β determined from 35 MeV pion DCX on the calcium isotopes,

$$|\alpha| = 13.5\ [\text{nb/sr}]^{\frac{1}{2}}, \quad (7a)$$

$$|\beta| = 56.0\ [\text{nb/sr}]^{\frac{1}{2}}, \quad (7b)$$

$$\psi = 134^\circ, \quad (7c)$$

Since systematics suggest that there is very little distortion at this pion energy we expect the α and β for calcium

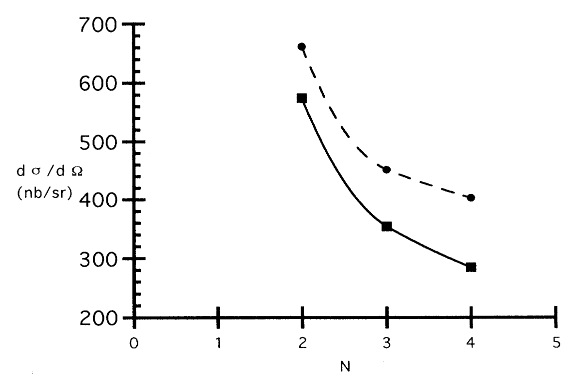

FIG. 4. The predicted 35 MeV (π^+, π^-) Ni cross sections at 25° to the ground state (black squares) vs the number of pairs of valence neutrons N using (3) with $|\gamma| = 1.3$ [nb/sr]$^{\frac{1}{2}}$, $|\beta| = 56.0$ [nb/sr]$^{\frac{1}{2}}$, and the relative phase $\Psi = 180°$. The black circles are a prediction with $|\gamma| = 0$.

isotopes to be roughly the same as for the nickel isotopes. The prediction for the DIAS cross section at 35 MeV has a very different shape compared to the cross section at 292 MeV. This is the result of the fact that β is much larger than α at this energy. From (1) we can see that, for N small, the cross section will decrease in this case and then increase for large N.

The transition to the ground state is plotted for the strong pairing limit ($\gamma = 0$) (dotted line) and for $\gamma = 1.3\ [\mathrm{nb/sr}]^{\frac{1}{2}}$, $\Psi = 180°$ (solid line). Of course γ will also vary with energy so this is not really a prediction but a suggestion that the larger cross section should make this transition easier to measure.

As stated earlier since there are only four nickel isotopes available as targets we cannot get a definitive test of generalized seniority for the DIAS transition. The tin isotopes, however, have eight isotopes as possible targets and would provide a more substantial test of generalized seniority. In Fig. 5 we plot the predicted DCX transition to the DIAS for 35 MeV pions at 25° using the amplitudes in (7). Because of the large number of pairs we see that the cross sections are much larger, making these measurements tractable; note that the scale is μb/sr whereas for the nickel isotopes it is nb/sr.

In summary, we conclude that the limited data on the DCX transition to the DIAS state is consistent with good generalized seniority in the nickel isotopes, but the limited data on DCX to the ground states of the neighboring zinc isotopes is ambiguous about the conservation of generalized seniority in these isotopes. Better data is needed and lower energies are suggested for larger cross sections, as well as the use of the tin isotopes as targets.

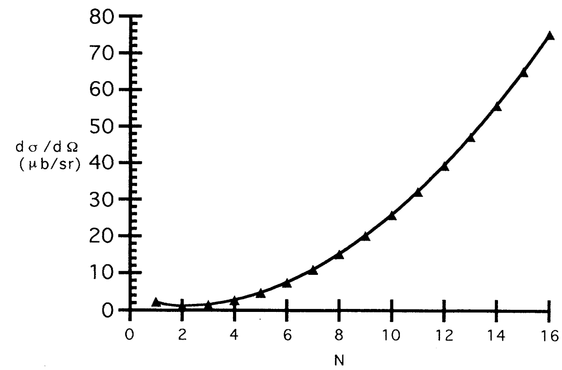

FIG. 5. The predicted 35 MeV (π^+, π^-) Sn cross sections at 25° to the DIAS (black diamonds) vs the number of pairs of valence neutrons N using (1) with $|\alpha| = 13.5\ [\mathrm{nb/sr}]^{\frac{1}{2}}$, $|\beta| = 56.0\ [\mathrm{nb/sr}]^{\frac{1}{2}}$, and the relative phase $\psi = 134°$.

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