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General behavior of double beta decay amplitudes in the quasiparticle random phase approximation

F. Krmpotić
Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de La Plata, C. C. 67, 1900 La Plata, Argentina
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Simple formulas for the $0^+ \to 0^+$ double beta decay matrix elements, as a function of the particle-particle strength g^{pp}, have been designed within the quasiparticle random phase approximation. The 2ν amplitude is a bilinear function of g^{pp}, and all 0ν moments behave as ratios of a linear function and the square root of another linear function of g^{pp}. It is suggested that these results are of general validity and that any modifications of the nuclear Hamiltonian or the configuration space cannot lead to a different functional dependence.

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The neutrinoless double beta decay ($0\nu\beta\beta$) is very interesting for several reasons. In the first place, this decay mode is viable only when the neutrino is a massive Majorana particle. As such, it constitutes a critical touchstone for various gauge models that go beyond the standard $SU(2)_L \times U(1)$ gauge model of electroweak interactions. Secondly, the neutrinos with nonzero masses have many interesting consequences for the history of the early Universe, in the evolution of stellar objects, and the supernova astrophysics. Thirdly, besides the issue of $m_\nu \neq 0$, there are other open questions in neutrino physics, and answers to which depend on $0\nu\beta\beta$ decay, such as: Why does nature favor only left-handed currents? Does the majoron exist? Yet, we shall not understand the $0\nu\beta\beta$ decay unless we understand the two neutrino double beta decay ($2\nu\beta\beta$). The last one is the rarest process observed so far in nature and offers a unique opportunity for testing the nuclear physics techniques for half-lives $\gtrsim 10^{20}$ yr. Thus, the comprehension of the $\beta\beta$ transition mechanism cannot but help advance knowledge of physics in general.

In recent years the quasiparticle random phase approximation (QRPA) has been the most popular method to deal with the problem of $0^+ \to 0^+$ double beta decay [1–9]. Within this model the $\beta\beta$-decay amplitudes are extremely sensitive to the interaction parameter in the particle-particle (PP) channel, usually denoted by g^{pp}. Independently of the nucleus that decays, of the residual interaction that is used, and of the configuration space that is employed, all the QRPA calculations done so far exhibit the following general features. (i) Close to the "natural" value for g^{pp} ($g^{pp} \cong 1$) the $2\nu\beta\beta$ moments have first a zero and later a pole at which the QRPA collapses. (ii) The zeros and poles of the $0\nu\beta\beta$ moments for the virtual states with spin and parity $J^\pi = 1^+$ are strongly correlated with the zeros and poles of the $2\nu\beta\beta$ moments. (iii) The $0\nu\beta\beta$ moments of multipolarity $J^\pi \neq 0^+, 1^+$ also possess zeros and poles but at significantly larger values of g^{pp}. (iv) As a function of g^{pp}, both the $2\nu\beta\beta$ and $0\nu\beta\beta$ moments always present similar shapes.

Figure 1 illustrates the behavior of the $0^+ \to 0^+$ $\beta\beta$ matrix elements for several nuclei. In the upper panel the $2\nu\beta\beta$ moments ($\mathcal{M}_{2\nu}$) are shown. The other two panels

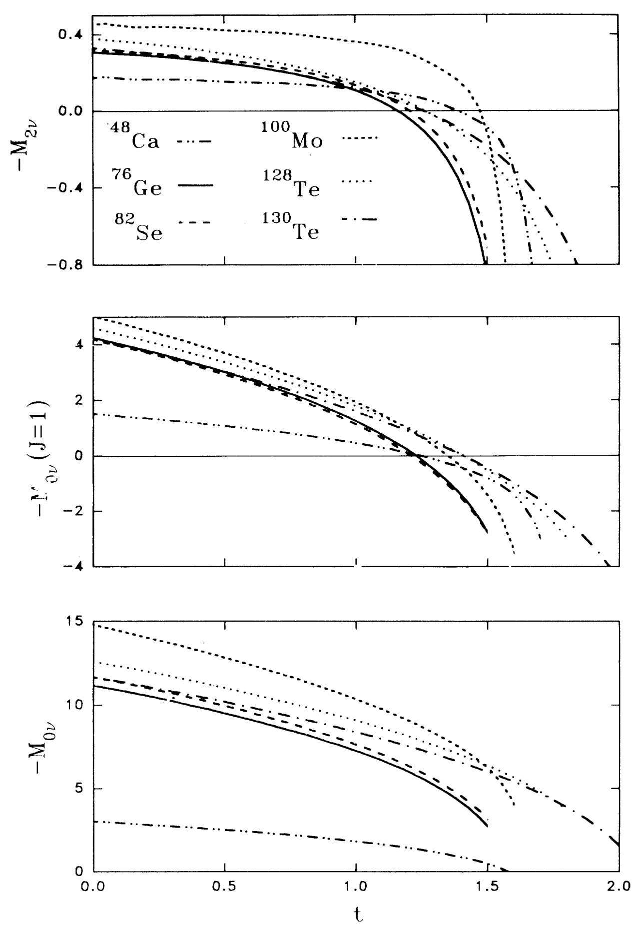

FIG. 1. Calculated double beta decay matrix elements $\mathcal{M}_{2\nu}$ (in units of $[\mathrm{MeV}]^{-1}$), $\mathcal{M}_{0\nu}(J^\pi = 1^+)$ and $\mathcal{M}_{0\nu}$, as a function of the particle-particle $S = 1$, $T = 0$ coupling constant t. The ^{48}Ca nucleus has been evaluated within $2\hbar\omega$ and $3\hbar\omega$ major oscillator shells. For the remaining systems I have adopted the oscillator shells $3\hbar\omega$ and $4\hbar\omega$ plus the $0h_{9/2}$ and $0h_{7/2}$ intruder orbitals from the $5\hbar\omega$ shell. The "physical values" of the parameter t (t_{sym}) are shown in the last row of Table I.

contain the $0\nu\beta\beta$ moments of multipolarity $J^\pi=1^+$ $[\mathcal{M}_{0\nu}(J^\pi=1^+)]$ and total $0\nu\beta\beta$ moments ($\mathcal{M}_{0\nu}$) induced by the neutrino mass mechanism. These results have been obtained with a δ force, using standard parametrization presented elsewhere [10]. Instead of the parameter g^{pp}, I use here the ratio between the triplet and singlet coupling strengths in the PP channel, i.e., $t=v_t/v_s$. Calculations with finite range interactions yield similar results [3–6].

More than once [7–9] we have pointed out that the $\beta\beta$ amplitudes go to zero within the QRPA because of the restoration of both the isospin and SU(4) symmetries. We have also suggested a physical criterion for fixing the PP coupling strength based on the maximal restoration of the SU(4) symmetry ($t=t_{\rm sym}$). Yet, the general characteristics mentioned above suggest the existence of some additional regularities, and the present concern reflects upon a global understanding of the $\beta\beta$ transition mechanism within the QRPA. Only in this way can one get full control of the calculations, which is one of the prerequisites for a reliable estimate of the nuclear matrix elements.

To begin with, I resort to the single mode model (SMM) description [9] of the $\beta\beta$ decays in the ^{48}Ca$\rightarrow^{48}$Ti and ^{100}Mo$\rightarrow^{100}$Ru systems. This is the simplest version of the QRPA, in which there is only one intermediate state for each J^π.

In the SMM the 0ν and 2ν moments for the $0^+\rightarrow0^+$ transitions read [9]

$$\mathcal{M}_{2\nu}=\mathcal{M}^0_{2\nu}\left[\frac{\omega^0}{\omega_{1^+}}\right]^2\left[1+\frac{G(1^+)}{\omega^0}\right], \tag{1}$$

$$\mathcal{M}_{0\nu}(J^+)=\mathcal{M}^0_{0\nu}(J^+)\frac{\omega^0}{\omega_{J^+}}\left[1+\frac{G(J^+)}{\omega^0}\right], \tag{2}$$

where $\mathcal{M}^0_{2\nu}$ and $\mathcal{M}^0_{0\nu}(J^+)$ are the corresponding unperturbed matrix elements. Here $G(J^+)\equiv G(pn,pn;J^+)$ are the PP matrix elements, ω^0 is the unperturbed energy, and ω_{J^+} are the perturbed energies. I will assume that the isospin symmetry is strictly conserved, in which case $\mathcal{M}_{0\nu}(0^+)\equiv0$. This statement is also valid for full calculations and therefore no further reference will be made to the intermediate states $J^\pi=0^+$. When the pairing factors are estimated in the usual manner, one gets

$$\omega=\omega^0\sqrt{1+F(34+9F/\omega^0)/25\omega^0+16G(1+F/\omega^0)/25\omega^0}\,, \tag{3}$$

and

$$\omega=\omega^0\sqrt{1+4F(45+F/\omega^0)/225\omega^0+G(270+172F/\omega^0+49G/\omega^0)/225\omega^0}\,, \tag{4}$$

for the single pair configurations $[0f_{7/2}(n)0f_{7/2}(p)]_{J^+}$ in ^{48}Ca and $[0g_{7/2}(n)0g_{9/2}(p)]_{J^+}$ in ^{100}Mo, respectively. Therefore, while the numerators in Eq. (2) depend only on the PP matrix elements, their denominators depend on the particle-hole (PH) matrix elements $F(J^+)\equiv F(pn,ph;J^+)$ as well. The numbers in the last two equations arise from the pairing factors. As illustrated in Fig. 2, the SMM is a fair first-order approximation for the $2\nu\beta\beta$ decays in ^{48}Ca and ^{100}Mo nuclei.

The role played by the ground state correlations (GSC) in building up Eqs. (1) and (2) can be summarized as follows: (a) The numerator, i.e., the factor $(1+G/\omega^0)$, comes from the interference between the forward and backward going contributions. These contribute coherently in the PP channel and totally out of phase in the PH channel. (b) The G^2 and F^2 terms in the denominator are very strongly quenched by the GSC, while the GF term is enhanced by the same effect. In particular, for ^{48}Ca the term quadratic in G does not contribute at all.

It can be stated therefore that, within the SMM and because of the GSC, the 2ν matrix element is mainly a bilinear function of $G(1^+)$. Besides, it passes through zero at $G(1^+)=-\omega^0$ and has a pole when $\omega_{1^+}=0$. Similarly, all $\mathcal{M}_{0\nu}(J^+)$ moments turn out to be quotients of a linear function of $G(J^+)$ and the square root of another linear function of $G(J^+)$. Both the zero and the pole of $\mathcal{M}_{0\nu}(1^+)$ matrix element coincide with those of the 2ν moment. One also should bear in mind that the magnitudes of the interaction matrix elements $G(J)$ and $F(J)$ decrease fairly rapidly when J increases. Thus the

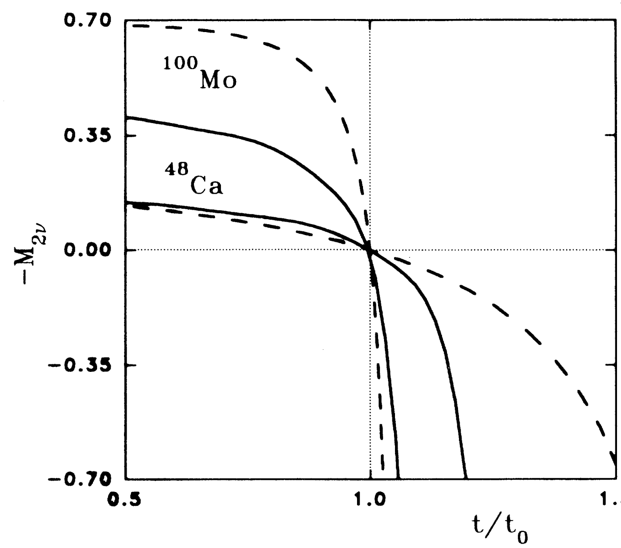

FIG. 2. The exact (solid lines) and SMM (dashed lines) matrix elements $\mathcal{M}_{2\nu}$ (in units of $[\mathrm{MeV}]^{-1}$), as a function of the coupling constant t/t_0 (defined in the text).

quenching effect, induced by the PP interaction, mainly concerns the allowed 0ν moment. For higher order multipoles it could be reasonable to expand the denominator in Eq. (2) in powers of $G(J^+)/\omega^0$ and to keep only the linear term. This term strongly cancels with a similar term in the numerator and the net result is a weak linear dependence of the $\mathcal{M}_{0\nu}(J^+\neq 1^+)$ moments on the PP strength. Obviously, for the last approximation to be valid, the parameter t (or g^{pp}) has to be small enough to keep ω_{1^+} real. Briefly, the SMM can account for all four points raised above, and leads to the following approximations for the dependence of the $\beta\beta$ amplitudes on the PP strength

$$\mathcal{M}_{2\nu} \cong \mathcal{M}_{2\nu}(t=0)\frac{1-t/t_0}{1-t/t_1} , \tag{5}$$

and

$$\mathcal{M}_{0\nu} \cong \mathcal{M}_{0\nu}(J^\pi=1^+;t=0)\frac{1-t/t_0}{\sqrt{1-t/t_1}} + \mathcal{M}_{0\nu}(J^\pi\neq 1^+;t=0)(1-t/t_2) , \tag{6}$$

where $t_1 \geq t_0$ and $t_2 \gg t_1$, and the condition $t \leq t_1$ is fulfilled. It is self-evident that these formulas do not depend on the type of residual interaction, and that analogous expressions are obtained for the $\beta\beta$ matrix elements when the parameter g^{pp} is used (with g^{pp}'s for t's).

The common behavior of the $\beta\beta$ moments for all nuclei, together with the similarity between the SMM and the full calculations for ^{48}Ca and ^{100}Mo (shown in Figs. 1 and 2, respectively), suggests to go a step further and try to express the exact calculations within the framework of Eqs. (5) and (6). At a first glance this seems a difficult task, because (i) the SMM does not include the effect of the spin-orbit splitting, which plays a very important role in the $\beta\beta$ decay through the dynamical breaking of the SU(4) symmetry, and (ii) the full calculations involve a rather large configuration space (of the order of 50 basis vectors). However, the reliability of formulas (5) and (6) is surprising. The results are presented in Table I. In the upper, middle, and lower panels I show the values of the parameters t_0, t_1, and t_2 that fit the $\beta\beta$ moments displayed in the same order in Fig. 1. I also list the values of the moments $\mathcal{M}_{2\nu}$, $\mathcal{M}_{0\nu}(J^\pi=1^+)$, and $\mathcal{M}_{0\nu}(J^+\neq 1^+)$ for $t=0$, together with the quantity $\mathcal{N}=\{\sum_{t=0}[\mathcal{M}_{\rm exact}(t)-\mathcal{M}_{\rm fit}(t)]^2\}^{1/2}$ that is an index of the goodness of the fit. The largest error occurs for ^{100}Mo. Still, even here it is not possible to distinguish visually the exact curves from the fitted ones. (This makes needless the exhibition of the adjusted curves.) In fact, for this nucleus the proposed formulas reproduce better the exact $\beta\beta$ moments than those obtained from the SMM. It is also gratifying that all three fits yield quite similar values for t_0 and t_1. The differences are at most of the order of 10%.

A comment regarding the full QRPA calculations might be appropriate. The matrix element $\mathcal{M}_{2\nu}$ can always be expressed by the ratio of two polynomials in $G(1^+)$ and $F(1^+)$ [see Eq. (8) of Ref. [8]]. For an n-dimensional configuration space these polynomials are of degrees $2n-1$ and $2n$, respectively. The above results seem to indicate that cancellations of the type (a) and (b) are likely to be operative to all orders, and that the linear terms in $G(1^+)$ are again the dominant ones. General expressions for the 0ν moments, as a function of the PP and PH matrix elements, are not known, but a similar cancellation may be taking place in these as well.

In summary, I have designed Eqs. (5) and (6) and verified that they nicely reproduce the full calculations of

TABLE I. The coefficients t_0, t_1, and t_2 and the matrix elements $\mathcal{M}_{2\nu}$, $\mathcal{M}_{0\nu}$ $(J^\pi=1^+)$, and $\mathcal{M}_{0\nu}$ $(J^\pi\neq 1^+)$ for $t=0$, in the parametrization of the 2ν and $0\nu\beta\beta$ moments. The quantity $\mathcal{N}$ is the norm of the residuals, i.e., the square root of the sum of squares of the residuals. The exact and fitted matrix elements are equal at $t=0$, and the strength t is varied, by steps of $\Delta t=0.1$, up to the collapse of the QRPA. The matrix elements $\mathcal{M}_{2\nu}$ are given in units of $[\mathrm{MeV}]^{-1}$. The values of the PP coupling strength, which lead to maximal restoration of the SU(4) symmetry ($t=t_{\rm sym}$), are shown in the last row.

	^{48}Ca	^{76}Ge	^{82}Se	^{90}Mo	^{128}Te	^{130}Te
$-\mathcal{M}_{2\nu}$	0.173	0.308	0.321	0.451	0.381	0.331
t_0	1.394	1.161	1.206	1.469	1.265	1.261
t_1	1.754	1.680	1.691	1.649	2.131	2.268
$\mathcal{N}$	3.26×10^{-2}	1.08×10^{-3}	7.44×10^{-5}	1.04×10^{-2}	2.31×10^{-3}	7.06×10^{-3}
$-\mathcal{M}_{0\nu}(J^\pi=1^+)$	1.506	2.242	4.179	5.015	4.599	4.182
t_0	1.244	1.230	1.211	1.346	1.407	1.408
t_1	1.765	1.693	1.720	1.741	2.228	2.364
$\mathcal{N}$	1.12×10^{-2}	4.87×10^{-3}	3.21×10^{-2}	2.21×10^{-1}	2.37×10^{-2}	6.34×10^{-2}
$-\mathcal{M}_{0\nu}(J^\pi\neq1^+)$	1.501	6.924	7.495	9.762	7.997	7.486
t_0	1.227	1.155	1.141	1.372	1.377	1.407
t_1	1.768	1.741	1.764	1.711	2.236	2.345
t_2	12.82	13.23	12.14	6.527	13.39	11.08
$\mathcal{N}$	1.92×10^{-2}	2.46×10^{-2}	2.20×10^{-2}	1.11×10^{-1}	1.68×10^{-2}	3.50×10^{-2}
$t_{\rm sym}$	$\cong1.50$	$\cong1.25$	$\cong1.30$	$\cong1.50$	$\cong1.40$	$\cong1.40$

the $\beta\beta$ matrix elements evaluated with a zero range force. I also feel that they are of general validity, and that any modification to the nuclear Hamiltonian or to the configuration space can only change the coefficients in these formulas, but will not lead to a different functional dependence. Thus, we possess now a global understanding of the $\beta\beta$ transition mechanism (and full control of the calculations) within the QRPA, which was the aim of this paper.

It should be stressed that for practical application one always has to perform the complete calculation in order to do the fit. The real advantages of the analytic formulas (5) and (6) are as follows: (1) they exhibit, in a very simple way, the main physics of the $\beta\beta$ decay in the QRPA model, and summarize the common features of the calculations done until now; and (2) they establish the potential and limits of the QRPA method, and give a hint of direction that should follow the future theoretical studies.

The pole at $t=t_1$ is the response of the QRPA to the nonphysical situation, in which the energy of the lowest virtual $J^{\pi}=1^+$ state becomes $\cong(E_i+E_f)/2$, where E_i and E_f are, respectively, the energies of the initial and final states. There is no reason in principle why this should not happen in a nuclear model calculation (for a sufficiently large value of t). But, within the QRPA approach the pole develops close to the "natural" value of t, which makes the $\beta\beta$ moments to vary rather abruptly in the physically relevant interval $t_0 \gtrsim t \gtrsim t_1$. Certainly, this is a weak point of the QRPA [11] and it is not clear yet how it could be circumvented.

A qualitative agreement, between the shell model and QRPA results for the $2\nu\beta\beta$ matrix elements in ^{48}Ca, has been reported [2,5]. When applied to medium and heavy nuclei, the shell model is always accompanied by a very severe truncation of the configuration space, in order to become tractable. Contrarily, the QRPA is a readily accessible and fully controlled approach, and as such it calls for further developments. Efforts in this direction have recently been done by extending the model to describe the 2ν decays to an excited final state [12], and by including the core polarization corrections to the effective interaction [13].

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