Nature of the ^{20}Na 2646-keV level and the stellar reaction rate for ^{19}Ne$(p,\gamma)^{20}$Na

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Careful comparison of ^{20}F and ^{20}Na, and available data on reactions leading to both, suggests that $J^{\pi}(2646)=3^{+}$. Other mirror identifications are indicated and quantities of astrophysical interest are calculated.

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The region of excitation energy just above 2.2 MeV in ^{20}Na is important because of the possibility of a resonance in the astrophysically interesting [1–3] reaction ^{19}Ne$(p,\gamma)^{20}$Na. Above the ^{19}Ne+p threshold at 2199 keV, the first known state [4–7] is at 2646 keV (see Fig. 1). This state, which is presumed to be the strongest (p,γ) resonance, is strongly populated [4–6] in ^{20}Ne$(^3\mathrm{He},t)^{20}$Na and has been variously and incorrectly identified [4–6] as the mirror of the 3173- or 3488-keV 1^+ states in ^{20}F and even [8] as the mirror of a supposed 1^- state at 3173 keV in ^{20}F.

Despite the large Coulomb shifts observed and expected for states containing significant $2s_{1/2}$ strength, the 3488-keV 1^+ state is too far away for ^{20}Na(2646) to be its mirror. In fact, the Coulomb shift calculated using a single-particle potential and weighted by the known [7] (d,p) spectroscopic factor indicates a mirror for the 3488-keV state at 3080 keV in ^{20}Na. The 2986-keV state in ^{20}Na has been assigned $J^{\pi}=1^{+}$ on the basis of the beta-delayed proton decay of ^{20}Mg [9] and via ^{19}Ne+p elastic scattering [10]. Consequently, we associate this state with the 3488-keV state in ^{20}F.

The 3173-keV state in ^{20}F is probably a 1^+ state and most likely has a six-particle–two-hole (6p-2h) configuration [11], i.e., $(sd)^6_{01}(1p)^{-2}_{10}$ (where the subscripts denote angular momentum J and isospin T). This state is weakly populated in the ^{20}Ne$(t,^3\mathrm{He})^{20}$F reaction [12] and in all other reactions surveyed [11,13–17], lending further support to its low-J and core-excited nature [9]. The 2646-keV state in ^{20}Na cannot be its mirror because such a state would be weakly populated in the ^{20}Ne$(^3\mathrm{He},t)^{20}$Na reaction and would also have a negligible Coulomb shift. The remaining candidates for the mirror of ^{20}Na(2646) are [7] a 3^+ state at 2966 keV and probable (3^-) and (4^-) states at 2865 and 2968 keV, respectively. Although the 2857-keV state in ^{20}Na has been assigned [4,5] $J^{\pi}=3^{+}$ on the basis of $(^3\mathrm{He},t)$ angular distributions [which would then imply negative parity for ^{20}Na(2646)], the fits to these data are not particularly compelling. With the exception of 0^- and 1^- states arising from coupling of a $p_{1/2}$ hole to the $\frac{1}{2}^+$ excited state of ^{21}Ne (and ^{21}Na), other negative-parity mirror pairs should have negligible Coulomb shifts. In fact, the shifts may be slightly negative, with states in ^{20}Na lying slightly above their ^{20}F counterparts. This expectation is borne out by the 2^- and 3^- states at 1309 (1338) and 1971 (1993) keV, respectively, in ^{20}F (^{20}Na). Therefore it is likely that the 996-keV level of ^{20}Na contains mirrors of both the 984-keV 1^- and 1057-keV 1^+ states of ^{20}F. Similarly, the three states in ^{20}Na at 1841, 1993, and 2064 keV probably contain mirrors of five ^{20}F levels: 1824-keV 5^+, 1844-keV 2^-, 1971-keV 3^-, 2044-keV 2^+, and 2194-keV 3^+. Consequently, the 2865-keV (3^-) state in ^{20}F is probably analogous to the 2857-keV state in ^{20}Na rather than to the 2646-keV state. By a similar argument, the 2968-keV (4^-) state in ^{20}F could be the mirror to the 3056-keV state in ^{20}Na. However, this latter state has been assigned $J^{\pi}=0^{+}$ from ^{19}Ne+p scattering [10]. The first known [7] 0^+ state in ^{20}F is at 3526 keV. A calculation of the Coulomb shift for this state shows it to be the analog of the 3056-keV state in ^{20}Na.

By process of elimination (assuming that no states remain to be identified below 3.5 MeV in ^{20}F), the only ^{20}F state whose mirror could be ^{20}Na(2646) is at 2966 keV (3^+). Correspondence with states above 3.5 MeV in ^{20}F would imply unphysically large Coulomb shifts. With this identification, the $(t,^3\mathrm{He})$ and $(^3\mathrm{He},t)$ cross sections are consistent and the Coulomb shift is appropriate. Hence, in what follows, we take this assignment as determined, viz., $J^{\pi}(2646)=3^{+}$. Our mirror assignments are summarized in Fig. 1.

The quantity of interest for astrophysical applications is the thermonuclear reaction rate

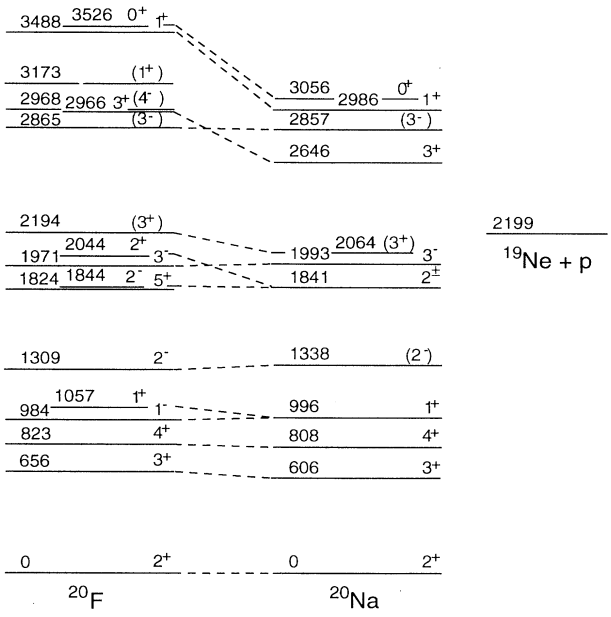

FIG. 1. Energy levels [6,7] of ^{20}F and ^{20}Na discussed in the text.

$$\langle \sigma v \rangle = \left[\frac{8}{\pi \mu} \right]^{1/2} (kT)^{-3/2} \int \sigma(E) E \exp(-E/kT) dE \ ,$$

where μ is the reduced mass and k is the Boltzmann constant. In general, the cross section $\sigma(E)$ contains contributions from resonant and nonresonant processes. For an isolated, narrow resonance at a center-of-mass energy $E_{\text{c.m.}}$, this expression reduces to

$$\langle \sigma v \rangle = (2\pi/\mu kT)^{3/2} \hbar^2 \omega\gamma \exp(-E_{\text{c.m.}}/kT) \ .$$

For a (p,γ) resonance, the resonance strength $\omega\gamma$ is given by

$$\omega\gamma = [(2J_f+1)/2(2J_i+1)] \Gamma_p \Gamma_\gamma / \Gamma \ .$$

Here J_f and J_i are the spins of the final and initial states, respectively, while Γ_p, Γ_γ, and Γ are the proton and gamma-ray partial and total widths of the resonance, respectively. Although the ^{19}Ne$(p,\gamma)^{20}$Na reaction has not been measured directly, it is possible to estimate resonance strengths using the known structure of the $A=20$ nuclei as a guide.

Resonant capture to the 2646-keV state ($E_{\text{c.m.}}=447$ keV, $J^\pi=3^+$) can proceed via a $1d_{5/2}$ proton coupled to the ground state of ^{19}Ne ($J^\pi=\frac{1}{2}^+$) or via a $2s_{1/2}$ proton coupled to the 238-keV first excited state ($J^\pi=\frac{5}{2}^+$) (in principle, other excited states could become important at extremely high temperatures). Proton single-particle widths of $\Gamma_{\text{SP}}(p_0)=9.7$ eV and $\Gamma_{\text{SP}}(p_1)=2.1$ eV were calculated within the framework of scattering from a real Woods-Saxon potential ($r_0=1.26$ $a=0.6$ fm). The $l=2$ spectroscopic factor for ^{20}F(2966) has been measured [15,18] in the ^{19}F$(d,p)^{20}$F reaction to be $S=0.054$. The corresponding shell-model value is $S=0.068$. For ^{19}Ne$^*+2s_{1/2}$, we take the shell-model value of $S=0.35$. Thus we obtain proton widths of $\Gamma_{p0}=0.52$ eV and $\Gamma_{p1}=0.73$ eV. Nothing is known of the γ decays of ^{20}Na(2646). Its mirror has a mean lifetime [7] of 60±40 fs, decaying primarily via $M1$'s to the 2_1^+, 3_1^+, and 4_1^+ levels. The shell-model lifetime for this state is 3.5 fs (see Table I), more than an order of magnitude shorter than the measured value, but consistent within the experimental uncertainty. [One prediction of the current work is that if the ^{20}F(2966) lifetime were to be better measured, a much smaller value should result.] Correcting for the E_γ^3 factor and applying a small N,Z correction in the $M1$ operator [19], the mean lifetime in ^{20}Na should be 5.5 fs, i.e., $\Gamma_\gamma=0.12$ eV.

A similar procedure may be used to extract resonance

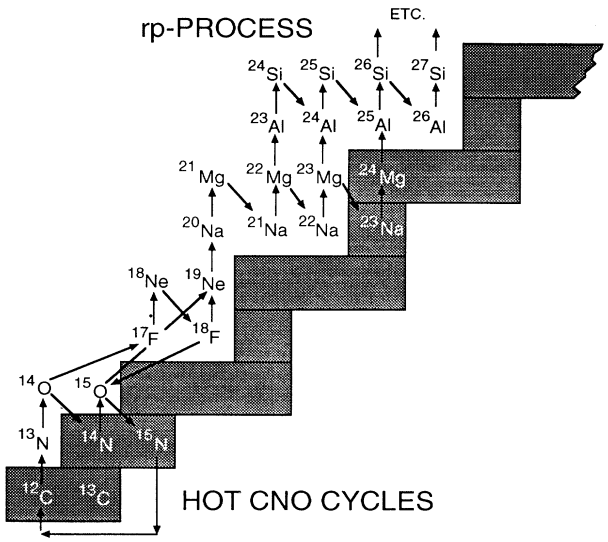

FIG. 2. The hot CNO cycles and early *rp* process. More detailed reaction networks may be found in Ref. [3] and references therein.

TABLE I. Expected $M1$ decays of ^{20}F(2966) and ^{20}Na(2646) from shell-model calculations.[a]

	^{20}F				^{20}Na			
J_f^π	E_f (MeV)	E_γ (MeV)	$B(M1)$	τ (fs)	E_f (MeV)	E_γ (MeV)	$B(M1)$	τ (fs)
2_1^+	0.0	2.966	0.285	7.6	0.0	2.646	0.300	10.2
3_1^+	0.656	2.310	0.160	28.7	0.606	2.040	0.181	36.9
4_1^+	0.823	2.143	0.676	8.5	0.808	1.838	0.523	17.5
Total				3.5				5.5

[a]The wave functions and effective operators used for the $M1$ matrix elements are described in Ref. [19]. The $E2$ contributions are too small to cause appreciable changes.

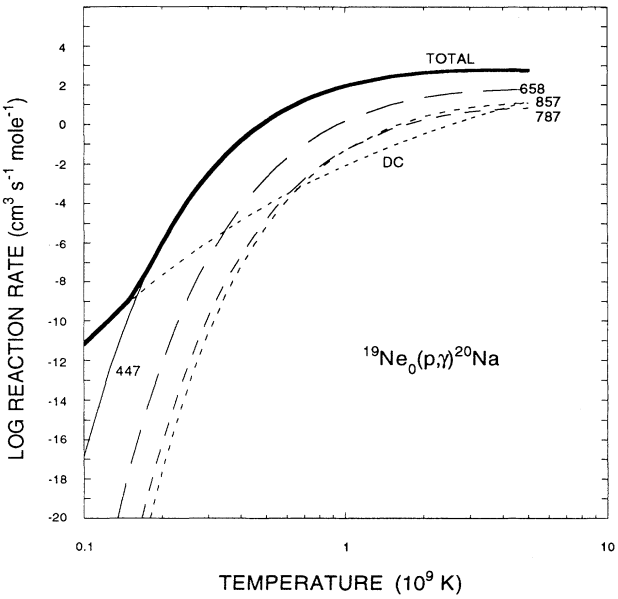

FIG. 3. Thermonuclear reaction rate for $^{19}Ne(p,\gamma)^{20}Na$ under the assumption that ^{19}Ne is in its ground state. Resonance contributions are labeled according to their energy and were calculated using the parameters found in Table II. The direct-capture rate (DC) is from Ref. [21].

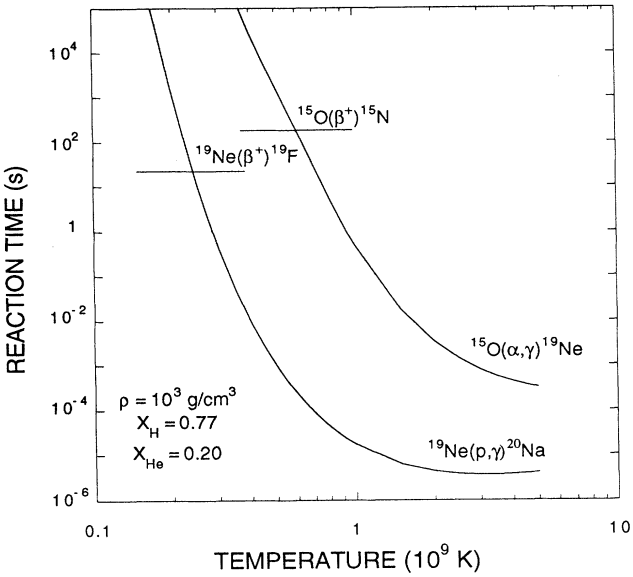

FIG. 4. Mean reaction times for $^{19}Ne(p,\gamma)^{20}Na$ and $^{15}O(\alpha,\gamma)^{19}Ne$ versus temperature for $X_H=0.77$ and $X_{He}=0.20$. For the purposes of illustration, a density of $\rho=10^3$ g/cm^3 was chosen. Mean lifetimes against β^+ decay are also indicated.

parameters for the other nearby states at 2857 keV ($E_{c.m.}=658$ keV, $J^\pi=3^-$), 2986 keV ($E_{c.m.}=787$ keV, $J^\pi=1^+$), and 3056 keV ($E_{c.m.}=857$ keV, $J^\pi=0^+$). In ^{20}F, neutron spectroscopic factors have been measured [15,18] for all three mirror states and mean lifetimes are tabulated [7] for the two positive-parity states. In the case of the 2857-keV state, we assume $\Gamma_p=\Gamma_\gamma$, which leads an upper limit on the resonance strength. For reasons that will be made clear below, we only consider capture involving the ground state of ^{19}Ne.

Explosive hydrogen burning, which accompanies supernovas and powers cataclysmic binaries, is expected to proceed via the hot CNO cycles or, for heavier nuclei, through the *rp* process [1–3] (Fig. 2). The former is considered to be the energy source for novas, whereas the latter is thought to power more energetic explosions (e.g., x-ray bursts). An *rp* process may be triggered directly from preexisting $A \geq 20$ nuclei or from lighter nuclei via several "breakout" reactions from the hot CNO cycles. The link between the hot CNO cycles and the *rp* process is provided primarily by the $^{15}O(\alpha,\gamma)^{19}Ne$ reaction [20] and to a lesser extent by the $^{18}F(p,\gamma)^{19}Ne$ and $^{18}Ne(\alpha,p)^{21}Na$ reactions. These breakout reactions will occur for temperatures $T>0.4\times10^9$ K (or $T_9>0.4$) and densities in excess of $\rho=10^3$ g/cm^3. An *rp* process may then be initiated by a subsequent $^{19}Ne(p,\gamma)^{20}Na$ reaction.

The rate of the $^{19}Ne(p,\gamma)^{20}Na$ reaction has been calculated using the resonance strengths listed in Table II and the contribution from nonresonant direct capture [21] and is shown in Fig. 3. For reactions involving ^{19}Ne in its *ground state,* the rate is dominated by the 447-keV resonance ($E_x=2646$ keV). However, by $T_9=1$, ^{19}Ne will be in its first excited state 15% of the time and in its second excited state 3% of the time. For $T_9>2$, the ground-state population falls below 50% of the total. Consequently, we have estimated the resonance strength for population of the 447-keV resonance from the first state of ^{19}Ne. The other resonances have strengths which appear to be determined primarily by the exit channel

TABLE II. Summary of resonance parameters.

E_x[a] (keV)	$E_{c.m.}$ (keV)	J^π	Γ_p (eV)	Γ_γ (eV)	$\omega\gamma$ (this study) (meV)	Ref. [4]	$\omega\gamma$ (literature) Ref. [5]	Ref. [6]
2646	447	3^+	$\Gamma_{p0}=0.52$	0.12	80	11.3	6.23	7
			$\Gamma_{p1}=0.73$		37			
2857	658	3^-	0.02	0.02[b]	≤ 20	19.2	15.95	20
2986	787	1^+	1.0×10^4	8×10^{-3}	2.2	28.8	51.19	70
3056	857	0^+	2.1×10^4	0.01	7.5	12	7.50	7.5

[a]Ref. [6].

[b]Assumed in order to give an upper limit on $\omega\gamma$.

(i.e., $\omega\gamma \approx \omega\Gamma_\gamma$) and, as a result, they should not change dramatically with temperature unless the proton width is drastically decreased (in which case they will become even less important). Assuming that these latter strengths are in fact unchanged by thermal excitation in ^{19}Ne, an approximate analytic expression for the reaction rate which includes the effects of thermal excitation to the first excited state (for population of the 2646-keV state) is

$$N_A\langle\sigma v\rangle = (5.45\times10^3)T_9^{-1.227} \times \exp(-4.378/T_9^{1.082})\ \mathrm{cm^3\,s^{-1}\,mol^{-1}}\ .$$

This approximation reproduces a more exact calculation to better than 10% over the temperature range $0.25 \le T_9 \le 2.0$ and once again the 2646-keV state is the major contributor to the total rate. For temperatures below this range, the ground-state rate is appropriate (with the contribution from direct capture). Above $T_9 \approx 2$, the population of higher-lying excited states cannot be ignored and so this approximation may not be accurate.

With our new calculations, the rate of the ^{19}Ne$(p,\gamma)^{20}$Na reaction is increased by about an order of magnitude over earlier results [4–6]. However, this increase is not astrophysically significant because the flow into the *rp* process is limited by the rate of the (slower) ^{15}O$(\alpha,\gamma)^{19}$Ne reaction. This point is illustrated in Fig. 4 where the mean reaction times for ^{19}Ne$(p,\gamma)^{20}$Na and ^{15}O$(\alpha,\gamma)^{19}$Ne are plotted versus temperature for hydrogen and helium mass fractions of $X_H = 0.77$ and $X_{He} = 0.20$, respectively. For $T_9 = 0.4$–2, the (p,γ) rate is 6.0–1.6 orders of magnitude greater than the (α,γ) rate (independent of density). In other words, once material is converted into ^{19}Ne, it will be rapidly passed along to higher masses.

Experimental verification of our results for the 2646-keV state are desirable, and here the increased resonance strength is significant because it leads directly to a corresponding increase in the expected count rate for the ^{1}H$(^{19}$Ne$,^{20}$Na$)\gamma$ reaction. Assuming a beam intensity of 1 particle nA, the 2646-keV state will be produced at a rate of 3860/h. In addition, our predictions of $J^\pi = 3^+$ and $\Gamma_\gamma/\Gamma = 0.09$ can be examined via indirect nuclear spectroscopy. Such studies are in progress.

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[1] R. K. Wallace and S. E. Woosley, Astrophys. J. Suppl. **45**, 389 (1981).
[2] M. Wiescher, J. Görres, F.-K. Thielemann, and H. Ritter, Astron. Astrophys. **160**, 56 (1986).
[3] A. E. Champagne and M. Wiescher, Annu. Rev. Nucl. Part. Sci. **42**, 39 (1992).
[4] L. O. Lamm, C. P. Browne, J. Görres, S. M. Graff, M. Wiescher, and A. A. Rollefson, Nucl. Phys. **A510**, 503 (1990).
[5] S. Kubono *et al.*, Z. Phys. A **331**, 359 (1989); Astrophys. J. **344**, 460 (1989).
[6] M. S. Smith, P. V. Magnus, K. I. Hahn, A. J. Howard, P. D. Parker, A. E. Champagne, and Z. Q. Mao, Nucl. Phys. **A536**, 333 (1992).
[7] F. Ajzenberg-Selove, Nucl. Phys. **A475**, 1 (1987).
[8] P. Descouvemont and D. Baye, Nucl. Phys. **A517**, 143 (1990).
[9] J. Görres *et al.*, Phys. Rev. C **46**, R833 (1992); A. Piechaczek *et al.*, in *Proceedings of the 6th International Conference on Nuclei Far from Stability,* Atomic Masses and Fundamental Constants 1992, edited by R. Neugart and A. Wöhr, Institute of Physics Conference Series 132 (IOP, Bristol, 1993).
[10] R. Coszach *et al.*, in *Proceedings of the 2nd International Symposium on Nuclear Astrophysics,* Karlsruhe, Germany, 1992, edited by F. Käppeler and K. Wissak (IOP, Bristol, 1993).
[11] R. Medoff, L. R. Medsker, S. C. Headley, and H. T. Fortune, Phys. Rev. C **14**, 1 (1976).
[12] N. M. Clarke, P. R. Hayes, M. B. Becha, C. N. Pinder, and S. Roman J. Phys. G **16**, 1547 (1990).
[13] H. T. Fortune and J. N. Bishop, Nucl. Phys. **A293**, 221 (1977).
[14] H. T. Fortune and J. D. Garrett, Phys. Rev. C **14**, 1695 (1976).
[15] H. T. Fortune and R. R. Betts, Phys. Rev. C **10**, 1292 (1974).
[16] G.-B. Liu and H. T. Fortune, Phys. Rev. C **37**, 1818 (1988).
[17] G.-B. Liu and H. T. Fortune, Phys. Rev. C **38**, 2134 (1988).
[18] H. T. Fortune, G. C. Morrison, R. C. Bearse, J. L. Yntema, and B. H. Wildenthal, Phys. Rev. C **6**, 21 (1972).
[19] B. A. Brown and B. H. Wildenthal, Nucl. Phys. **A474**, 290 (1987).
[20] P. V. Magnus, M. S. Smith, A. J. Howard, P. D. Parker, and A. E. Champagne, Nucl. Phys. **A506**, 332 (1990).
[21] K. Langanke, M. Wiescher, W. A. Fowler, and J. Görres, Astrophys. J. **301**, 629 (1986).