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Nuclear and hypernuclear particle-hole states in the relativistic σ-ω mean field theory: Nuclear matter formalism

Joseph Cohen
Institute of Nuclear and Particle Physics, Department of Physics, University of Virginia, Charlottesville, Virginia 22901
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Studies of the closed core response to a perturbation in the Dirac Hamiltonian (due to an out-of-core baryon) are extended to particle-hole states in nuclear matter. Both isoscalar nucleon-particle–nucleon-hole states and Λ-particle–nucleon-hole hypernuclear states are considered. Core-response effects on the nuclear baryon current and the vector and scalar densities are studied, using both a perturbative treatment and linear response theory. While no essentially new physical insight is encountered by considering particle-hole states, there are many new and interesting applications. In particular, we mention applications to hypernuclei, meson-nucleus (especially K^+) interactions, meson photoproduction on nuclei, nuclear reactions, and nuclear transitions in inelastic scattering and weak interactions in nuclei. We point out interest in core-response effects related to nuclear transitions, where new physical insight is expected.

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I. INTRODUCTION

There has recently been increasing interest in relativistic effects arising in models of the nucleus based on the Dirac equation with strong scalar and vector potentials [1,2]. In particular, attention has been focused on areas for which relativistic predictions differ significantly from those obtained in the traditional framework of nonrelativistic nucleons [3]. However, an unambiguous experimental signature of the large potentials and other aspects of relativistic dynamics has been difficult to find.

The reduced nucleon effective mass, $M_N^* = M_N + S$, where S is the scalar potential, yields enhanced single-particle currents in the nuclear medium. At first glance this would appear to have immediate consequences on experimental observables related to probes that couple to the nuclear convection current. Nuclear magnetic moments (especially for nuclei with one nucleon or hole away from a closed shell) provide a well-known example [1]: the enhanced relativistic single-particle convection currents result in large deviations from the Schmidt values for closed shell ± 1 nuclei. (This result is based on using the simplest single-particle picture, where the closed-shell core remains spherical and does not respond to the valence nucleon. The magnetic moment of the nucleus is then determined by the valence nucleon.) Experimentally, however, one finds *isoscalar* magnetic moments in reasonable agreement with the Schmidt values, and clearly not enhanced by as much as 100% according to the predictions of these naive relativistic considerations. A careful analysis has, indeed, shown that such enhancements disappear when a full and consistent treatment of the nuclear system as a whole is carried out [at low momenta (Q)] [4,5]. As a result, an interesting insight into the nature of the relativistic mean field theory has been gained.

This effect is a result of the response of the nuclear core to the valence nucleon outside of this core. The simple single-particle picture, discussed above, is inadequate to describe the ground-state *current* of closed shell ± 1 nuclei in the self-consistent mean field approximation (MFT). In particular, the current due to the valence particle only is a poor approximation to the full (isoscalar) baryon current for a core-plus-valence-nucleon system. The valence nucleon is a source of additional meson fields, to which the core responds. The dynamical modification of the closed-core states must be included, and the subsequent core contribution is of the same magnitude as the (extreme single-particle) valence nucleon contribution. For the nuclear vector currents, the pertinent core response effect arises (in the language of the MFT) from a nonvanishing three-vector (space components) ω field. (It is worthy of note that the core response can also be realized through the σ field, for example, in the case of the scalar density ρ_s. This case will be discussed later in this paper.) In nuclear matter, where the unperturbed (inert core) system is a filled Fermi sphere of nucleons, the static core response to a valence nucleon involves only the mixing of positive- and negative-energy (unperturbed) wave functions; it is studied using linear response theory. *This is a static response that does not occur in a nonrelativistic theory.* The net result is that relativistic *nuclear* convection currents, in contrast with the *single-particle* currents, are not enhanced by the factor of approximately M/M^*. Indeed, nuclear convection currents are found to be insensitive to the value of M^* at low momentum transfer Q, and no significant difference occurs between relativistic and nonrelativistic predictions (e.g., for magnetic moments) [6]. (Currently, only the isoscalar nuclear currents can be treated satisfactorily in the context of the relativistic σ-ω model [5].)

It is worthy of note that the core response is *not* a correction to the fully self-consistent MFT (Hartree)[1]

[1]In general, we use the terms "Hartree" and "MFT" interchangeably here, as has become customary in recent years [2,5,7]; they will, however, differ somewhat in Sec. II C.

solution for the complete (core-plus-extra-particle) system. It only provides a correction to the current obtained in the extreme single-particle picture, based on a spherically symmetric core plus an independent valence nucleon. (The latter is not a self-consistent approach. It is, therefore, the lack of self-consistency in this oversimplified extreme independent-particle picture that must be corrected.) The core response features will be automatically incorporated in a fully self-consistent treatment of the entire system; in this case one has to generate self-consistently the MFT Dirac spinors for the complete (core-plus-valence) system. Using this new set of wave functions, all MFT calculations can again be carried out without explicit reference to negative-energy states. (Of course, the exact self-consistent states can each be expanded in terms of the complete set of closed-core spinors, requiring both *positive and negative* energy solutions of the latter; this is just an immediate consequence of completeness.) The exact core-plus-valence problem is difficult to solve since spherical symmetry is lost. Such solutions have been provided for regular nuclei [8]. However, we feel that there is considerable physical insight to be gained from the alternative perturbative and linear-response approaches which we shall follow here. This approach is also particularly convenient and suitable for applications to hypernuclei, which are discussed in this work.

The alternative approach to the full self-consistent solution, which we adopt in this work, uses the single-particle nucleon wave functions of the closed-core system as a starting point for describing the nonspherically symmetric system. Self-consistency now implies that the valence particle (hole) cannot be treated independently of the inert core, and the closed-core Dirac spinors will get modified. This approach, based on linear response theory, incorporates self-consistency approximately; the valence nucleon is treated as an external source of meson mean fields, and the core response does not act back on it. The differences between the linear response and the fully self-consistent calculations are of order $1/A$, and the two approaches are therefore equivalent in nuclear matter. Furthermore, the linear response approach is interesting because the pertinent physical effects are clearly identified. Although relativistic and nonrelativistic models yield similar results for the isoscalar nuclear currents at low Q, there is a fundamental difference between these two approaches. In the relativistic picture, the isoscalar Schmidt values are obtained after a strong cancellation between the valence convection current and the contribution from the modified core (this cancellation is not accidental and has its roots in the basic feature of the model: two large potentials of opposite sign, almost balancing each other to yield the small binding energies of nucleons in nuclei [6]), while they arise directly from the valence current alone in the nonrelativistic shell model [isoscalar (nonrelativistic) core polarization and other many-body corrections are small].

Consequently, one has to look elsewhere for experimental signatures of the relativistic nuclear model. Cohen and Furnstahl [7] have pointed out that the above distinction between relativistic and nonrelativistic dynamics leads to two distinct predictions for hypernuclear magnetic moments. Unlike the purely nuclear case, core-response corrections to a Λ-hyperon result in a net effect on the hypernuclear current because the Λ is an isoscalar particle. (The Λ couplings to the meson fields are different from those of the nucleon.) Thus, the effect of the strong fields in relativistic models might be observed by introducing a strange particle into the system.

In this paper we describe the corresponding situation for particle-hole (p-h) states in nuclear matter and Λ hypernuclei, extending the formalism of Refs. [5,7] for such cases. (It has been shown in several works that a finite nucleus treatment does not appreciably change the Fermi gas results, especially when a local-density approximation is used; see, e.g., Furnstahl [5] and Ichii *et al.* [4].) We study the total nuclear baryon current and the vector and scalar nuclear densities (the vector density is frequently referred to as the baryon density, and is just the conventional density discussed in nonrelativistic nuclear physics). As we shall see, no new insight into the pertinent underlying dynamics is gained by considering p-h states instead of one particle outside a closed core. However, the p-h treatment has some practical significance and will be important for a number of problems discussed in Sec. III, such as K^+-nucleus scattering, meson photoproduction, weak neutral currents in nuclei, or nuclear excitations by means of inelastic scattering of various probes. We shall also point out that core-response effects in nuclear transitions are an interesting subject of further study. Furthermore, when Ref. [7] was published we emphasized [7] that our calculations were performed for a closed-shell nuclear core plus an extra Λ hyperon. Here we show that similar effects will, in principle, occur for a closed-shell core with a nucleon hole plus a Λ particle (we shall refer to such a state as a Λ-h excitation); however, practical considerations in carrying out hypernuclear experiments favor a measurement performed for the original system (Λ hyperon outside a closed shell) considered in our original calculation, Ref. [7] (even this measurement is a very difficult one). We therefore view this paper as a follow-up on past publications, extending previous treatments and clearing some relevant issues which have been left incomplete in earlier discussions in the literature.

Recent interest has focused on the large ω-Λ phenomenological tensor coupling; see [18–22]. While pertaining more to finite nuclear systems than to a nuclear matter limit, this seems to be an important enough issue to merit a discussion in this paper. The large tensor component in the $\omega\Lambda\Lambda$ vertex appears naturally in a quark model for the Λ hyperon; the same model also yields a small ωNN tensor vertex coupling, in agreement with the nuclear σ-ω MFT Lagrangian. The large ω-Λ tensor coupling is important for resolving the problem of the small spin-orbit interaction in Λ hypernuclei within the Dirac approach, a central issue in hypernuclear structure studies. It also has important implications on magnetic moment calculations for hypernuclei.

We start this work with a discussion based on perturbation theory (Sec. II B). We then show that the same results can be obtained within the linear response theory and demonstrate that the first-order perturbation theory treatment is equivalent to one-ring random phase approximation (RPA). Summing rings to all orders we obtain the full RPA nuclear response effect (Sec. II C). Some of the more formal comments necessary for a rigorous derivation of our results are grouped together and presented in a special section (II D), which may be skipped by most readers without loss of clarity. All the formalism is generalized to include applications to hypernuclei and we study the linear core-response effects on the nuclear currents and densities. Useful and interesting applications of our results to a number of nuclear problems are discussed and suggested in Sec. III, which also contains a summary of our formal results and a further discussion of some physical aspects of the linear core response.

II. FORMALISM AND APPLICATIONS

A. The σ-ω Lagrangian in the mean field approximation (MFT)

We work along the formal lines of Furnstahl and Serot [5], who deal with the case of one nucleon outside a closed shell. Here we consider adding a particle (N or Λ) and a nucleon hole to the filled Fermi sea sphere of nucleons as a model, e.g., of the ground state of a hypernucleus. (The formal development presented here for the Λ is, indeed, valid for any other exotic spin-$\frac{1}{2}$ baryon B embedded in the nucleus.) Our starting point is, therefore, the mean field approximation to the σ-ω model. The Lagrangian density, minimally extended to include Λ hyperons, is

$$\begin{aligned}\mathcal{L}_{\rm MPT}=&\bar{\psi}_N[\gamma_\mu(i\partial^\mu-g_v^N V^\mu)-(M_N-g_s^N\phi_0)]\psi_N\\&+\bar{\psi}_\Lambda[\gamma_\mu(i\partial^\mu-g_v^\Lambda V^\mu)-(M_\Lambda-g_s^\Lambda\phi_0)]\psi_\Lambda\\&-\tfrac{1}{2}m_s^2\phi_0^2+\tfrac{1}{2}m_v^2V_\mu V^\mu+\mathcal{L}_{\rm EM}\ ,\end{aligned}\tag{1}$$

where we have included both nucleons and Λ in the presence of a scalar field ϕ_0 and a vector field $V^\mu=(V_0,\mathbf{V})$ (note that the maintaining the three-vector component of V^μ is crucial in our discussion). In Eq. (1), m_s and m_v are masses of the scalar and vector mesons and $\mathcal{L}_{EM}$ is the electromagnetic Lagrangian [2]. The meson-baryon coupling constants in Eq. (1) differ for nucleons and hyperons [7]. The MFT equations of motion for the baryons and mesons are obtained following the usual procedures [2]. Positive and negative energy spinor solutions to the baryon equations, $u(\lambda,\mathbf{k})$ and $v(\lambda,\mathbf{k})$, are readily constructed for nuclear matter in momentum space. The Lagrangian of Eq. (1) does not include the phenomenological ω-Λ tensor coupling which has recently been shown to be important. We delay the discussion of this issue to Sec. III.

(It is true that the relativistic nuclear theory does not always match the precision of the older, well-established nonrelativistic theory. There are many-body effects that require further study, and at times the agreement between theory and experiment calls for further improvements. However, the properties dealt with in much detail in this paper, namely, densities and electromagnetic properties, are accounted for to a rather impressive precision in the regular nuclear Dirac theory, especially when applied to isoscalar properties. Moreover, our formalism is expected to provide a basis for further studies along these lines, and the special appeal of such formalism is indicated in the following.)

B. Particle-hole state: Perturbative treatment

We consider a particle-hole state in nuclear matter in a large volume Ω. We start with a discussion based on perturbation theory, in order to clarify the more formal treatment. Our unperturbed (inert core) system is a Fermi sphere of nucleons for which a solution is given in the Hartree approximation (with $\mathbf{V}=0$) [2].

The closed nuclear core (cc) gives the zeroth-order fields:

$$\phi_0^{(\rm cc)}=\frac{1}{\Omega}\frac{g_s^N}{m_s^2}\sum_{\mathbf{k},\lambda}^{k_p}\bar{u}_N(\lambda,\mathbf{k})u_N(\lambda,\mathbf{k})\ ,\tag{2}$$

$$V_0^{(\rm cc)}=\frac{1}{\Omega}\frac{g_v^N}{m_v^2}\sum_{\mathbf{k},\lambda}^{k_p}\bar{u}_N(\lambda,\mathbf{k})\gamma^0u_N(\lambda,\mathbf{k})\ ,\tag{3}$$

$$\mathbf{V}^{(\rm cc)}=\frac{1}{\Omega}\frac{g_v^N}{m_v^2}\sum_{\mathbf{k},\lambda}^{k_p}\bar{u}_N(\lambda,\mathbf{k})\boldsymbol{\gamma}u_N(\lambda,\mathbf{k})=0\ .\tag{4}$$

Upon adding a particle (momentum $\mathbf{p}$) and a hole (momentum $\mathbf{q}$), the mean fields are changed by

$$\delta\phi_0=\frac{1}{\Omega}\frac{1}{m_s^2}[g_s^B\bar{u}_B(\lambda,\mathbf{p})u_B(\lambda,\mathbf{p})-g_s^N\bar{u}_N(\lambda',\mathbf{q})u_N(\lambda',\mathbf{q})]=\frac{1}{\Omega}\frac{1}{m_s^2}\left[g_s^B\frac{M_B^*}{E_p^{B*}}-g_s^N\frac{M_N^*}{E_q^{N*}}\right]\ ,\tag{5}$$

$$\delta V_0=\frac{1}{\Omega}\frac{1}{m_v^2}[g_v^B\bar{u}_B(\lambda,\mathbf{p})\gamma^0u_B(\lambda,\mathbf{p})-g_v^N\bar{u}_N(\lambda',\mathbf{q})\gamma^0u_N(\lambda',\mathbf{q})]=\frac{1}{\Omega}\frac{1}{m_v^2}(g_v^B-g_v^N)\ ,\tag{6}$$

$$\delta\mathbf{V}=\mathbf{V}=\frac{1}{\Omega}\frac{1}{m_v^2}[g_v^B\bar{u}_B(\lambda,\mathbf{p})\boldsymbol{\gamma}u_B(\lambda,\mathbf{p})-g_v^N\bar{u}_N(\lambda',\mathbf{q})\boldsymbol{\gamma}u_N(\lambda',\mathbf{q})]=\frac{1}{\Omega}\frac{1}{m_v^2}\left[g_v^B\frac{\mathbf{p}}{E_p^{B*}}-g_v^N\frac{\mathbf{q}}{E_q^{N*}}\right]\ ,\tag{7}$$

where the baryon B can be N (a nucleon) or Λ, $M_B^*=M_B-g_s^B\phi_0$, and $E_k^{B*}=[k^2+M_B^{*}]^{1/2}$. Note that the $u(\lambda,\mathbf{k})$'s are the spinor solutions for the state $(\lambda,\mathbf{k})$ of the unperturbed Dirac equation. It is evident from Eq. (6) that a p-h state does not change V_0 for a nucleon-hole state ($B=N$), *but produces a change* proportional to $g_v^B-g_v^N$ in V_0 *for* $B\neq N$; this is consistent with the role of V_0 as a level of reference (shifting the zero of the energy) for measuring single-particle energies. It is also worthy of note that the Λ, which is not Pauli blocked [9], can occupy (unlike the nucleon) any single-particle state, including $\mathbf{p}=0$.

In the present treatment, the closed-core nucleons will be affected by the new fields $\delta\phi_0$, δV_0, and $\delta\mathbf{V}$ [Eqs. (5)–(7)], but the appropriate changes in the core do not act back on the p-h states. (Likewise, the core occupation numbers are left unchanged when the p-h states are added.) This approximates the fully self-consistent treatment to order $1/A$ [5]. The two treatments are therefore equivalent for nuclear matter, as already noted above.

The perturbation in the single-particle mean-field Dirac Hamiltonian due to the new fields for a nucleon or Λ, denoted here as B', is

$$\delta h = -\gamma_0(g_s^{B'}\delta\phi_0)+g_v^{B'}\delta V_0-\boldsymbol{\alpha}\cdot(g_v^{B'}\delta\mathbf{V})$$

$$= -\frac{1}{\Omega}\left[\frac{g_s^{B'}}{m_s^2}\left[g_s^B\frac{M_B^*}{E_p^{B*}}-g_s^N\frac{M_N^*}{E_q^{N*}}\right]\gamma^0-\frac{g_v^{B'}}{m_v^2}(g_v^B-g_v^N)+\frac{g_v^{B'}}{m_v^2}\boldsymbol{\alpha}\cdot\left[g_v^B\frac{\mathbf{p}}{E_p^{B*}}-g_v^N\frac{\mathbf{q}}{E_q^{N*}}\right]\right] \tag{8}$$

where $\boldsymbol{\alpha}=\gamma^0\boldsymbol{\gamma}$. The complete Hamiltonian is $H=H_0+\delta h$, where H_0 represents the closed-core, unperturbed Hamiltonian. We note that δh separates into two parts, one for the particle and the other one for the hole. To avoid possible confusion we note, for example, that the change in a nucleonic Dirac Hamiltonian as a result of a Λ-hole excitation is obtained for $B'=N$, $B=\Lambda$.

For nuclear matter the perturbations are static (time independent) and uniform (independent of $\mathbf{x}$), and their Fourier transforms thus have only $\mathbf{k}=0$ components. Matrix elements of δh vanish between unperturbed wave functions with different momenta, and the perturbing Hamiltonian cannot connect p-h states to the core ground state. Nonzero positive energy matrix elements of δh are diagonal, and will not contribute to any modification of the wave functions in first-order perturbation theory [10]. However, δh has nonvanishing matrix elements between a positive energy state $u(\lambda,\mathbf{k})$ and a negative energy state $v(\lambda,-\mathbf{k})$.

The first-order correction to the core single-particle spinor wave functions $u_N(\lambda,\mathbf{k})$ due to the perturbation δh is, therefore, given by

$$\delta u_N(\lambda,\mathbf{k})=\sum_{\lambda'}\frac{v_N^\dagger(\lambda',-\mathbf{k})\delta h u_N(\lambda,\mathbf{k})}{\epsilon_N^{(+)}(\mathbf{k})-\epsilon_N^{(-)}(\mathbf{k})}v_N(\lambda',-\mathbf{k}) , \tag{9}$$

where the energy denominator is equal to $2E_k^{N*}$ (the unperturbed energies appear here; E_k^{N*} is the same for positive and negative energy states).

There will consequently be changes in nuclear quantities such as the total baryon current density $\mathbf{j}^{(0)}=\mathbf{j}^{(cc)}+\mathbf{j}_{\text{p-h}}$, which has a core and a p-h contribution. We examine it first and then look at the scalar and vector densities.

The current density $\mathbf{j}$ is given by

$$\mathbf{j}=\frac{1}{\Omega}\left[\left[\sum_{\mathbf{k},\lambda}^{k_p}\bar{u}_N(\lambda,\mathbf{k})\boldsymbol{\gamma}u_N(\lambda,\mathbf{k})\right]+\bar{u}_B(\lambda,\mathbf{p})\boldsymbol{\gamma}u_B(\lambda,\mathbf{p})-\bar{u}_N(\lambda',\mathbf{q})\boldsymbol{\gamma}u_N(\lambda',\mathbf{q})\right] . \tag{10}$$

It is related to $\mathbf{V}$ in the MFT through

$$\mathbf{j}=\frac{m_v^2}{g_v^N}\mathbf{V}^{(cc)}+\frac{m_v^2}{g_v^B}\delta\mathbf{V}_B(\mathbf{p})-\frac{m_v^2}{g_v^N}\delta\mathbf{V}_N(\mathbf{q}) ,$$
$$\mathbf{V}=\frac{g_v^N}{m_v^2}\mathbf{j}^{(cc)}+\frac{g_v^B}{m_v^2}\mathbf{j}_B(\mathbf{p})-\frac{g_v^N}{m_v^2}\mathbf{j}_N(\mathbf{q}) , \tag{11}$$

using Eqs. (4) and (7); $\delta\mathbf{V}$ and $\mathbf{j}_{\text{p-h}}$ have been separated into their particle and hole components, using a self-evident notation.

The combined results in Eqs. (4) and (7) represent the so-called *zeroth-order* contribution (using core basis states) to the total baryon current, which is the *enhanced single-particle current*. Self-consistency requires, however, that we do not stop at the zeroth order. [Indeed, the zeroth-order contribution is based on spinor wave functions calculated *self-consistently for the closed core system,* not for the new, perturbed system. We should certainly look for the contribution of the ω meson, which is absent from Eqs. (4) and (7). In addition, we should expect a dependence on $(g_v^{B'}-g_v^N)V_0$, since the particle B' is measured relative to a background of nucleons.] Going now to the first-order correction in the perturbation series we find

$$\mathbf{j}^{(1)}=\frac{1}{\Omega}\sum_{\mathbf{k},\lambda}^{k_p}[\delta\bar{u}_N(\lambda,\mathbf{k})\boldsymbol{\gamma}u_N(\lambda,\mathbf{k})+\bar{u}_N(\lambda,\mathbf{k})\boldsymbol{\gamma}\delta u_N(\lambda,\mathbf{k})] , \tag{12}$$

with δu_N given by Eq. (9). This change is of the same order in $1/\Omega$ $(1/A)$ as the zeroth-order contribution from the valence particle and hole, because every core particle contributes coherently.

Substituting Eq. (9) into (12), the total change in the baryon current density to first order can be evaluated using the explicit expressions for the spinors (all of the positive and negative energy spinors are unperturbed). It is more elegant and fruitful, however, to follow Furnstahl and Serot's [5] use of projection operators for the core particles. Spin sums can be converted to traces and we find that

$$\mathbf{j}^{(0)}+\mathbf{j}^{(1)}=\frac{1}{\Omega}\left[\frac{\mathbf{p}}{E_p^{B*}}-\frac{\mathbf{q}}{E_q^{N*}}\right]-\frac{1}{\Omega}\sum_k^{k_F}\text{Tr}\left\{[\Lambda_-(-\mathbf{k})\boldsymbol{\gamma}\Lambda_+(\mathbf{k})+\Lambda_+(\mathbf{k})\boldsymbol{\gamma}\Lambda_-(-\mathbf{k})]\frac{\gamma^0\delta h}{2E_k^{N*}}\right\} . \tag{13}$$

In Eq. (13) the projection operators $\Lambda_\pm$ for a particle of positive or negative energy are analogous to the standard free-particle Dirac case and are given by

$$\Lambda_+(\mathbf{k})=\sum_\lambda u_N(\lambda,\mathbf{k})\bar{u}_N(\lambda,\mathbf{k})=\frac{E_k^{N*}\gamma^0-\mathbf{k}\cdot\boldsymbol{\gamma}+M_N^*}{2E_k^{N*}} ,$$

with a corresponding expression for the negative energy counterpart:

$$\Lambda_-(\mathbf{k})=-\sum_\lambda v_N(\lambda,\mathbf{k})\bar{v}_N(\lambda,\mathbf{k})=\frac{-E_k^{N*}\gamma^0+\mathbf{k}\cdot\boldsymbol{\gamma}+M_N^*}{2E_k^{N*}} .$$

We have written Eq. (13) explicitly in order to demonstrate that, in the present treatment, the first-order correction is also divided (similarly to $\mathbf{j}^{(0)}$) into separate particle and hole contributions. This follows from Eq. (8) and shows what characteristic features we can expect from the full core response: the corrections affect the enhanced single-body particle and hole current separately and independently. (Recall that this result is derived for nuclear matter.) Their individual effects are similar to the corresponding one for an extra particle outside a closed core [4,5].

Equation (13) is evaluated using standard trace theorems [11]. The contribution to the sum over the core states ($\sum_{\mathbf{k}}^{k_F}$) from the term of $\gamma^0\delta h$ proportional to g_s^N/m_s^2 vanishes, since it amounts to $\sum_{\mathbf{k}}^{k_F}\mathbf{k}=0$. The term originating from δV_0 vanishes by orthogonality. Therefore, the perturbative change in the baryon current density is solely a result of the three-vector part of the ω field. Thus,

$$\begin{aligned}\mathbf{j}^{(0)}+\mathbf{j}^{(1)}&=\frac{1}{\Omega}\left[\frac{\mathbf{p}}{E_p^{B*}}-\frac{\mathbf{q}}{E_q^{N*}}\right]+\frac{\eta}{\Omega^2}\sum_{\mathbf{k}}^{k_F}\frac{g_v^N}{m_v^2E_k^{N*3}}\left\{\left[\mathbf{k}\cdot\left[g_v^B\frac{\mathbf{p}}{E_p^{B*}}-g_v^N\frac{\mathbf{q}}{E_q^{N*}}\right]\right]\mathbf{k}-E_k^{N*2}\left[g_v^B\frac{\mathbf{p}}{E_p^{B*}}-g_v^N\frac{\mathbf{q}}{E_q^{N*}}\right]\right\}\\&=\frac{1}{\Omega}\left[\frac{\mathbf{p}}{E_p^{B*}}\left[1-\frac{g_v^Ng_v^B}{m_v^2}\frac{\rho_v}{E_{k_F}^{N*}}\right]-\frac{\mathbf{q}}{E_q^{N*}}\left[1-\frac{g_v^{N2}}{m_v^2}\frac{\rho_v}{E_{k_F}^{N*}}\right]\right]\end{aligned}\tag{14}$$

(note that $B'=N$). In Eq. (14) isospin degeneracy is included *for the core,* so that a spin-isospin degeneracy coefficient η (=4 for symmetric nuclear matter) has been introduced into the core contribution. The last expression on the right-hand side of Eq. (14) has been obtained by transforming from a sum over $\mathbf{k}$ to an integral; we have used the notation $E_{k_F}^{N*}=(k_F^2+M_N^{*2})^{1/2}$ for the Fermi energy and $\rho_v=(\eta/6\pi^2)k_F^3$ for the nucleon (vector) density. (As we shall momentarily see, ρ_v is not modified by the core response.) The enhanced particle and hole currents are separately corrected by the core response (similar individual corrections have been found for the case of one particle outside a closed core; see Ref. [5] for the nucleon case, and Ref. [7] for the Λ).

Using the finite Hartree parameters [2] we find for the nucleon ($B=N$) $(g_v^{N^2}/m_v^2)\rho_v/E_{k_F}^{N*}=0.62$, thus

$$\frac{1}{E_{k_F}^{N*}}\left[1-\frac{g_v^{N^2}}{m_v^2}\frac{\rho_v}{E_{k_F}^{N*}}\right]=\frac{0.65}{E_{k_F}^N}$$

or

$$\frac{1}{M_N^*}\left[1-\frac{g_v^{N^2}}{m_v^2}\frac{\rho_v}{E_{k_F}^{N*}}\right]=\frac{0.70}{M_N} .$$

Results for $B=\Lambda$ depend on the value of g_v^Λ which will be discussed in detail later (see Sec. III B 1); it is typically around $0.3g_v^N$ to $0.7g_v^N$, so the pertinent hypernuclear correction is also large. Note that in a consistent first order calculation, the first order perturbation theory correction and the effective mass enhancement ($1/M_B^*$ or $1/E_{k_F}^{N*}$) cancel each other to a high degree if the lowest order only is considered for both the vector and scalar fields (i.e., in M_B^* and in the core response). This result is also true in the more traditional "nonrelativistic" sense using the concept of meson exchange pair currents, as discussed in Sec. III A, and stems from gauge invariance. The important point is that the single-particle enhancement is no longer present. However, the factor of 0.65 (0.70) in the above estimate (where we used M_B^* to all orders in ρ_s, but only the lowest order in ρ_v) is not our final answer: Since the first order correction is large we cannot be satisfied with the lowest order result. In the next section we study higher orders of the core response, again following Furnstahl and Serot [5].

In discussing the effects of the core response on characteristic nuclear quantities it is also necessary to examine the pertinent consequences for the scalar and vector nuclear densities [2,12]. These densities are related to the MFT fields ϕ_0 and V_0, respectively, via relations similar to Eq. (11) (with obvious modifications). It is important to note that not all nuclear observables are affected by the core response. We have shown in [10] that, to order $1/\Omega$, the single-particle energies are correctly described by the closed-core Hartree-MFT solutions. Calculating ρ_v, the timelike component of the total baryon current, we find that

$$\begin{aligned}\rho_v&=\frac{1}{\Omega}\left[\left[\sum_{\mathbf{k},\lambda}^{k_F}\bar{u}_N(\lambda,\mathbf{k})\gamma^0u_N(\lambda,\mathbf{k})\right]+\bar{u}_B(\lambda,\mathbf{p})\gamma^0u_B(\lambda,\mathbf{p})-\bar{u}_N(\lambda',\mathbf{q})\gamma^0u_N(\lambda',\mathbf{q})\right]\\&=\frac{1}{\Omega}(A+1-1)=\frac{A}{\Omega} ,\end{aligned}\tag{15}$$

which is obviously the correct and final answer. (Isospin degeneracy is again included for the core.) Calculating the first order perturbative correction explicitly along the lines of Eqs. (12), (9), and (13) we indeed find that this correction vanishes identically (and not just in the limit $\Omega\to\infty$ as for the single-particle energy case discussed in [10]).

Unlike the case of the vector density, core response corrections do affect the scalar density ρ_s. The zeroth-order contribution (using core basis states) to the total baryon current, including the p-h state, is

$$\rho_s^{(0)}=\frac{1}{\Omega}\left[\left[\sum_{\mathbf{k},\lambda}^{k_F}\bar{u}_N(\lambda,\mathbf{k})u_N(\lambda,\mathbf{k})\right]+\bar{u}_B(\lambda,\mathbf{p})u_B(\lambda,\mathbf{p})-\bar{u}_N(\lambda',\mathbf{q})u_N(\lambda',\mathbf{q})\right]$$

$$=\frac{1}{\Omega}\left[\left[\sum_{\mathbf{k},\lambda}^{k_F}\frac{M_N^*}{E_k^{N*}}\right]+\frac{M_B^*}{E_p^{B*}}-\frac{M_N^*}{E_q^{N*}}\right]. \tag{16}$$

Equation (16) (for ρ_s) differs from Eq. (15) (for ρ_v): the p-h state makes a contribution to ρ_s; furthermore, core response corrections are of the same order in $1/\Omega$ as the p-h contribution, because every core particle contributes. The first order perturbation correction to ρ_s, along the lines of Eqs. (12), (9), and (13), is

$$\rho_s^{(1)}=\frac{\eta}{\Omega^2}\frac{g_s^N}{m_s^2}\left[g_s^B\frac{M_B^*}{E_p^{B*}}-g_s^N\frac{M_N^*}{E_q^{N*}}\right]\left[\sum_{\mathbf{k}}^{k_F}\frac{\mathbf{k}^2}{E_k^{N*3}}\right], \tag{17}$$

where the spin-isospin degeneracy coefficient has been introduced again as in Eq. (14).

It is important to note that the core-response correction for ρ_s, is realized through the σ field [and not via the ω field as in Eq. (14)]. Rigorously this is not unexpected, as our subsequent discussion will prove (see Secs. II C and II D). In the MFT, the response to a perturbation is mediated by a meson of the same quantum numbers as the probe; moreover, at $Q=0$ no scalar-vector mixing occurs. We believe that this is an intuitive demonstration of how closely the scalar density is related to the scalar meson field, while the vector current is associated with the vector meson field. Here this is formally a result of the Lorentz-scalar structure of this quantity, along with a vanishing mixed scalar-vector correction. But the scalar-vector correction does not vanish in general (namely, for a nonzero momentum transfer, $Q\neq 0$, or for a finite system). These remarks will be better and more precisely understood following our discussion in Sec. II C, where we provide an extension of the present results to all orders, using linear response theory.

For nuclear matter, the sums in Eqs. (16) and (17) can be replaced by integrals [as in Eq. (14)]. These integrals can be done analytically and the final result is

$$\rho_s^{(0)}+\rho_s^{(1)}=\eta\frac{M_N^*}{4\pi^2}\left[k_FE_{k_F}^{N*}-M_N^{*2}\ln\frac{k_F+E_{k_F}^{N*}}{M_N^*}\right]+\frac{1}{\Omega}\left[\frac{M_B^*}{E_p^{B*}}-\frac{M_N^*}{E_q^{N*}}\right]$$
$$-\frac{\eta}{\Omega}\frac{1}{2\pi^2}\frac{g_s^N}{M_s^2}\left[g_s^B\frac{M_B^*}{E_p^{B*}}-g_s^N\frac{M_N^*}{E_q^{N*}}\right]\left[\frac{1}{2}k_FE_{k_F}^{N*}+\frac{M_N^{*2}k_F}{E_{k_F}^{N*}}-\frac{3}{2}M_N^{*2}\ln\frac{k_F+E_{k_F}^{N*}}{M_N^*}\right], \tag{18}$$

where the major (core) contribution appears on the first line, and the p-h and core-response corrections are of order $O(1/\Omega)$. The fundamental difference between the results of Eqs. (18) and (14) is a consequence of the vanishing (nonvanishing) core contribution to the current density $\mathbf{j}$ (the scalar density ρ_s); the lowest nonvanishing contribution to $\mathbf{j}$ is of order Ω^{-1} (or A^{-1}).

Using finite hartree parameters [2], the first line of Eq. (18) is $0.93\rho_v$. For nucleons we write the second plus third line as

$$\frac{1}{\Omega}\left[\frac{M_N^*}{E_p^{N*}}-\frac{M_N^*}{E_q^{N*}}\right](1-\xi),$$

and we find that $\xi=0.11$ for symmetric nuclear matter ($\eta=4$). This core-response correction factor of the p-h contribution to ρ_s is much smaller than the corresponding effect in Eq. (14). It is an approximately 10% modification to the $O(1/A)$ valence correction, and is not expected to have any significance in nuclear physics calculations. We shall discuss this subject in more detail in Secs. II C and III.

C. Particle-hole state: Linear response theory

1. General introduction

Since the effects of the modified core can be large, it is necessary to go beyond the first-order result in perturbation theory. This is efficiently achieved using Green's function methods. In this approach, the MFT (Hartree) result corresponds to the self-consistent summation of tadpole contributions to the baryon self-energy. Calculating the Hartree-MFT propagator directly and exactly for the core plus p-h system can be difficult; alternatively one can start with the MFT-Hartree solution for the closed core alone, and then modify this system (in Refs. [5,7] an extra particle is added; here we add a particle and hole to this system). Assuming an inert core with a superimposed p-h state we find the enhanced current $\mathbf{j}^{(0)}$ of Eq. (14). However, the fields produced by the particle and the hole modify the propagator of the core. The total core linear response is determined by the polarization insertion (the RPA-type ring at zero energy) computed with the nucleon Hartree-MFT propagator. In the present discussion for nuclear matter, where the perturbations are static and uniform, we need the response functions at zero four-momentum transfer.

We shall demonstrate that our previous perturbation theory result can be identified with the lowest order ring contribution (Π) to the linear response function Π_∞ at zero momentum transfer. To first order in the ring summation we indeed recover Eq. (14). Summing the rings to all orders, we obtain the full linear response. This formalism provides, therefore, an efficient tool for extending the perturbation-theory results of Sec. II B to all orders once the Green's functions are known.

The treatment presented here is again based on Ref. [5], applied to a p-h state and extended to include the possibility of a hypernuclear production as in the previous section.

2. *The relativistic nuclear Hartree propagators and self-energies*

In order to better motivate our treatment we first outline the exact formalism for the system of interest. (Our actual work will, however, be carried out within the linear response scheme, as explained above.) For the purpose of this brief, explanatory discussion, we first assume a system of nucleons only. In the Green's function approach, the Hartree approximation in nuclear matter is obtained by the self-consistent summation of the self-energy tadpoles in the baryon propagator. In a general, deformed system, the analytic form of the Hartree propagator in nuclear matter is, for nucleons,

$$\begin{aligned}
{}_HG^N(k) &= (\gamma_\mu\kappa^\mu + M_N^*)\left[\frac{1}{\kappa_\lambda\kappa^\lambda - M_N^{*2}+i\delta} + 2\pi i\delta(\kappa_\lambda\kappa^\lambda - M_N^{*2})\theta(\kappa^0)n_\kappa\right] \\
&= {}_HG_F^N(k) + {}_HG_D^N(k) \\
&= (1-n_\kappa)\frac{\Lambda_+(\boldsymbol{\kappa})}{\kappa^0 - E_\kappa^{N*}+i\delta} + n_\kappa\frac{\Lambda_+(\boldsymbol{\kappa})}{\kappa^0 - E_\kappa^{N*}-i\delta} - \frac{\Lambda_-(-\boldsymbol{\kappa})}{\kappa^0 + E_\kappa^{N*}-i\delta} \\
&= {}_HG_p^N(k) + {}_HG_h^N(k) + {}_HG_v^N(k)\ .
\end{aligned} \tag{19}$$

The following notation is used in Eq. (19):

$$\kappa^\mu = k^\mu + {}_H\Sigma_N^{v\mu},\quad M_N^* = M_N + {}_H\Sigma_N^s\ ,$$

$$E_\kappa^{N*} = (\boldsymbol{\kappa}^2 + M_N^{*2})^{1/2},\quad E_N(k) = \pm E_\kappa^{N*} - {}_H\Sigma_N^{v0}\ .$$

The Hartree self-energies (${}_H\Sigma^s, {}_H\Sigma^v$) are related to the mean scalar and vector meson fields by [see also the discussion following Eq. (22)]

$${}_H\Sigma_N^s = -g_s^N\phi_0$$

and

$${}_H\Sigma_N^{v\mu} = -g_v^N V^\mu\ ,$$

so the other familiar expressions for M^* and κ^μ based on Eq. (1) are obtained. The function n_κ is the occupation function that defines the Fermi surface. For a Fermi sphere, $n_\kappa \to n_k \equiv \theta(k_F - |\mathbf{k}|)$. In this paper, where we deal with a p-h excitation of the core, the momentum distribution is no longer spherical; the Fermi surface can be determined by minimizing the mean-field energy density at fixed baryon and momentum densities [5].

We have written the Green's function, Eq. (19), in two alternative forms [2,13]. In the first line it is shown as a sum of Feynman and density-dependent components. The first component resembles the free-particle Feynman propagator (describing the propagation of virtual positive- and negative-energy baryons, or baryons and antibaryons). This is the only piece that remains as the baryon density $n_\kappa \to 0$ (or $k_F \to 0$). The second term is density dependent and allows (at a finite density) for the propagation of holes in the Fermi sea and corrects the propagation of positive-energy baryons to account for the Pauli exclusion principle. The density-dependent piece shifts poles corresponding to occupied positive-energy states from the lower half of the complex k^0 plane to its upper half.

We note [13] that the baryons are "dressed," implying that their energy, mass, and Dirac spinor wave functions are modified by the baryon self-energy ${}_H\Sigma_B$. [In the free case, the virtual dressing of the particles is absorbed into the physical masses and coupling constants. In the many-body system, the interactions change the free Feynman propagator, as shown by Eq. (19) (e.g., $k \to \kappa$, $M_N \to M_N^*$), and the result is an additional many-body effect. The nucleon in the medium is regarded as a "bare" nucleon that gets "dressed" as a result of its interactions with the medium. The self-energy enters the Dyson equation which relates the bare and dressed nucleon propagators.]

In the alternative form (third line) of Eq. (19), the κ^0 dependence is isolated in simple poles. The propagator is divided into physically motivated pieces according to the type of each pole: p (particle) for $\kappa^0 = E_\kappa^{N*}$ and an unoccupied state (a factor $1-n_\kappa$); h (hole) for $\kappa^0 = E_\kappa^{N*}$ and an occupied state (a factor n_κ); v (negative energy state or an antibaryon) for $\kappa^0 = -E_\kappa^{N*} < 0$. The projection operators of Eq. (19) are obtained as before, replacing $\mathbf{k}$ by $\boldsymbol{\kappa}$ in the standard nuclear core expressions [cf. Eq. (13)] [5]. We note that the expression used for ${}_HG^N$ is not Lorentz covariant, but is suitable and more illuminating for our purposes here.

The exact Green's function for the system of interest yields the Hartree self-energies for nucleons:

$$
\begin{aligned}
{}_H\Sigma_N^s &= i\frac{g_s^{N^2}}{m_s^2}\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[{}_HG^N(k)]e^{ik^0\zeta} ,\\
{}_H\Sigma_N^{v\mu} &= i\frac{g_v^{N^2}}{m_v^2}\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma^\mu\, {}_HG^N(k)]e^{ik^0\zeta} ,
\end{aligned} \tag{20}
$$

and the baryon current density:

$$
j^\mu = -i\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma^\mu\, {}_HG^N(k)]e^{ik^0\zeta} \tag{21}
$$

using the formalism of Ref. [2]. The current density j^μ is related to the vector self-energy through [see also Eq. (11)]

$$
j^\mu = -\frac{m_v^2}{g_v^{N^2}}\, {}_H\Sigma_N^{v\mu} . \tag{22}
$$

Later in this section we present and use Hartree self-energies for the general case of a baryon B (in this work we are interested in the cases $B=N$ or $B=\Lambda$, but the formalism is valid for any spin-$\frac{1}{2}$ baryon B; of course, the isoscalar nature of the underlying many-body theory requires some care regarding isospin).

The $e^{ik^0\zeta}$ factors in the integrands above allow the integration contours to be closed in the upper half plane, eliminating contributions from ${}_HG_p^N$ to the self-energies and the baryon current. The other two terms, ${}_HG_h^N+{}_HG_v^N$, yield contributions from both occupied positive-energy states and occupied states in the negative-energy sea.

A short discussion of the relativistic σ-ω MFT is necessary here in order to make the present section more intelligible and self-contained. In the MFT, only density-dependent contributions to the integrals will be retained, namely, those terms proportional to n_κ. For tadpole diagrams this implies that only the occupied positive-energy states contribute (stemming from ${}_HG_h^N$). One is actually integrating over real baryon states within the Fermi sea only. Thus, all terms arising from integrals over ${}_HG_F^N$ are dropped, and only contributions from the density-dependent part of the Green's function, ${}_HG_D^N$, are considered. The MFT results can be derived by summing the tadpole diagrams self-consistently in nuclear matter, retaining only the contributions from nucleons in the filled Fermi sea. Here, the replacement of the full Hartree Green's function by its density-dependent part defines the MFT values of the self-energies: $\Sigma_N^s=-g_s^N\phi_0$ and $\Sigma_N^{v\mu}=-g_v^NV_0\delta^{\mu 0}$, where $\delta^{\mu 0}$ results from rotational invariance when the system is spherically symmetric. [We note that the full relativistic Hartree approximation (RHA) [2] involves divergent integrals over the occupied negative-energy sea. Infinities are removed using a renormalization procedure. No counterterms are needed to render ${}_H\Sigma_N^{v\mu}$ finite, and the RHA result for this term is identical to that of the MFT [2]. Counterterms are necessary, however, to render ${}_H\Sigma_N^s$ finite. The result is a transcendental equation [2] for the RHA scalar self-energy where the first term is similar to the MFT result (and is proportional to ρ_s) and the second term is a finite "vacuum fluctuation" correction. Such corrections are outside the scope of our treatment here.]

In cases where two Hartree propagators appear under the integral (ring diagrams, for example) *there will be contributions from the negative energy states even in the MFT.* This can happen when the density-independent part, ${}_HG_v^N$, of one propagator multiplies the density-dependent part of the other. We have already seen that the mixing of positive- and negative-energy states is crucial in the present treatment [Eqs. (9), (13), (17)], and the same will be true in this section.

Using this prescription we can perform the k^0 integration for the vector and scalar self-energies. We are now left with three-dimensional integrals, which can be converted into sums by putting the system in a large box. Writing the projection operator in terms of the single-particle Dirac spinors we recover the MFT expressions (the meson ground-state expectation values) [5].

3. Core modifications in linear response theory with applications to nuclear currents and densities

The *exact* expressions, Eqs. (19)–(22), are difficult to solve, especially when a different kind of baryon (here we are mainly interested in a Λ hyperon) is embedded in the nuclear medium. As explained above we follow the approach of Sec. II B and start with the simpler (and known [2]) Hartree solution for the closed core itself. This solution is then corrected by adding to it the contribution of the valence particle hole and their effect on the core.

The analytic form of the Hartree propagator for the core nucleons in the nuclear matter limit is [cf. Eq. (19)]

$$
\begin{aligned}
{}^c_HG^N(k) &= (1-n_k)\frac{\Lambda_+(\mathbf{k})}{\kappa^0-E_k^{N*}+i\delta}+n_k\frac{\Lambda_+(\mathbf{k})}{\kappa^0-E_k^{N*}-i\delta}\\
&\quad -\frac{\Lambda_-(\mathbf{k})}{\kappa^0+E_k^{N*}-i\delta}\\
&= {}^c_HG_p^N(k)+{}^c_HG_h^N(k)+{}^c_HG_v^N(k).
\end{aligned} \tag{23}
$$

Note that we have left κ^0 in the denominators, because the time component ($\mu=0$) of the vector self-energy is nonzero even for a spherical system (while the space components $\mu=1,2,3$ vanish and thus $\boldsymbol{\kappa}\rightarrow\mathbf{k}$). The time component of the vector self-energy is, however, different in the two cases represented by Eqs. (19) and (23) since the latter is calculated for a closed core while the former is for the nonspherical, core + p-h system. As we shall see, for a constant time component of the self-energy (or V_0) the integration over κ^0 can be shifted to an integration over k^0, and the two give equivalent results. The occupation function for the core is $n_k=\theta(k_F-|\mathbf{k}|)$, as discussed following Eq. (19).

We now add, as in Sec. II B, a particle (with momentum $\mathbf{p}$) and a hole (with momentum $\mathbf{q}$). Their propagation will take place through the meson fields. In the extreme (and inappropriate) approximation of an inert core, the meson fields are fixed and the propagator for the new system is obtained by merely shifting the pole for the (previously unoccupied) valence state to the upper half k^0 plane, while eliminating the corresponding contribution of the (previously occupied) hole state. This description of the new occupation status of the valence p-h state is formally obtained by adding a correction $\delta_H G$ to the Green's function of the core.

The additional function needed to shift the pole for a particle (B) of momentum $\mathbf{p}$ and a hole (N) of momentum $\mathbf{q}$ is obtained from the Green's function [cf. Eq. (19)]. We keep only the density-dependent part (which is the only one necessary here, as we shall see) and expand the $\delta((k^0)^2-E^{*2})$ function using the standard formula. Then, with the θ function and the definition of the projection operator, and inserting a δ distribution in the three-momentum variable, we obtain the following correction to the core propagator:

$$\begin{aligned}[\delta_H G(k)]_{\rm val} &= {}^{\rm val}_{H}G^{BN}(k)\\ &= 2\pi i[\delta(k^0-E_p^{B^*})\tfrac{1}{2}\delta_{\mathbf{p},\mathbf{k}}\Lambda_+^{\rm B}(\mathbf{p})-\delta(k^0-E_q^{N^*})\tfrac{1}{2}\delta_{\mathbf{q},\mathbf{k}}\Lambda_+(\mathbf{q})]\\ &= [\delta_H G^B(k)-\delta_H G^N(k)]_{\rm val}\\ &= {}^{\rm val}_{H}G^B(k)-{}^{\rm val}_{H}G^N(k)\ . \end{aligned} \tag{24}$$

Here the single-particle spinors are determined by the *core* potentials, as was the case in Sec. II B. The valence particle (hole) is equally distributed over the two possible values of the spin projection λ.

Note that Eq. (24) implies independent corrections from the baryon (B) and the hole (N), reflecting their individual interactions with the core. The baryon and the hole do not interact with each other in this approximation; this would be the meaning of a particle-hole state in the present work. [In order to study the effect of the mutual p-h interaction (as well as the effect of the corrected core back on the p-h states) it would appear to be necessary to carry out an *exact* Hartree calculation of the pertinent nuclear system. Although the effect is clearly of order $O(1/A)$, its actual magnitude is unclear at this point.]

At this level of an inert core plus valence p-h, the interaction of the valence particle and hole with the core, through the meson fields, is included to all orders. But the core response is not included (as discussed in Secs. II B and I). We call this our zeroth order (using core basis states). We can calculate the three-vector current density, Eq. (21), for this zeroth order using ${}^{c}_{H}G^N+{}^{\rm val}_{H}G^{BN}$ as the approximate propagator for the complete system:

$$\begin{aligned}\mathbf{j}(0) &= -i\int\frac{dk^0}{2\pi}\frac{1}{\Omega}\sum_{\mathbf{k}}\mathrm{Tr}\{\boldsymbol{\gamma}[{}^{c}_{H}G^N(k)+{}^{\rm val}_{H}G^{BN}(k)]\}e^{ik^0\zeta}\\ &= 0+\frac{1}{\Omega}\left[\frac{\mathbf{p}}{E_p^{B*}}-\frac{\mathbf{q}}{E_q^{N*}}\right]\ . \end{aligned} \tag{25}$$

This result is identical to our previous zeroth order, enhanced single-particle current [see Eqs. (13) and (14)], as expected. (In a more realistic, finite-nucleus treatment, the valence particle is subject to lower nuclear-medium densities than the hole; it would thus have a larger effective mass, namely, closer to the free mass. Consequently, the relative enhancements will be larger for the hole than for the particle in this model.)

This picture for the core+p-h system is not self-consistent, as already noted, because the core is not inert and the valence particle and hole generate fields that act on the core nucleons. This is in complete analogy with the results of Sec. II B, where Eqs. (5)–(7) show the modifications in the mean fields due to the particle and the hole; their effects on the core nucleons [Eqs. (8) and (9)] and on the nuclear current [Eq. (14)] and densities [Eqs. (15) and (18)] to first order in perturbation theory have been derived. In the present approach the additional (p-h) fields modify the core propagator ${}^{c}_{H}G^N(k)$, Eq. (23).

In the spirit of the present treatment, the valene particle and hole will be considered as external sources of meson fields acting on the core. In nuclear matter these fields are static and uniform, and the corresponding changes in the Hartree self-energies are obtained by using the correction to the Green's function, Eq. (24). Changes in the self-energies for baryons of type B' due to a valence baryon B where $B',B=N$, or Λ are, for example,

$$i\frac{g_s^{B'}g_s^B}{m_s^2}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[{}^{\rm val}_{H}G^B(k)]\ ,$$

for the scalar self-energy, with corresponding expressions for the vector quantity. In this notation, substituting $B'=N$ and $B=\Lambda$ yields the changes in the nucleonic Hartree self-energies as a result of a valence Λ, and so on [see also Eq. (8) regarding the introduction of B']. Thus, the appropriate definitions for the incremental changes in the self-energies are [cf. Eqs. (20)]:

$$\begin{aligned}[\delta_H\Sigma^s]_{\rm val} &= {}^{\rm val}_{H}\Sigma^s\\ &= i\frac{g_s^N}{m_s^2}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[g_s^B\,{}^{\rm val}_{H}G^B(k)-g_s^N\,{}^{\rm val}_{H}G^N(k)]\end{aligned}$$

and

$$\begin{aligned}[\delta_H\Sigma^{v^\mu}]_{\rm val} &= {}^{\rm val}_{H}\Sigma^{v^\mu}\\ &= i\frac{g_v^N}{m_v^2}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma^\mu(g_v^B\,{}^{\rm val}_{H}G^B(k)\\ &\qquad -g_v^N\,{}^{\rm val}_{H}G^N(k))]\ . \end{aligned} \tag{26}$$

(Note that $B'=N$.) Evaluating the integrals we find

$$[\delta_H \Sigma^s]_{\rm val} = -\frac{g_s^N}{m_s^2}\frac{1}{\Omega}\left[g_s^B \frac{M_B^*}{E_p^{B*}} - g_s^N \frac{M_N^*}{E_q^{N*}}\right] = -g_s^N \delta\phi_0 , \tag{27}$$

$$[\delta_H \Sigma^{v^0}]_{\rm val} = -\frac{g_v^N}{m_v^2}\frac{1}{\Omega}(g_v^B - g_v^N) = -g_v^N \delta V_0 , \tag{28}$$

and

$$[\delta_H \boldsymbol{\Sigma}^v]_{\rm val} = -\frac{g_v^N}{m_v^2}\frac{1}{\Omega}\left[g_v^B \frac{\mathbf{p}}{E_p^{B*}} - g_v^N \frac{\mathbf{q}}{E_q^{N*}}\right] = -g_v^N \delta\mathbf{V} . \tag{29}$$

These results have been obtained by returning to a discrete set of linear momentum states in a very large box of volume Ω, as done before (see [12] or Eq. (25), for example). The results in Eqs. (27)–(29) agree with those obtained in Sec. II B, Eqs. (5)–(7).

Next, we look at the modification of the core propagator using linear response theory. The linear response of the system is determined by polarization insertions, or rings, created with the core Green's function ${}^c_H G^N$. In our case, where the nuclear matter limit implies static and uniform perturbations, the response functions are required at a vanishing four-momentum transfer ($Q=0$) only.

Prior to calculating the full RPA response, let us first look at the lowest order case. This will be shown below to be fully equivalent to the first-order perturbations corrections obtained in Sec. II B. At this order, the corresponding change in the baryon current of the core is

$$j^{(1)\mu} = -\Pi^\mu(0)[\delta_H \Sigma^s]_{\rm val} + \Pi^{\mu\nu}(0)[\delta_H \Sigma_v^\mu]_{\rm val} . \tag{30}$$

A rigorous derivation of Eq. (30) is provided in Sec. II D, where formal comments omitted from the present discussions are provided for the interested reader. In Eq. (30), the so-called scalar-vector and the vector-vector polarization insertions are

$$\Pi^\mu(Q) = i\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma^\mu\, {}^c_H G^N(k+Q)\, {}^c_H G^N(k)] \tag{31}$$

and

$$\Pi^{\mu\nu}(Q) = i\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma^\mu\, {}^c_H G^N(k+Q)\gamma^\nu\, {}^c_H G^N(k)], \tag{32}$$

respectively. The two core propagators in Eqs. (31) and (32) yield an RPA (linear-response) ring, with one σ and one ω meson lines for the scalar-vector polarization insertion, or two ω meson lines for the vector-vector one. Note that due to the vector character of j^μ, only vector polarization insertions are required. However, the full polarization insertion has a scalar-scalar piece as well; this will be discussed later.

We have already explained that in the present work, where we study static and uniform perturbations, we only need the $Q=0$ values of the polarization insertions. To evaluate Eqs. (31) and (32) we just follow exactly the discussion of Ref. [5], because only the core (nucleons only) propagator is involved.

Substituting the three terms [see Eq. (23)] for each core propagator into the integrands in Eqs. (31) and (32), we find nine terms in each integrand. These may be denoted by xy, where $x,y=p,h$ or v. At $Q=0$ the ph and hp terms vanish before any integration is carried out, since the factor $n_k(1-n_k)=0$. We emphasize again that this is a result of the pure static and uniform nature of the perturbation. We then integrate over k^0; terms with both poles in the same half-plane (pp, hh, vv, hv, and vh) do not contribute. Thus, only two terms, pv and vp, contribute in our case. As in Sec. II B [Eqs. (9), (12), and (17)], only the mixing of positive- and negative-energy states is involved.

Consistent with the MFT, we must keep only the density-dependent component of the integrands. This requires the substitution $1-n_k \to -n_k$. The integrals now include only the occupied states, and the omitted terms are the vacuum fluctuation corrections [2]. After the k^0 integration is carried out we are left with three-dimensional momentum-space integrals over traces identical to the ones needed in Sec. II B [for the evaluation of Eqs. (13) and (17)]. Evaluating the required traces and integrals we find

$$\Pi^\mu(0)=0 ,$$
$$\Pi^{\mu\nu}(0) = \begin{cases} -\dfrac{\rho_v}{E_{k_F}^{N*}}\delta_{ij} \equiv \Pi_T\delta_{ij}, & \mu=i,\nu=j \\ 0, & \text{otherwise} \end{cases} \tag{33}$$

(these results are valid only when the core is a filled Fermi sphere). In Eq. (33) $\rho_v=(\eta/6\pi^2)k_F^3$, as in Eq. (14); we have also included in η the isospin degeneracy factor *for the core.* Equation (33) and the discussion of the foregoing paragraphs provide a picture whereby the linearized core response will be obtained through a series of RPA NN rings with two ωNN vertices; such rings as shown in Fig. 1. This is a type of static response that does not occur in a nonrelativistic nuclear many-body theory, and is similar to what we have found in Sec. II B, Eqs. (13) and (14). It appears here in a rather transparent way, as a result of the self-consistency imposed [2] on the theory.

The lowest-order correction to the baryon current can now be evaluated using Eq. (30) and then be added to the zeroth order, Eq. (25), to yield the total baryon current at this level of our calculation:

$$\mathbf{j}^{(0)}+\mathbf{j}^{(1)} = \frac{1}{\Omega}\left[\frac{\mathbf{p}}{E_p^{B*}}\left[1+\frac{g_v^N g_v^B}{m_v^2}\Pi_T\right] - \frac{\mathbf{q}}{E_q^{N*}}\left[1+\frac{g_v^{N^2}}{m_v^2\Pi_T}\right]\right] . \tag{34}$$

Substituting Π_T, Eq. (33), into Eq. (34), we recover the results of Sec. II B, Eq. (14). The two approaches lead to identical results, as discussed above. The first-order perturbation-theory result of Sec. II B can now be identified with the lowest-order ring contribution to the

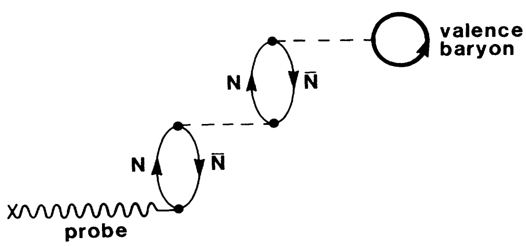

FIG. 1. Typical contribution to the relativistic core-response correction applied to a valence-baryon current. The rings indicate the mixing of positive (N) and negative ($\bar{N}$) energy core states. (Only the Pauli-blocked negative energy states are included; see Sec. III A). The heavy solid line represents a valence-baryon wave function calculated with the core mean meson fields and the dashed line is the ω- or σ-meson propagator, depending on the nature of the probe [see Eqs. (35) and (42)]. The full linear-response correction is obtained by summing the rings to all orders. Each valence baryon current is individually corrected by the core response in our model.

linear response function at a vanishing momentum transfer. The linear response theory, however, provides a more efficient means of going beyond the first-order perturbation theory result.

As discussed in Sec. II B, the perturbation of the core is large, and one cannot stop at the lowest order. Summing the perturbative contribution to all orders, we obtain the full linear response for the many-body system. This is practically obtained by a summation to all orders of the polarization insertions or rings. In nuclear matter this is obtained simply as a geometric series, yielding the full RPA response function (the interested reader is again referred to Sec. II D for a rigorous discussion):

$$\Pi_\infty^{(T)} = \Pi_T + \Pi_T \frac{g_v^{N^2}}{m_v^2} \Pi_T + \cdots = \Pi_T \left[1 + \frac{g_v^{N^2}}{m_v^2} \Pi_\infty^{(T)} \right] = \frac{\Pi_T}{1 - (g_v^{N^2}/m_v^2)\Pi_T} . \tag{35}$$

This function describes the full linearized response of the nuclear core to the static, uniform perturbation of the p-h state, probed by means of the baryon current. Note that only vector mesons are included here since the scalar mesons do not contribute to this linear response [$\Pi^\mu(0)=0$ in Eq. (33)]. Furthermore, the vector-meson propagators assume a very simple form in Eq. (35), because they do not transfer any finite momentum.

The total baryon current (including the RPA linearized core response contribution) can now be obtained by replacing Π_T by $\Pi_\infty^{(T)}$ in Eq. (34):

$$\mathbf{j} = \frac{1}{\Omega} \left[\frac{\mathbf{p}}{E_p^{B*}} \left[1 + \frac{g_v^N g_v^B}{m_v^2} \Pi_\infty^{(T)} \right] - \frac{\mathbf{q}}{E_q^{N*}} \left[1 + \frac{g_v^{N^2}}{m_v^2} \Pi_\infty^{(T)} \right] \right]$$
$$= \frac{1}{\Omega} \left\{ \frac{\mathbf{p}}{E_p^{B*}} \left[1 + \frac{g_v^B}{g_v^N} \frac{g_v^{N^2}}{m_v^2} \Pi_T \left[1 - \frac{g_v^{N^2}}{m_v^2} \Pi_T \right]^{-1} \right] - \frac{\mathbf{q}}{E_q^{N*}} \left[1 - \frac{g_v^{N^2}}{m_v^2} \Pi_T \right]^{-1} \right\}$$
$$= \frac{1}{\Omega} \left[\frac{\mathbf{p}}{E_p^{B*}} \left[1 - \frac{g_v^N}{m_v^2} \Pi_T (g_v^N - g_v^B) \right] - \frac{\mathbf{q}}{E_q^{N*}} \right] \left[1 - \frac{g_v^{N^2}}{m_v^2} \Pi_T \right]^{-1} . \tag{36}$$

In our model, the particle (B) and hole (nucleon) currents are individually corrected by the core response, and each effect is of the type depicted in Fig. 1. Using the finite Hartree parameters [2] in Eq. (35) we find for nucleons $-(g_v^{N^2}/m_v^2)\Pi_\infty^{(T)}=0.38$ [$(g_v^{N^2}/m_v^2)\Pi_T = -(g_v^{N^2}/m_v^2)\rho_B/E_{k_F}^{N*} = -0.62$]. Thus, $\Pi_\infty^{(T)}$ differs considerably from Π_T. For this reason we could not stop our calculation at the first order, but had to go to higher orders. Furthermore, $(1/M_N^*)(1+(g_v^{N^2}/m_v^2)\Pi_\infty^{(T)}) = 0.62/M_N^* \approx 1/M_N$, so the single-particle $1/M^*$ enhancement is no longer present. The *relativistic* RPA nuclear response restores the *nonrelativistic* value of the single-particle current when evaluated to all orders; this is not true for the lowest-order perturbative correction, as discussed in Sec. II B [following Eq. (14)]. A similar conclusion also holds for $B=\Lambda$ with a reasonable g_v^Λ/g_v^N ratio (of the order of 0.5, as discussed later): the total RPA corrected Λ current is virtually equal to the nonrelativistic value.

The total core correction for the baryon B (embedded in the nuclear medium) differs, though, by a factor of g_v^B/g_v^N from the correction found for nucleons only. We note that the response to the particle (B) results from both the spacelike part of the ω meson and from

$(g_v^B - g_v^N)V_0$. The latter does not contribute in the pure nucleon-only case, so a Λ hyperon (or any other baryon which is distinguishable from the nucleon) introduces new many-body relativistic effects, not present for regular nuclei. *This provides a potentially fruitful way of probing the effects of the strong fields present in relativistic nuclear models.* Such attitude has been adopted in Refs. [7,14].

As in Sec. II B we are also interested in studying the effects of the core response on the scalar and vector nuclear densities [2,12]. [Results up to the first order in perturbation theory are presented in Eqs. (15)–(18).]

We obtain the vector density ρ_v, as the zeroth component of the baryon current, Eqs. (21) and (22). At the zeroth-order level (inert core, plus valence contributions, using the core basis states) we find (including isospin degeneracy for the core)

$$\rho_v^{(0)} = -i\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma^0({}^{c}_{H}G^N(k) + {}^{\mathrm{val}}_{H}G^{BN}(k))]e^{ik^0\zeta}$$

$$= \frac{\eta}{(2\pi^3)}\int_0^{k_F} d^3k + \frac{1}{\Omega}\sum_{\mathbf{k}}(\delta_{\mathbf{p},\mathbf{k}} - \delta_{\mathbf{q},\mathbf{k}})$$

$$= \frac{1}{\Omega}(A+1-1) = \frac{A}{\Omega} \ . \tag{37}$$

The traces needed for evaluating $\rho_v^{(0)}$ are identical to those required for Eq. (28). [Equation (28) and the valence contributions to Eq. (37) differ only by constant multiplicative factors.] The result, Eq. (37), is identical to our previous one, Eq. (15), and is obviously the correct and final answer for ρ_v.

Our last statement will now be formally proven within the linear response theory used in this section. We look at the first-order correction first; since $\Pi^{\mu}(0)=0$ and $\Pi^{0\nu}(0)=0$, it follows from Eq. (30) that $\rho_v^{(1)}=0$. This result is also valid when the polarization insertions are integrated to all orders in linear response theory (some necessary formal details are provided in Sec. II D of the present work and in Appendix A of Ref. [5], but this result is also intuitively clear based on our discussion so far). Thus, there are no core response (RPA) corrections to ρ_v, and this quantity is correctly given by the inert-core plus valence contributions alone.

We turn now to the scalar density ρ_s. While the vector polarization insertions [Eqs. (31) and (32)] were required for studying the current, j^{μ}, here we need the scalar quantities. The missing part, namely, the scalar-scalar polarization insertion, is given in lowest order by

$$\Pi^{(0)}(Q) = i\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}[{}^{c}_{H}G^N(k+Q){}^{c}_{H}G^N(k)] \ , \tag{38}$$

yielding an RPA NN ring with two σNN vertices.

As explained above, only the value of $\Pi^{(0)}$ at $Q=0$ is required here. It is evaluated in the same way as done for Eqs. (31) and (32); we end up with a trace identical to the one needed for the evaluation of Eq. (17) in Sec. II B. Thus,

$$\Pi^{(0)}(0) \equiv \Pi_s$$

$$= -\frac{\eta}{2\pi^2}\left[\frac{1}{2}k_F E_{k_F}^{N*} + \frac{M_N^{*2}k_F}{E_{k_F}^{N*}} - \frac{3}{2}M_N^{*2}\ln\frac{k_F + E_{k_F}^{N*}}{M_N^*}\right] , \tag{39}$$

where the required integral has already been evaluated in order to get the perturbation-theory result of Eq. (18), and isospin degeneracy for the core has been included in the factor η [as in Eq. (33)].

We proceed along the lines of the calculation for j^{μ}; ρ_s is proportional to the scalar hartree self-energy [cf. Eqs. (22) and (11)]. At the zeroth-order level (core+valence contributions) we find

$$\rho_s^{(0)} = -i\int \frac{dk^0}{2\pi}\frac{1}{\Omega}\sum_{\mathbf{k}}\mathrm{Tr}[{}^{c}_{H}G^N(k) + {}^{\mathrm{val}}_{H}G^{BN}(k)]e^{ik^0\zeta} \ , \tag{40}$$

which gives exactly the same result as Eq. (16) of Sec. II B. We now calculate the lowest-order linear response correction (one ring) where

$$\rho_s^{(1)} = \Pi^{\mu}(0)[\delta_H\Sigma_{\mu}^{v}]_{\mathrm{val}} - \Pi^{(0)}(0)[\delta_H\Sigma^S]_{\mathrm{val}} \tag{41}$$

[cf. Eq. (30)]. Using the results of Eqs. (39) and (33) for the scalar-scalar and scalar-vector polarization insertions, and substituting $[\delta_H\Sigma^s]_{\mathrm{val}}$ from Eq. (27) we recover the results of Eqs. (17) and (18), Sec. II B.

Next, we shall obtain the full linear response by summing the perturbative first-order contribution to all orders. This extension of our first-order result to all orders is analogous to our previous treatment of the current $\mathbf{j}$ in Eqs. (35) and (36). In nuclear matter it is once again obtained as a geometric series, yielding the full RPA response function

$$\Pi_{\infty}^{(0)} = \Pi_s + \Pi_s\frac{g_s^{N^2}}{m_s^2}\Pi_s + \cdots$$

$$= \Pi_s\left[1 + \frac{g_s^{N^2}}{m_s^2}\Pi_{\infty}^{(0)}(0)\right]$$

$$= \frac{\Pi_s}{1-(g_s^{N^2}/m_s^2)\Pi_s} \ . \tag{42}$$

Equation (42) describes the full linearized response of the nuclear core to the static, uniform perturbation of the p-h state, probed by means of the scalar density. Only scalar mesons are included here: the vector mesons do not contribute in this case as $\Pi^{\mu}(0)=0$. There exists a strong similarity between Eqs. (42) and (35).

The nuclear scalar density for the system, including the inert core, valence p-h, and RPA linearized core response contributions, is obtained by [cf. Eq. (36)]

$$\rho_s = \frac{\eta}{\Omega}\sum_{\mathbf{k}}^{k_F}\frac{M_N^*}{E_k^{N*}} + \frac{1}{\Omega}\left[\frac{M_B^*}{E_p^{B*}}\left[1+\frac{g_s^N g_s^B}{m_s^2}\Pi_\infty^{(0)}(0)\right] - \frac{M_N^*}{E_q^{N*}}\left[1+\frac{g_s^{N^2}}{m_s^2}\Pi_\infty^{(0)}(0)\right]\right]$$

$$= \frac{\eta}{(2\pi^3)}\int_0^{k_F}\frac{M_N^*}{E_k^{N*}}d^3k + \frac{1}{\Omega}\left\{\frac{M_B^*}{E_p^{B*}}\left[1+\frac{g_s^B}{g_s^N}\frac{g_s^{N^2}}{m_s^2}\Pi_s\left[1-\frac{g_s^{N^2}}{m_s^2}\Pi_s\right]^{-1}\right] - \frac{M_N^*}{E_q^{N*}}\left[1-\frac{g_s^{N^2}}{m_s^2}\Pi_s\right]^{-1}\right\}$$

$$= \frac{\eta}{(2\pi^3)}\int_0^{k_F}\frac{M_N^*}{E_k^{N*}}d^3k + \frac{1}{\Omega}\left[\frac{M_B^*}{E_p^{B*}}\left[1-\frac{g_s^N}{m_s^2}\Pi_s(g_s^N-g_s^B)\right] - \frac{M_N^*}{E_q^{N*}}\right]\left[1-\frac{g_s^{N^2}}{m_s^2}\Pi_s\right]^{-1} . \quad (43)$$

In our model, the particle (B) and hole (nucleon) contributions to the scalar density are individually corrected by the core response. Each effect is of the type depicted in Fig. 1, mediated by the scalar meson and involving the scalar-scalar polarization insertions summing to all orders.

Using the finite Hartree parameters [2] in Eqs. (39) and (42) we find for nucleons

$$-\frac{g_s^{N^2}}{m_s^2}\Pi_\infty^{(0)}(0)=0.10 \quad \left[\frac{g_s^{N^2}}{m_s^2}\Pi_s=-0.11\right] .$$

Thus, $\Pi_\infty^{(0)}(0)$ and Π_s are numerically very close to each other, $\Pi_\infty^{(0)}(0)/\Pi_s=0.90$ [the small first-order correction, Eqs. (17) and (18), obviously remains small when summed to all orders within the RPA]. The core-response correction is only around 10% of the valence contribution, which is itself a $1/A$ correction to the scalar density of the core. Note the basic difference between the baryon current density j and the scalar density ρ_s, which is a consequence of the vanishing (nonvanishing) total contribution form the closed core to the former (the latter). Although it is not expected that the core response will have any non-negligible effect in nuclear physics, the $1/A$ valence correction may prove to be important when a precise comparison between theory and experiment is called for. One such case we are aware of, namely, K^+-nucleus scattering, will be discussed briefly in Sec. III.

D. The relativistic Hartree propagator and RPA polarization insertions: Formal comments

In this section we provide the formal discussion necessary for a rigorous derivation of the results of Sec. II as well as new, additional ones. The material in this section is required for a complete and exact understanding of our results, but much more than intuitive insight has already been provided by our preceding discussions. Consequently, the reader may skip these formal comments without loss of clarity.

Our discussion is based on Appendix A of Ref. [5] (hereinafter referred to as A), extended again to include a p-h state with a baryon which may be distinguishable from the (rest of the) nucleons. The total subsequent change in the Hartree propagator is

$$\delta_H G(k)=[\delta_H G(k)]_{\text{val}}+[\delta_H G(k)]_{\text{RPA}} . \quad (44)$$

In Eq. (44), $[\delta_H G(k)]_{\text{val}}$ describes the new occupied status of the valence p-h state and is given by Eq. (24), while $[\delta_H G(k)]_{\text{RPA}}$ accounts for the modifications of the core arising from the p-h state. The main difference between our discussion here and that of A lies in the valence part.

The scalar-vector and vector-vector polarization insertions [Eqs. (31) and (32)] have been introduced in the context of the current j^μ. The scalar-scalar quantity is given to lowest order in Eqs. (38) and (39). In general we need the full polarization insertion including all three components. Coupled equations for the full polarization insertions to all orders, $\Pi_\infty^{(0)}$, Π_∞^μ, and $\Pi_\infty^{\mu\nu}$, were given by Chin [15], and are also presented in A. Under the conditions $\Pi^\mu(0)=0, \Pi^{\mu\nu}(0)\propto\delta_{ij}$ which apply in this work, one finds that the full polarization insertions (denoted as above by the subscript ∞) *at* $Q=0$ are obtained from

$$\Pi_\infty^{(0)}(0)=\Pi^{(0)}(0)+\Pi^{(0)}(0)\frac{g_s^{N^2}}{m_s^2}\Pi_\infty^{(0)}(0) , \quad (45a)$$

$$\Pi_\infty^\mu(0)=0 , \quad (45b)$$

for the scalar-scalar and scalar-vector cases, and

$$\Pi_\infty^{\mu\nu}(0)=\Pi^{\mu\nu}(0)-\Pi^\mu{}_\sigma(0)\frac{g_v^{N^2}}{m_v^2}\Pi_\infty^{\sigma\nu}(0) , \quad (45c)$$

for the vector-vector case. Equations (45c) and (45a) are identical to Eqs. (35) and (42); the three types of polarization insertions are decoupled under the above conditions.

Using these relations we now obtain an expression for $[\delta_H G(k)]_{\text{RPA}}$. As shown in A, this correction to the Green's function has a contribution from the valence self-energies, Eqs. (26)–(29), and from the linear response of the core (expressed in terms of the polarization insertions):

$$[\delta_H G(k)]_{\rm RPA}={}^c_H G^N(k)\left[\left[1+\frac{g_s^{N2}}{m_s^2}\Pi_\infty^{(0)}(0)-\frac{g_v^{N2}}{m_v^2}\gamma_\mu\Pi_\infty^\mu(0)\right][\delta_H\Sigma^s]_{\rm val}-\left[\gamma^\nu+\frac{g_s^{N2}}{m_s^2}\Pi_\infty^\nu(0)-\frac{g_v^{N2}}{m_v^2}\gamma_\mu\Pi_\infty^{\mu\nu}(0)\right][\delta_H\Sigma_\nu^v]_{\rm val}\right]{}^c_H G^N(k)\ . \tag{46}$$

We note that $[\delta_H\Sigma^{s,v}]_{\rm val}$ are independent of k [cf. Eqs. (27)–(29)]. The total correction to the core Green's function within linear response theory can be obtained by substituting Eqs. (24) and (46) into Eq. (44), and used for calculating corrections to j^μ, M_B^*, etc., as done in the previous section.

The connection between $\delta_H G$ and the physical quantities of interest in this work may become clearer by examining the resulting changes in the self-energies:

$$\begin{aligned}\delta_H\Sigma^s&=i\frac{g_s^N}{m_s^2}\int\frac{d^4k}{(2\pi)^4}{\rm Tr}[g_s\delta_H G(k)]e^{ik^0\zeta}\ ,\\ \delta_H\Sigma^{v^\mu}&=i\frac{g_v^N}{m_v^2}\int\frac{d^4k}{(2\pi)^4}{\rm Tr}[g_v\gamma^\mu\delta_H G(k)]e^{ik^0\zeta}\ ,\end{aligned}\tag{47}$$

where the generic $g_{s,v}$ can be the nucleon (N) or the baryon (B) coupling constants as necessary [see, e.g., Eq. (26)]. The contributions from $[\delta_H G(k)]_{\rm val}$ are given by Eqs. (26)–(29); with $[\delta_H G(k)]_{\rm RPA}$ included as well, we get the result presented in A:

$$\begin{aligned}\delta_H\Sigma^s&=[\delta_H\Sigma^s]_{\rm val}+[\delta_H\Sigma^s]_{\rm RPA}\\ &=\left[1+\frac{g_s^{N2}}{m_s^2}\Pi_\infty^{(0)}(0)\right][\delta_H\Sigma^s]_{\rm val}\\ &\quad-\frac{g_s^{N2}}{m_s^2}\Pi_\infty^\mu(0)[\delta_H\Sigma_\mu^v]_{\rm val}\end{aligned}\tag{48}$$

and

$$\begin{aligned}\delta_H\Sigma^{v^\mu}&=[\delta_H\Sigma^{v^\mu}]_{\rm val}+[\delta_H\Sigma^{v^\mu}]_{\rm RPA}\\ &=\frac{g_v^{N2}}{m_v^2}\Pi_\infty^\mu(0)[\delta_H\Sigma^s]_{\rm val}\\ &\quad+\left[g^{\mu\nu}-\frac{g_v^{N2}}{m_v^2}\Pi_\infty^{\mu\nu}(0)\right][\delta_H\Sigma_\nu^v]_{\rm val}\ .\end{aligned}\tag{49}$$

In deriving Eqs. (48) and (49) one uses the expressions for the full polarization insertions, Eqs. (45a) and (45c). Note that, using Eq. (22) as well as (28), (29), and (11), the expression for $\delta_H\Sigma^{v^\mu}$ is identical to Eqs. (25), (37), (30), and (35) (Sec. II C) for the nuclear current and vector densities; similarly $\delta_H\Sigma^s$, Eq. (49), is similar to Eqs. (40)–(43) for the scalar density.

Substituting the expressions for $\delta_H\Sigma^{s,v}$ [Eqs. (48) and (49)] into Eq. (46) we can formally simplify the expression for $[\delta_H G(k)]_{\rm RPA}$. Then, using Eq. (44) we obtain for the total change in the Hartree propagator

$$\delta_H G(k)=[\delta_H G(k)]_{\rm val}+{}^c_H G^N(k)(\delta_H\Sigma^s-\gamma_\mu\delta_H\Sigma^{v^\mu}){}^c_H G^N(k)\ . \tag{50}$$

Equation (50) is an integral equation for $\delta_H G(k)$, because $\delta_H\Sigma^{s,v}$ are defined through $\delta_H G$. Substituting Eq. (50) into the definitions of the changes in the baryon self-energies, Eqs. (47), we find

$$\delta_H\Sigma^s=[\delta_H\Sigma^s]_{\rm val}+\frac{g_s^{N2}}{m_s^2}[\Pi^{(0)}(0)\delta_H\Sigma^s-\Pi^\mu(0)\delta_H\Sigma_\mu^v]$$

and (51)

$$\delta_H\Sigma^{v^\mu}=[\delta_H\Sigma^{v^\mu}]_{\rm val}+\frac{g_v^{N2}}{m_v^2}[\Pi^\mu(0)\delta_H\Sigma^s-\Pi^{\mu\nu}(0)\delta_H\Sigma_\nu^v]\ .$$

These are coupled, transcendental equations for the changes in the baryon self-energy as a result of adding the p-h state to the core. The equations decouple when the core system is a filled Fermi sphere, $\Pi^\mu(0)=0$ and $\Pi^{\mu\nu}(0)\propto\delta_{ij}$. Then,

$$\delta_H\Sigma^s=[\delta_H\Sigma^s]_{\rm val}\left[1-\frac{g_s^{N2}}{m_s^2}\Pi_s\right]^{-1}\quad\text{[cf. Eq. (42)]}\ , \tag{52a}$$

$$\delta_H\Sigma^{v^0}=[\delta_H\Sigma^{v^0}]_{\rm val}\ , \tag{52b}$$

and

$$\delta_H\Sigma^{v^i}=[\delta_H\Sigma^{v^i}]_{\rm val}\left[1-\frac{g_v^{N2}}{m_v^2}\right]^{-1}\quad\text{[cf. Eq. (35)]}\ . \tag{52c}$$

As noted in A, the RPA-corrected propagator ${}^c_H G^N(k)+\delta_H G(k)$ is still only an approximation to the full Hartree Green's function for the core +p-h system. The results obtained here are exact in the nuclear matter limit, while additional contributions [of higher order in $1/\Omega$ $(1/A)$] are expected for finite, small nuclei. Such calculations have recently been reported for regular nuclei [8]; we are unaware of such developments for hypernuclei.

A brief summary of our results so far is given in the next section, where we also discuss some of their possible applications. The importance of the phenomenological $\omega\Lambda\Lambda$ tensor vertex is discussed in Sec. III B, where we apply the theory to hypernuclei.

III. SUMMARY AND DISCUSSION: APPLICATIONS AND CONCLUSIONS

This section contains a discussion of useful applications of our results to a number of nuclear problems. In order to make it somewhat self-contained we also provide a brief summary here of our formal steps and a further discussion of some physical aspects of the linear core response.

A. Introduction and brief summary

Accurate MFT (Hartree) predictions of nuclear properties for the non-closed-core case (e.g., one particle outside a closed shell, or a p-h state added to the core as considered in this paper), where spherical symmetry is lost, require an extension of the closed-core, ground-state Hartree calculations to the non-spherically-symmetric cases. This would imply a direct solution of the corresponding self-consistent MFT equations for such systems, and represents the most direct approach to pursue. However, such self-consistent solutions are technically more difficult to achieve, since the simplicity of the equations and wave functions for the closed core is destroyed. This is a result of the breakdown of spherical symmetry.

The alternative approach, adopted and described in this paper, uses the single-particle nucleon wave functions of the closed-shell system as a starting point for describing the modified system. Self-consistency now implies that the valence particle (hole) cannot be treated independently of the inert core and provide an external perturbation to the filled core. The additional meson fields generated by the p-h are considered as static external fields acting on the nucleons in the core. Thus, the closed-core Dirac nucleons get modified. In a perturbative calculation in nuclear matter, it has been shown that the important effect of the added p-h is to cause a mixing between the positive-energy core wave functions and their negative energy counterparts with the same momentum. This correction due to mixing of negative-energy states can be summed to all orders using linear response theory (RPA rings). This summation is implicit in the full, self-consistent Hartree solution for the complete system, and is a purely relativistic type of response that does not occur in a nonrelativistic nuclear theory. Even small changes in the single-particle Hamiltonian can imply significant changes in some nuclear properties if all the core particles contribute coherently. The differences between the linear response and the self-consistent calculation for the full system are of order $1/A$, and the two approaches are therefore equivalent in nuclear matter. The special interest in the linear response approach lies in the clear identification of the pertinent physical effects.

In Sec. II B we have shown, using perturbation theory, that it is incorrect to use the closed-core solutions in calculating certain properties of the complete core+valence system. Indeed, the core response to the valence p-h state leads to important corrections in some nuclear observables. In the case of the baryon current **j**, for example, these corrections are as large as the zeroth-order valence result. Consequently, the inclusion of higher orders of the core response is evidently called for. This task has been carried out, within the linear response theory, in Sec. II C.

Core-response corrections essentially restore the nonrelativistic value of the single-particle current, canceling the M_B/M_B^* enhancement in Eqs. (7), (13), and (25). No core-response corrections exist for the nuclear vector density ρ_v; $O(1/A)$ modifications for the scalar density, ρ_s, arises from the valence p-h contribution and the RPA response. This difference between ρ_s and **j** is a consequence of the nonvanishing (vanishing) core contribution to the former (the latter). It is important to note that the core RPA response correction to ρ_s is realized through the scalar field (and not via the vector field as for **j**).

An interesting point to note is that it is possible to view the relativistic linearized core response effect as a consequence of Pauli blocking of certain $N\bar{N}$ rings, namely, those involving a nucleon in an already occupied positive-energy state. In this context we recall that the full relativistic Hartree approximation (RHA) [2] includes contributions from the filled Dirac sea of negative-energy states (antinucleons) to the nucleon self-energy; in the MFT such contributions are neglected. Thus, RHA linear response would include contributions involving all possible negative-energy configurations, which can be divided into a vacuum polarization part (sometimes called the Feynman part) minus the part of the vacuum response that is Pauli blocked in the nuclear medium (the density-dependent part). The latter part arises from the filled positive-energy states, and involves the Pauli blocking of $N\bar{N}$ excitations that could otherwise put the nucleon into one of these states. The vacuum polarization contributions have been neglected in the MFT linear response, as discussed in Sec. II C. We can therefore regard the consistent MFT linear response in nuclear matter as a Pauli-blocking effect [16]. This interesting interpretation also emerges from Eqs. (31), (32), and (38) upon substituting Eq. (23) *and* $1-n_k \rightarrow -n_k$, as in Sec. II C 3.

For completeness we note that the linear core response to valence states is sometimes called "backflow." This rubric arises from an interpretation of the core response as an actual retrograde coherent motion therefrom, or a screening by the vector interaction that slows the valence particles down. We have avoided this terminology in the present work since it is unnecessary and even confusing, as indicated recently by Noble [4].

It is interesting to note that nonrelativistic magnetic moment predictions for closed-shell $\pm$one nucleon, including the σ- and ω-meson exchange two-body pair currents (MEC) corrections, show that the Dirac isoscalar magnetic moments are enhanced by the σ exchange and reduced by the ω exchange. This has been shown [17] by Blunden, by Ichii *et al.*, and by Delorme and Towner to lowest order in perturbation theory by calculating exchange current corrections using Feynman pair diagrams, and is a general result of gauge invariance. In order to render a comparison with the MFT meaningful, the scalar and vector exchange currents are assumed to originate from effective fields, which do not have to correspond uniquely and exactly to the same mesons considered in free-space physical processes. This is contrary to the usual procedures in exchange currents calculations. Thus, neither free ω mass and couplings to nucleons nor the parameters derived in the various meson-exchange potentials are used. One assumes instead, in the spirit of the MFT, that the sum of the scalar and vector potentials is very small and negative (binding). Furthermore, the MEC calculations of Blunden do not invoke phenomenological short-range correlations between nucleons, nor hadronic form factors. (These have the effect of reducing the heavy-meson pair graphs contributions.) The large degree of cancellation between the scalar and vector exchange currents leaves only a small net contribution from these mesons. Here the direct (Hartree) matrix elements (as opposed to the exchange) yield the major contributions, while the Fock terms are small in this case. The nonrelativistic and relativistic nuclear baryon currents in nuclear matter are equivalent to lowest order in the mesonic mean fields, differing from the free convection current (i.e., $\mathbf{q}/M_N$ in the case of a nucleon) by just a small binding energy contribution. However, the meson exchange pair currents contributions are of a relativistic origin, and amount to a perturbative addition of relativistic corrections; the term "nonrelativistic" applies to the exchange currents only in the sense of a nonrelativistic reduction. Since we have compared here a nonrelativistic and a relativistic result, we note that the latter case does not allow for a model-independent separation of "orbital" and "spin" contributions to the magnetic moments.

Moreover, it has been pointed out by Bentz *et al.* [4] that no enhancement of the nuclear current due to the reduced bound-baryon effective mass is expected, based on the Ward-Takahashi identity in a general framework (not restricted to the σ-ω model). For isoscalar nuclear currents in the Hartree approximation to the σ-ω model, the Ward-Takahashi identity results in the RPA core-response correction discussed in Sec. II C.

B. Applications

Several important and interesting applications of our results will be discussed and suggested in this section; some will be looked into in depth, while the rest will be only briefly mentioned. They include relativistic studies of hypernuclei (our main subject of study here), K^+-nucleus interactions, meson (especially π^0 and K^+) photoproduction on nuclei, photonuclear processes: photon- or electron-induced nucleon knockout (γ,p) or $(e,e'p)$ and proton radiative capture (p,γ), inelastic scattering of various probes off nuclei, and weak interactions in nuclei. Since some of these applications comprise the subject matter of a separate paper, they will only be described briefly here. We hope to address such issues in detail in future works. Other applications discussed here, however, are theoretically attractive but in practice yield very small effects which cannot be detected experimentally. These cases will also be discussed and explained.

1. Hypernuclear currents and densities

In this section we consider a hypernuclear system consisting of a closed core of nucleons plus a Λ particle–nucleon-hole state (i.e., a Λ hyperon which has been produced on a closed-shell nucleus). Possible production mechanisms are the strangeness exchange reaction (K^-,π^-) or the associated production reactions (π^+,K^+), (γ,K^+), (p,K^+). This section therefore involves the application of the σ-ω model to hypernuclear physics, which is an area of great interest in nuclear science [23]. A number of authors [7,14,24] have applied Dirac phenomenology to hypernuclear studies and have obtained interesting results. In particular, Cohen and Furnstahl [7] have studied the core-response effect on the magnetic moments of closed-shell plus Λ hypernuclei, and found a difference between relativistic (MFT) and nonrelativistic (Schmidt) predictions. The difference arises directly from the core response and provides a useful opportunity for an experimental test of the underlying dynamical model. Due to its isoscalar nature, the Λ hyperon represents a very suitable baryon for studies within the σ-ω model, where only isoscalar currents can be handled. Furthermore, the large (and poorly known) isovector many-body corrections are not expected to be important for the case, and the Λ contribution to the nuclear magnetic moment results only from the anomalous Λ magnetic moment $\mu_\Lambda=-0.613\mu_N$. These observations make Λ hypernuclei a very attractive tool for studying the applications of the relativistic σ-ω model.

The formalism presented in this work enables us to deal with closed-core plus Λ-h hypernuclei. Toward this end we note that numerical values of the coupling constants of the Λ hyperon to the σ and ω fields are required in places such as Eq. (36) or Eq. (43). From the phenomenology of hypernuclear binding energies and level splittings we know that the Λ couplings to the mean scalar and vector fields are considerably smaller than the corresponding nuclear values. Thus, one may reasonably conclude that $g^\Lambda_{s,v}/g^N_{s,v}<1$. Furthermore, practitioners of hypernuclear Dirac theory usually assume $g_s^\Lambda/g_s^N \simeq g_v^\Lambda/g_v^N$, that is, both Dirac scalar and vector potentials for the Λ scale by the same factor from the nucleonic ones. Our qualitative results do not depend on this assumption, and we could adopt different ratios for the scalar and vector coupling constants; however, this assumption is not unreasonable, since the large degree of cancellation between scalar and vector potentials in the MFT demands that they remain close in magnitude.

In order to achieve consistency with the empirics [25], a possible choice for the coupling constants of the Λ hyperon is

$$\frac{g_s^\Lambda}{g_s^N}=\frac{g_v^\Lambda}{g_v^N}=0.4 \; . \tag{53}$$

It is interesting to note that this choice of couplings is also found to be required to correctly account for the hypernuclear binding in the work of Glendenning and Moszkowski [71]. We are unable to comment on this result at this time, however, it is possible that a more detailed Hartree-Fock calculation could resolve the issue [72]. These values [Eq. (53)] are based on extending the pure scalar+vector σ-ω MFT to achieve a Dirac theory of hypernuclei using the Lagrangian of Eq. (1). For regular nucleons this model adequately describes the bulk properties of nuclear matter. When applied to finite nuclei, the pure scalar+vector model also reproduces the nonrelativistic standard shell model with a central potential $V_c^N \simeq -53$ MeV and a spin orbit $V_{so}^N \simeq 17$ meV. Although finite nuclear theory is, for the most part, outside the scope of this work, a meaningful discussion of hypernuclei necessitates a digression here; moreover, the main applications of the theory presented in Sec. II are for finite nuclear many-body phenomena.

The above values, Eq. (53), for $g_{s,v}^{\Lambda}$ are significantly smaller than any naive quark model predictions for the σ,ω meson-Λ coupling constants. The simple quark-counting prediction with the s quark as a spectator is

$$g_m^{\Lambda} \simeq \tfrac{2}{3} g_m^N \quad (m=\sigma \text{ or } \omega) .$$

However with such high couplings the central and spin-orbit potentials are predicted to be much larger than the empirical ones, by up to a factor of 4. Most authors felt compelled to assume the much smaller ratio, Eq. (53), although we know of no theoretical justification for such a small ratio.

In contrast to the nonrelativistic shell model for *ordinary nuclei,* the σ-ω model explains the large spin-orbit and shallow central potentials naturally in its nonrelativistic limit. For hypernuclei, on the other hand, the Λ-nuclear spin-orbit interaction is small and the nonrelativistic shell model seems to be a perfectly satisfactory starting point. In this case, the pure scalar+vector Dirac model yields a much too large spin-orbit interaction. This is a result of the connection between the central and spin-orbit potentials in the Dirac approach. However, recent experiments consistently indicate a very low limit on the Λ-hypernuclear spin-orbit potential [26]. It is impossible to accommodate a reasonable central potential, capable of producing a bound Λ hypernucleus, along with a very small (perhaps vanishing) spin-orbit potential. A number of papers [18–22] have recently raised the possibility of a large $\omega\Lambda\Lambda$ tensor vertex. When included in the MFT σ-ω Lagrangian as an additional phenomenological term, this tensor vertex transforms the complicated Λ-nuclear single-particle Dirac equation with scalar, vector, and tensor potentials into a simple nonrelativistic shell model wave equation with just a shallow central potential in the nonrelativistic limit [21]. Predictions for Σ and Ξ hypernuclei have also been made [21].

We now present a brief summary of the approach outlined above, in a quark model [19] that goes beyond the broken SU(3) flavor symmetry by extrapolating the chiral invariant vector coupling of the QCD Lagrangian to long distances starting from a Fierz transformation of its quark action. The model includes relativistic effects of the interacting quark in terms of its small Dirac wave function from some confinement model (in a bag or constituent quark model). For the nucleon, the scalar meson vertex is given by [19]

$$g\sqrt{2}\langle 1\rangle_N = 3gF_0^-(q^2)\sqrt{2}\,\bar{u}_N(p')u_N(p) \tag{54a}$$

in the bag model (BM), or, in the constituent quark model (CQM):

$$g\sqrt{2}\langle 1\rangle_N = 3g\sqrt{2}\left[1-\frac{\alpha}{4m_q^2}\right]\left[1+\frac{\alpha}{4m_q^2}\right]^{-1}\bar{u}_N(p')u_N(p)\exp(Q^2/6\alpha) . \tag{54b}$$

Here g is an overall meson-quark coupling constant, p,p' are nucleon momenta, Q is the four-momentum transfer, and m_q is the constituent quark mass. The following overlap integrals will be used here:

$$F_0^{\pm}(Q^2) = 4\pi\int_0^R dr\, r^2[g^2(r)\pm f^2(r)]j_0(Qr) \tag{55a}$$

and

$$F_1(Q^2) = 16\pi\frac{M}{q}\int_0^R dr\, r^2 g(r)f(r)j_1(Qr) , \tag{55b}$$

where R is the bag radius and M the baryon mass of the relevant meson-baryon vertex, and $f(r)$ and $g(r)$ are the lower and upper components of the Dirac spinor.

The nucleon-vector meson vertex is given by

$$\begin{aligned}\langle\gamma^\mu(\tau)^T\rangle_N &= \left[1-\frac{Q^2}{4M_N^2}\right]^{-1}\bar{u}_N(p')\left[\left[x_TF_0^+ + \frac{Q^2}{8M_N^2}y_TF_1\right]\gamma^\mu - (x_TF_0-\tfrac{1}{2}y_TF_1)\frac{i}{2M_N}\sigma^{\mu\nu}Q_\nu\right](\tau_N)^T u_N \\ &\equiv \bar{u}_N(p')\left[F_{1T}\gamma^\mu + \frac{i}{2M_N}F_{2T}\sigma^{\mu\nu}Q_\nu\right](\tau_N)^T u_N(p)\end{aligned} \tag{56a}$$

in the BM, and

$$\langle\gamma^\mu(\tau)^T\rangle_N = \left[1-\frac{Q^2}{4M_N^2}\right]^{-1}\bar{u}_N(p')\left[\left[\Gamma_{0T}-\frac{Q^2}{2M_N}\Gamma_T\right]\gamma^\mu-(\Gamma_{0T}-2M_N\Gamma_T)\frac{i}{2M_N}\sigma^{\mu\nu}Q_\nu\right](\tau)^T u_N(p) , \tag{56b}$$

in the CQM. In Eqs. (56)

$$x_T=3^{1-T},\quad y_T=-2(\tfrac{5}{3})^T\ , \tag{57}$$

and

$$\Gamma_{0T}=\left[1+\frac{\alpha}{4m_q^2}\right]^{-1}3^{1-T}\left[1+\frac{\alpha}{4m_q^2}\left[1+\frac{Q^2}{9\alpha}\right]\right]\times\exp\left[\frac{Q^2}{6\alpha}\right], \tag{58a}$$

$$\Gamma_T=\left[1+\frac{\alpha}{4m_q^2}\right]^{-1}\left[\frac{5}{3}\right]^T\frac{1}{3m_q}\exp\left[\frac{Q^2}{6\alpha}\right]. \tag{58b}$$

Also, τ (τ_N) is the quark (nucleon) isospin operator, and $T=0$ or 1; obviously $T=0$ for the ω meson. The ω meson couples via $ig\gamma^\mu$ to each of the three valence u or d quarks, so its coupling to the *nucleon* is $ig\langle\gamma^\mu\rangle_N$. The ω-nucleon vector coupling constant in this model is given by

$$g_{\omega NN}=gF_{10}(Q^2=0)=3g\quad(\text{BM})\ , \tag{59a}$$

$$g_{\omega NN}=g\Gamma_{00}(Q^2=0)=3g\quad(\text{CQM})\ . \tag{59b}$$

The ω-nucleon tensor coupling is given, respectively, by

$$f_{\omega NN}=gF_{20}(Q^2=0)=-3gF_0^+(Q^2=0)+gF_1(Q^2=0)\quad(\text{BM})\ , \tag{60a}$$

$$f_{\omega NN}=-g\left[\Gamma_{00}(Q^2=0)-2M_N\Gamma_0(Q^2=0)\right]=g\left[-3+\left[1+\frac{\alpha}{4m_q^2}\right]^{-1}\frac{2M_N}{3m_q}\right]\quad(\text{CQM})\ . \tag{60b}$$

Fitting m_q to the measured nucleon mass and α to the known value for the axial-vector coupling constant $g_A=1.25$, we find a fairly large tensor-to-vector ratio:

$$f_{\omega NN}/g_{\omega NN}=-0.47\ , \tag{61a}$$

while from the nucleon anomalous magnetic moments [21]

$$f_{\omega NN}/g_{\omega NN}=-0.09\ . \tag{61b}$$

This small value, Eq. (61b), is in good agreement with the hadronic phenomenology [27]. Of course, arguments can be given pro and con a fit to the measured nucleon magnetic moment values. It has been shown [21] that the result (61b) is stable relative to small changes in the nucleon isovector and isoscalar magnetic moments and that the procedure leading to Eq. (61b) also yields a tensor-to-vector ratio for the ρNN coupling which is closer to phenomenological values.

The small tensor-to-vector ratio of Eq. (61b) agrees well with the MFT Lagrangian for nucleons. For the Λ hyperon we adopted [21] the *uds* basis, where quarks are treated as distinguishable and the u,d quarks are explicitly antisymmetrized. Assuming no admixture of strange quark content in the ω (or the σ), the Okubo-Zweig-Iizuka (OZI) rule implies that the mesons will couple only to the u and d quarks. The $\omega\Lambda\Lambda$ vertex is consequently given by

$$\langle\Lambda|g\gamma^\mu|\Lambda\rangle=2g\left[1-\frac{Q^2}{4M_\Lambda^2}\right]^{-1}F_0^+\bar{u}_\Lambda(p')\times\left[\gamma^\mu-\frac{i}{2M_\Lambda}\sigma^{\mu\nu}Q_\nu\right]u_\Lambda(p) \tag{62}$$

yielding the ratio

$$f_{\omega\Lambda\Lambda}/g_{\omega\Lambda\Lambda}=-1\ . \tag{63}$$

This ratio is independent of a fitting procedure to a particular set of data, in contrast to the nucleon case, Eqs. (61). It is indeed, *independent of the magnetic moment of the* Λ, which comes from the σ-matrix element of the s quark and does not tell us anything about the value Eq. (63). (See Ref. [21] for more details.)

When applied to hypernuclei, these results bring about considerable changes compared with the pure scalar+vector model. The latter come about because of the ratio $f_{\omega\Lambda\Lambda}/g_{\omega\Lambda\Lambda}$, Eq. (63), being both large and precisely equal to -1, relative to the small nucleon value, Eq. (61b). The strong $\omega\Lambda\Lambda$ tensor coupling adds an extra term to the MFT Lagrangian,

$$-\frac{f_{\omega\Lambda\Lambda}}{4M_\Lambda}\bar{\psi}_\Lambda\sigma^{\mu\nu}F_{\mu\nu}\psi_\Lambda\ , \tag{64}$$

where $F_{\mu\nu}=\partial_\mu V_\nu-\partial_\nu V_\mu$. Using Eq. (63) and the *on-shell* Gordon decomposition in momentum space

$$\gamma_\mu-\frac{i}{2M_\Lambda}\sigma_{\mu\nu}Q^\nu=\frac{(p+p')_\mu}{2M_\Lambda}\ , \tag{65}$$

where p,p' are the initial and final Λ four-momenta, Eqs. (63) and (64) result in a pure convection current coupling [Eq. (65)] between the Λ and the vector mean field. The single-particle Dirac equation for a Λ embedded in a hypernuclear system is

$$\left\{i\gamma_\mu\partial^\mu-\left[M_\Lambda+g_V^\Lambda\frac{(p+p')^\mu}{2M_\Lambda}V_\mu-g_s^\Lambda\phi_0\right]\right\}\psi_\Lambda=0\ . \tag{66}$$

As shown in Refs. [20,21], this resolves the outstanding problem of the small Λ-hypernuclear spin-orbit interaction. The final result is [21] a transformation from a complicated Λ-hypernuclear single-particle equation with scalar, vector, and tensor potentials to a simple Dirac equation virtually equivalent to the nonrelativistic shell model with only a shallow central potential. (Note that this discussion relies on the intensive nature of the observable, where only the time component of the four-vector potential is important.)

In Ref. [21] the $\omega\Lambda\Lambda$ tensor coupling, Eq. (64), was derived from a quark model. In a purely hadronic theory such terms are sometimes put by hand into the Lagrangian, in analogy with magnetic-moment contributions of Pauli type representing the interaction of the electromagnetic field with the anomalous magnetic moment of the baryon [cf. Eq. (67) below]. It has been realized that such contributions should emerge (in terms of hadronic degrees of freedom) as higher-order perturbative corrections in the field theoretical calculations [28]. However, a complete theory along these lines has never been worked out; indeed, very similar words were written 40 years ago [29] in very similar circumstances. It is important to keep in mind, however, that the magnetic moments and tensor couplings should not be treated as tree-level contributions in the strict hadronic theory sense.

The preceding discussion may question the necessity of a Λ-Dirac model. However, in a hypernucleus one deals with nucleons and the hyperon simultaneously, and the Dirc approach provides a natural framework for dealing with both, without *ad hoc* forces. Thus, we consider the results of Ref. [21] for Λ hypernuclei as a success of the Dirac-MFT, because such a consistent and natural description for both nucleons an a hyperon does not seem to exist in the nonrelativistic shell model.

With the ω-Λ tensor coupling included, the larger values for g_m^Λ predicted from the quark model are no longer ruled out, although a further decrease in the experimental limit on the Λ hypernuclear spin-orbit interaction, if found in the future, is likely to change this situation. Thus, we will consider values for g_m^Λ/g_m^N in the range of 0.4 [Eq. (53)] to 0.7 (quark model values) reasonable. However, our qualitative results will not depend on these particular numbers.

Since the core response effects are small for the single-particle energies we shall deal with the magnetic moments and scalar densities of hypernuclei. We start with a pure scalar+vector theory (without the tensor couplings) and then examine the effect of the tensor terms on the results. Magnetic moments of closed-shell plus one Λ hyperon have been given by Cohen and Furnstahl [7], while those of closed-shell $\pm$ one *isoscalar* nucleon are presented in Ref. [5].

In the present framework, where the Λ hyperon and hole currents are separately corrected by the core response in nuclear matter, the *total isoscalar effective electromagnetic current* [2] in the hypernuclear ground state is [cf. Eq. (36)]

$$\langle \mathbf{J}(\mathbf{x})\rangle = -\tfrac{1}{2}U_N^\dagger(\mathbf{x})\boldsymbol{\alpha}U_N(\mathbf{x})\left[1-\frac{g_v^{N^2}}{m_v^2}\Pi_T\right]^{-1} - \frac{\kappa_s}{2M_N}\vec{\nabla}x\{U_N^\dagger(\mathbf{x})\beta\boldsymbol{\Sigma}U_N(\mathbf{x})\}$$
$$-\tfrac{1}{2}U_\Lambda^\dagger(\mathbf{x})\boldsymbol{\alpha}U_\Lambda(\mathbf{x})\frac{g_v^\Lambda}{g_v^N}\left[1+\frac{m_v^2}{g_v^{N^2}}\Pi_T^{-1}\right]^{-1} + \frac{\mu_\Lambda}{2M_N}\vec{\nabla}x\{U_\Lambda^\dagger(\mathbf{x})\beta\boldsymbol{\Sigma}U_\Lambda(\mathbf{x})\}\ , \tag{67}$$

in a pure scalar+vector theory (without tensor couplings *yet*). Here $U_B(\mathbf{x})$ ($B=\Lambda,N$) is the Hartree single-particle solution for the Λ or N in the meson fields of the closed-shell core [2], the isoscalar nucleon anomalous magnetic moment is $\kappa_s=(\kappa_p+\kappa_n)/2$, $\Pi_T\equiv-\rho_v/E_{k_F}^{N^*}$ from Eq. (33), $\boldsymbol{\alpha}$ and β are the usual Dirac matrices, and

$$\boldsymbol{\Sigma}=\begin{bmatrix}\sigma & 0\\ 0 & \sigma\end{bmatrix}.$$

Note that the Λ has no charge, so its valence current is not detected by an electromagnetic probe, but we must include the relativistic core response current, which modifies the electromagnetic current and the magnetic moment. The anomalous magnetic moment operators themselves are not modified in the present model [30]; the corresponding *elementary* contributions to the nuclear current in Eq. (67) are consequently not appreciably modified in the present model, because the Dirac matrix $\beta\boldsymbol{\Sigma}$ does not mix upper and lower components. Note that in a nonrelativistic shell-model picture, the Λ contribution to the total current results only from the anomalous component.

Based on systematics of regular nuclei, where the Schmidt values for the magnetic moments agree with measurements for even-even or odd-odd nuclei in the neighborhood of doubly-magic nuclei, we expect even-mass hypernuclei to be well described by the simple configuration $|j_N\otimes j_\Lambda\rangle_j$. The magnetic moments of ΛN^{-1} (or ΛN) states are obtained by coupling a nuclear moment (μ_N) and a Λ-hyperon moment (μ_Λ). The total hypernuclear spin J is obtained by coupling a nuclear spin j_N with a Λ-hyperon spin. If the hyperon is assumed to be in the $s_{1/2}$ state (which is, of course, a very reasonable assumption for the purpose of determining magnetic moments via weak-decay mechanisms), then the hypernuclear ground state is simply described by $[(j_N)^{\pm 1}\otimes\Lambda s_{1/2}]_J$.

Magnetic moments of the $N^{-1}\Lambda$, $N\Lambda$ systems are given by [31]

$$\langle\mu(J=j_n-\tfrac{1}{2})\rangle=\frac{2j_N-1}{2j_N}\frac{2j_N+2}{2j_N+1}\langle\mu_N\rangle-\frac{2j_N-1}{2j_N+1}\langle\mu_\Lambda\rangle\ ,$$
$$\langle\mu(J=j_N+\tfrac{1}{2})\rangle=\langle\mu_N\rangle+\langle\mu_\Lambda\rangle\ , \tag{68}$$

where $\langle\mu_N\rangle$ is the *nuclear* magnetic moment of the j_N state and $\langle\mu_\Lambda\rangle$ the Λ-hyperon moment of the $s_{1/2}$ state. A similar result can be obtained by applying the generalized Landé formula (see Eqs. (9.16) and (9.21) in Ref. [32]).

Because of the numerical values of μ_p,μ_n,μ_Λ, we find that the magnetic moments of $p\Lambda$ systems are nonnegative, while those of $n\Lambda$ systems are nonpositive. For $N\Lambda$ systems the Schmidt lines of $J=j_n+\frac{1}{2}$ are straight lines on a Schmidt plot of $\langle\mu\rangle$ against j_N, due to the additivity of magnetic moments of such stretched states, Eq. (68), considering the Schmidt lines of regular odd-mass nuclei with spin $j_N = l_N + \frac{1}{2}$.

Using Eqs. (68) and (36) it is now possible to calculate the MFT results for the magnetic moments of Λ-hole hypernuclear states. From Eq. (36) we find that the particle and hole currents are individually corrected by the core response; this observation depends on the approximations used in our model, which we believe to be justified for hypernuclear Λ-h systems. In particular, neglecting the two-particle Λ-h interaction, each core-response correction is of the type depicted in Fig. 1. We could use, therefore, previous results by Cohen and Furnstahl [7] and by Furnstahl and Serot [5] (for a Λ hyperon or for a nucleon outside a closed shell, respectively) in Eqs. (68). However, it can be reasonably argued that such a calculation would be an empty exercise from the experimental point of view, since hole-state magnetic moments are neither very well described by the Schmidt model nor by the "valence-plus-core" relativistic MFT. To a large extent, this comes about because of the isovector component of the nucleon magnetic moment; the MFT is primarily an isoscalar theory. We therefore use here the experimental values for the nuclear contribution to the magnetic moment [$\langle \mu_N \rangle$ in Eq. (68)]. Since isoscalar magnetic moments are well described by the MFT, one might also consider the (isoscalar) combinations of Λ particle—neutron hole plus Λ particle—proton hole. However these are produced at two different reactions [(K^-,π^-) or (π^+,K^+) for the Λ particle–neutron hole and (p,K^+), $(e,e'K^+)$, (γ,K^+) or a reaction of a similar type for the Λ particle–proton hole] requiring two separate hypernuclear experiments, increasing considerably the experimental error. The hyperon contribution, on the other hand, is believed to be reliable since the Λ is purely isoscalar.

Magnetic moment contributions for Λ particle–neutron-hole states, where the initial nuclei are ^{16}O, ^{40}Ca, ^{90}Zr, and ^{208}Pb are given in Table I. The Λ single-particle is assumed to be the $1s_{1/2}$, because unlike nucleons the Λ is not Pauli excluded from any of the nuclear occupied states; the hypernuclear lifetime ($\sim 10^{-10}$ sec) is large enough to allow the hyperon to reach the $1s_{1/2}$ state. We note that in certain cases, such as $_\Lambda{}^{16}$O and $_\Lambda{}^{208}$Pb, *large differences occur between the relativistic and nonrelativistic predictions,* resulting from the core response to the Λ hyperon (which is a purely relativistic effect). These results are certainly encouraging; however, the pertinent measurements are difficult to carry out and would require a major experimental effort [33]. This is the only case we are aware of where a careful relativistic calculation provides a prediction which is different from the nonrelativistic one for a pure scalar + vector model.

The possibility of the strong tensor coupling of the ω meson to Λ, Eqs. (63) and (64), affects the magnetic properties of hypernuclei within the framework of the minimal $\sigma+\omega$ model. Gattone, Chiapparini, and Izquierdo [22] have recently calculated numerically the effect of the tensor Lagrangian term, Eq. (64), on the results of Ref. [7]. They find that the resulting predicted magnetic moments of Λ hypernuclei are restored back to the nonrelativistic (single particle) predictions, when the Λ single-particle state is assumed to be $1s_{1/2}$. On the other hand, Gattone *et al.* find relativistic theoretical modifications of the Λ hypernuclear magnetic moments for higher Λ single-particle states, similar to Ref. [7].

While this effect is beyond the scope of this work (which is limited to nuclear matter) we believe that the present work will be incomplete without a proper discussion of the tensor coupling. Without repeating the calculation of Gattone *et al.* [22] we can provide a physical insight into their nuclear results in an analytical and intuitive manner [34]. We will have to deal with a finite system; a compact and self-contained derivation will be provided in the ensuing discussion.

Toward this end we start with the *nucleon* Dirac equation obtained from the Lagrangian of Eq. (1):

$$[-i\boldsymbol{\alpha}\cdot\vec{\nabla}+\beta(M_N-g_s^N\phi_0)+g_V^N V_0-\boldsymbol{\alpha}\cdot g_V^N \mathbf{V}]\psi_N(\mathbf{r}) = E_N\psi_N(\mathbf{r}) . \quad (69)$$

A standard manipulation [35] gives

$$2(E_N-g_V^N V_0)\psi_N^\dagger(\mathbf{r})\boldsymbol{\alpha}\psi_N(\mathbf{r}) = -i\psi_N^\dagger(\mathbf{r})(\overleftarrow{\nabla}-\vec{\nabla})\psi_N(\mathbf{r})+\vec{\nabla}\times[\psi_N^\dagger(\mathbf{r})\boldsymbol{\Sigma}\psi_N(\mathbf{r})]-2g_V^N\psi_N^\dagger\psi_N . \quad (70)$$

TABLE I. Hypernuclear magnetic moments (in units of μ_N) predicted in the present model for Λ-particle–nucleon-hole states with $J=j_N+\frac{1}{2}$ [see Eq. (68)]. The Λ single-particle state is assumed to be $1s_{1/2}$ (see text). With the free Λ-hyperon magnetic moment $\mu_\Lambda=-0.613\mu_N$, the nonrelativistic Schmidt value $\langle\mu_\Lambda\rangle=-0.613\mu_N$ as well.

Hypernucleus	Λ Contribution	Nuclear contribution (measured)	Total magnetic moment	Nonrelativistic prediction
$^{16}_{\Lambda}$O	−0.648	0.719	0.071	0.106
$^{16}_{\Lambda}$N	−0.648	−0.283	−0.931	−0.896
$^{40}_{\Lambda}$Ca	−0.665	1.022	0.357	0.409
$^{40}_{\Lambda}$K	−0.665	0.391	−0.274	−0.222
$^{90}_{\Lambda}$Y	−0.676	−0.137	−0.813	−0.750
$^{208}_{\Lambda}$Pb	−0.681	0.583	−0.098	−0.03

The first two terms on the right-hand side of Eq. (70) are convection and spin contributions, while the third one results from the spacelike vector potential **V**. Note that $f_{\omega NN}\cong 0$ and there is no other spin contribution in the nucleon case.

The spinors ψ_N in Eqs. (69) and (70) are the solutions for the bound state single-particle wave functions of the entire system (nuclear core + hyperon), not just the closed-core (spherical) problem. Equation (70) can be compared with the Gordon decomposition, Eq. (65), for free, on-shell nucleons. Gattone *et al.* [22] use Eq. (65) in an approximate finite nucleus calculation. Since for infinite nuclear matter $u_N^\dagger u_N=(E_\kappa^{N^*}/M_N^*)\bar{u}_N u_N$, Eq. (70) can be reasonably described as the Gordon decomposition with scalar and vector interactions included. It explicitly contains the spin, orbital, and vector potential contributions.

The three-vector **V** in Eqs. (69) and (70) is related to the baryon current, see Eq. (11). Although it is exact only for nuclear matter we *assume* its validity here. Thus, the total *nucleon* contribution to the baryon current density can be expressed as

$$\mathbf{j}_N=\sum_N \psi_N^\dagger\boldsymbol{\alpha}\psi_N$$
$$=\sum_N\frac{1}{2(E_N-g_V^NV_0)}[i\psi_N^\dagger(\overleftarrow{\nabla}-\overrightarrow{\nabla})\psi_N+\overrightarrow{\nabla}\times(\psi_N^\dagger\boldsymbol{\Sigma}\psi_N)]$$
$$-\sum_N\frac{g_V^N\mathbf{V}}{E_N-g_V^NV_0}\psi_N^\dagger\psi_N\ . \tag{71}$$

In order to proceed we now assume that the energy denominators can be taken outside of the sums, using an average energy $\bar{E}_N$. The first term vanishes for closed-shell nuclear configurations (with total orbital and spin angular momenta equal to 0). Moreover, in calculating nuclear magnetic moments the orbital contribution would vanish for closed-shell, $L=0$ configurations $[\mu\propto\int d\mathbf{r}\frac{1}{2}\mathbf{r}\times\mathbf{j}(\mathbf{r})]$. Only the hole contribution $\mathbf{j}_h$ is left.

The last, nonvanishing term in Eq. (71) gives

$$\mathbf{j}_N=-g_V^N\mathbf{V}\frac{\rho_N}{\bar{E}_N-g_V^NV_0}-\mathbf{j}_h\ , \tag{72}$$

where ρ_N is the vector density of nucleons [cf. Eq. (15)]. Note that $\mathbf{j}_N$ (and **V**) does not vanish since the spinors ψ_N in Eq. (71) are solutions of the many-body equation (69) for the whole-nucleus model (and not just the spherically closed core).

We can now use Eq. (11) to solve for $\mathbf{j}_N$. Since the contributions of the hole ($\mathbf{j}_h$) and the hyperon ($\mathbf{j}_\Lambda$) are independent in this model, the present argument is best demonstrated by focusing on the hyperon part alone. (The hole part is not affected by the present mechanism and remains virtually unmodified relative to nonrelativistic predictions [5].) Thus, the pertinent contribution is

$$\mathbf{j}_N=-\left[1+\left[\frac{g_V^N}{m_V}\right]^2\frac{\rho_N}{\bar{E}_N-g_V^NV_0}\right]^{-1}\frac{g_V^Ng_V^Y}{m_V^2}\times\frac{\rho_N}{\bar{E}_N-g_V^NV_0}\mathbf{j}_Y\ . \tag{73}$$

This result is similar to that obtained in Ref. [7], since the total baryon current

$$\mathbf{j}_B=\mathbf{j}_N+\mathbf{j}_Y=\left[1+\left[\frac{g_V^N}{m_v}\right]^2\frac{\rho_N}{\bar{E}_N-g_V^NV_0}\right]^{-1}\times\left[1+\frac{g_V^N}{m_V^2}(g_V^N-g_V^N)\frac{\rho_N}{\bar{E}_N-g_V^NV_0}\right]\mathbf{j}_Y \tag{74}$$

[cf. Eq. (67)]. This result gave rise to a relativistic effect when compared with the nonrelativistic extreme single-particle Schmidt theory. The main advantage of the present derivation is that it allows us to account the $\omega\Lambda\Lambda$ tensor coupling (which is not possible in a model of infinite nuclear matter).

The additional tensor coupling, Eqs. (63) and (64), modifies the Λ vector current

$$\psi_\Lambda^\dagger\boldsymbol{\alpha}\psi_\Lambda=\frac{1}{2(E_\Lambda-g_\Lambda^VV_0)}[i\psi_\Lambda^\dagger(\overleftarrow{\nabla}-\overrightarrow{\nabla})\psi_\Lambda+\overrightarrow{\nabla}\times(\psi_\Lambda^\dagger\boldsymbol{\Sigma}\psi_\Lambda)] \tag{75}$$

by adding to it the tensor term

$$\frac{f_{\omega\Lambda\Lambda}}{g_V^\Lambda}\frac{1}{2M_\Lambda}\overrightarrow{\nabla}\times(\psi_\Lambda^\dagger\beta\boldsymbol{\Sigma}\psi_\Lambda)\ . \tag{76}$$

This extra (tensor) part, Eq. (76), is obviously expected to modify our previous results (Table I and Ref. [7]) for the hypernuclear magnetic moments.

For clarity of presentation we will deal with an s-state Λ (no orbital contributions) first, and then with $l>0$ Λ single-particle states. (The s-state Λ is, of course, our main interest here.) Such s states are described in terms of real functions, so the first term on the right-hand side of Eq. (75), namely, the one containing $(\overleftarrow{\nabla}-\overrightarrow{\nabla})$, does not contribute to the Λ baryon current.

Since the Λ is electrically neutral, it contributes to the hypernuclear electromagnetic current only through its anomalous magnetic moment Λ (no orbital contributions)

$$\mathbf{j}_\Lambda^{\rm em}(\mathbf{r})=\frac{\mu_\Lambda}{2M_N}\overrightarrow{\nabla}\times[\psi_\Lambda^\dagger(\mathbf{r})\beta\boldsymbol{\Sigma}\psi_\Lambda(\mathbf{r})]\ . \tag{77}$$

This contribution is very close to the Schimdt value with deviations of the order of $O(f_\Lambda^2/g_\Lambda^2)$. The closed core contribution

$$\mathbf{j}_{\rm core}^{\rm em}(\mathbf{r})=1/2\sum_{\rm core}\psi_N^\dagger\boldsymbol{\alpha}\psi_N$$

is identical in form to the nuclear baryon current in Eqs. (71)–(76).

Our previous results (Table I and Ref. [7]) were obtained when only a pure-vector $\omega\Lambda\Lambda$ coupling was used in the closed core current $\mathbf{j}_{\rm core}^{\rm em}$. With the presence of the tensor term, Eq. (76), the closed core contribution to the magnetic moment is

$$\mu_{\rm core}=-\frac{1}{4}\frac{g_V^N g_V^\Lambda}{m_V^2}\int d^3r\left[1+\left[\frac{g_V^N}{m_v}\right]^2\frac{\rho_N(r)}{\bar{E}_N-g_V^N V_0(r)}\right]^{-1}\frac{\rho_N(r)}{\bar{E}_N-g_V^N V_0(r)}$$
$$\times\mathbf{r}\times\left\{\frac{1}{2[E_\Lambda-g_V^\Lambda V_0(r)]}\{i\psi_\Lambda^\dagger(\mathbf{r})(\overleftarrow{\nabla}-\vec{\nabla})\psi_\Lambda(\mathbf{r})+\vec{\nabla}\times[\psi_\Lambda^\dagger(\mathbf{r})\mathbf{\Sigma}\psi_\Lambda(\mathbf{r})]\}+\frac{f_{\omega\Lambda\Lambda}}{g_V^A}\frac{1}{2M_\Lambda}\vec{\nabla}\times(\psi_\Lambda^\dagger\beta\mathbf{\Sigma}\psi_\Lambda)\right\}. \quad (78)$$

In Eq. (78), it is possible to replace the density ρ_N by the vector potential using $\rho_N=(m_V^2/g_V^N)V_0$.

For an s-wave Λ the term containing $\overleftarrow{\nabla}-\vec{\nabla}$ vanishes. Using the methods of Ref. [35] once more, and neglecting terms $O(v^2/c^2)$ (where v is the typical, average velocity of the Λ in the nucleus), we find

$$\mu_{\rm core}\cong-\frac{1}{4}g_V^\Lambda\int d^3r\left[1+g_V^N\frac{V_0(r)}{\bar{E}_N-g_V^N V_0(r)}\right]^{-1}\frac{V_0(r)}{\bar{E}_N-g_V^N V_0(r)}\mathbf{r}\times\left\{\frac{1}{2E_\kappa^{\Lambda *}}\vec{\nabla}\times[\psi_\Lambda^\dagger(\mathbf{r})\mathbf{\Sigma}\psi_\Lambda(\mathbf{r})]\right.$$
$$\left.+\frac{f_{\omega\Lambda\Lambda}}{g_V^\Lambda}\frac{1}{2M_\Lambda}\vec{\nabla}\times\frac{M_\Lambda^*}{E_\Lambda^*}[\psi_\Lambda^\dagger(\mathbf{r})\mathbf{\Sigma}\psi_\Lambda(\mathbf{r})]\right\}. \quad (79)$$

Using Eq. (63), and noting that the derivatives ($\vec{\nabla}\times$ operator) limit the values of $\mathbf{r}$ to the nuclear surface where differences between M_Λ and M_Λ^* are small, we find next that

$$\mu_{\rm core}\cong 0 .$$

This interesting result means that for a Λ hyperon in the $1s$ state, and in the presence of $\omega\Lambda\Lambda$ tensor coupling, the core contribution to the hypernuclear magnetic moment is nonexistent or small and the expected result is simply the Schmidt single-particle value. This result is in agreement with the numerical calculations of Ref. [22]; here we have provided the physical insight into those numerical results.

For higher orbital angular momentum ($l_\Lambda>0$) states, orbital contributions do arise from the term involving the $(\overleftarrow{\nabla}-\vec{\nabla})$ operator in the hyperon current. Note that this term does not contribute to the Λ electromagnetic current since the Λ is electrically neutral, but it contributes to the Λ baryon current and to the total nuclear electromagnetic current for $l_\Lambda>0$. The pertinent orbital contributions are large and their net contribution to the electromagnetic current is always negative [7,22]. In order to understand further the results of Ref. [22] we point out that the last two terms of Eq. (78) (namely, the anomalous—or tensor—type terms) still mutually cancel for $l_\Lambda>0$. Thus, *the nonvanishing contribution* comes from the first term (involving the $\overleftarrow{\nabla}-\vec{\nabla}$ operator). However, this term is also *the only contributor* to a nuclear matter or a local-density approximation (LDA) calculation such as Ref. [7]: no magnetic-moment-type contributions exist in nuclear matter.

Thus, *for* $l_\Lambda>0$ there is a great similarity between the nuclear matter (or LDA) and the finite-nucleus calculations when the latter includes the effect of the $\omega\Lambda\Lambda$ tensor coupling. We therefore expect the LDA results of Ref. [7] to be in good agreement with $l_\Lambda>0$ finite-nucleus calculations of Ref. [22]. The core contribution is negative in both cases and its magnitude is also similar, with differences at a level which can be fully expected when comparing an LDA with a finite-nucleus calculation.

Using similar steps we have also derived the dynamical electromagnetic matrix element between states differing only in the hyperon orbital. The same techniques that led to Eqs. (70), (71), (75), (76), and (78) yield the total electromagnetic matrix element for the closed nucleon core+Λ:

$$\langle f|\mathbf{j}|i\rangle=\langle f|\mathbf{j}_\Lambda^{\rm em}|i\rangle+\langle f|\mathbf{j}_{\rm core}^{\rm em}|i\rangle$$
$$=\frac{\mu_\Lambda}{2M_N}\vec{\nabla}\times[\psi_{\Lambda_f}^\dagger(\mathbf{r})\beta\mathbf{\Sigma}\psi_{\Lambda_i}(\mathbf{r})]-\frac{1}{2}g_V^\Lambda\frac{V_0(r)}{\bar{E}_N-g_V^N V_0(r)}$$
$$\times\left\{\frac{i}{E_{\Lambda_f}+E_{\Lambda_i}-2g_V^\Lambda V_0(r)}\{\psi_{\Lambda_f}^\dagger(\mathbf{r})(\overleftarrow{\nabla}-\vec{\nabla})\psi_{\Lambda_i}(\mathbf{r})+\vec{\nabla}\times[\psi_{\Lambda_f}^\dagger(\mathbf{r})\mathbf{\Sigma}\psi_{\Lambda_i}(\mathbf{r})]\}\right.$$
$$\left.+\frac{f_{\omega\Lambda\Lambda}}{g_V^\Lambda}\frac{1}{2M_\Lambda}\vec{\nabla}\times[\bar{\psi}_{\Lambda_f}(\mathbf{r})\mathbf{\Sigma}\psi_{\Lambda_i}(\mathbf{r})]\right\}. \quad (80)$$

Based on arguments leading to Eqs. (75), (76), and (78), the contribution to the Λ *baryon* current ($\mathbf{j}_\Lambda$) is very small [i.e., the last two terms in Eq. (25) cancel each other]; we find that our observations for the hypernuclear magnetic moments also hold for the dynamical electromagnetic current matrix element.

The preceding discussion would be valid in the presence of a nucleon hole as well, since the hole and hyperon currents are independent in this model.

We have thus demonstrated in an intuitive, analytical manner that the results of Ref. [22] can be understood from those of Ref. [7] upon adding $\omega\Lambda\Lambda$ tensor coupling. We have considered finite nucleus, bound-state wave functions for the *full* system (nuclear core plus hyperon) and used generalizations for finite nuclei of the Gordon decomposition, analyzing full nuclear currents (including the induced core contribution). The approximate spin independence of the Λ baryon current depends, of course, on the precise numerical value of Eq. (63). This value would differ for other hypernuclei [21]. However, experimental studies of the currents of the more exotic hypernuclei (Σ, Ξ, etc., hypernuclei) are not expected in the foreseeable future.

Another possible application of the present results involves the scalar density, ρ_s. The modification of the closed-core quantity, $\rho_s^{(\text{cc})}$, by the hypernuclear Λ-hole state is seen from Eq. (43) to be

$$\rho_s^{(\text{hypernuclear})}-\rho_s^{(\text{cc})} =\frac{1}{\Omega}\left\{\frac{M_\Lambda^*}{E_p^{\Lambda *}}\left[1+\frac{g_s^\Lambda}{g_s^N}\frac{g_s^{N^2}}{m_s^2}\Pi_s\left[1-\frac{g_s^{N^2}}{m_s^2}\Pi_s\right]^{-1}\right] -\frac{M_N^*}{E_q^{N*}}\left[1-\frac{g_s^{N^2}}{m_s^2}\Pi_s\right]^{-1}\right\} . \tag{81}$$

In order to obtain a numerical estimate we now assume that the Λ hyperon is at $p=0$, while the hole is at $q\cong k_F$. This situation corresponds to a nucleon close to the Fermi surface, which is converted into a Λ at the lowest possible hyperon single-particle state. Equation (81) then gives

$$\rho_s^{(\text{hypernuclear})}-\rho_s^{(\text{cc})}=\frac{1}{\Omega}\left[1-\frac{g_s^{N^2}}{m_s^2}\Pi_s\left[1-\frac{g_s^\Lambda}{g_s^N}\right] -\frac{M_N^*}{E_{k_F}^{N*}}\right]\left[1-\frac{g_s^{N^2}}{m_s^2}\Pi_s\right]^{-1} . \tag{82}$$

The numerical input used is similar to that at the end of Sec. II C 3, following Eq. (43). Since $(g_s^{N^2}/m_s^2)\Pi_s=-0.11$, the core-response correction to $\rho_s^{(\text{cc})}$ is small. This has already been pointed out to be the case for nucleons [following Eq. (43)]; although core-response corrections are somewhat larger for hypernuclei, the changes in ρ_s come mainly from the valence contributions. The overall effect is of order $1/A$ but is physically interesting and theoretically attractive. Unfortunately, the actual magnitude of this effect is too small to be studied experimentally. Indeed, we find

$$\rho_s^{(\text{hypernuclear})}-\rho_s^{(\text{cc})}=\frac{0.16}{\Omega} .$$

The finite Hartree result [2] for ρ_v at the center of a heavy nucleus is about 0.15 fm^{-3}, and this is very close to the nuclear matter result. We shall therefore use $1/\Omega=0.15/A\ \text{fm}^{-3}$ to obtain

$$\rho_s^{(\text{hypernuclear})}-\rho_s^{(\text{cc})}\simeq\frac{-0.024}{A}\ \text{fm}^{-3} .$$

This is evidently a very small effect, even for a relatively low $A\simeq 10$.

Note that for the case of a closed-shell nuclear core plus an extra Λ hyperon (such as ${}^{17}_{\Lambda}\text{O}$ or $\text{A}^{209}_{\Lambda}\text{Pb}$) the effect on ρ_s is much larger,

$$\delta\rho_s=\frac{0.96}{\Omega}\simeq\frac{0.14}{A}\ \text{fm}^{-3} ,$$

because one does not have to subtract the hole contribution. However, even this correction to ρ_s is too small to be measured.

In principle, a potentially useful way of measuring ρ_s is by studying meson scattering. As discussed later, the pion is not a sensitive enough probe for this purpose, but K^+ scattering is a promising candidate when high-precision hadronic measurements are called for. Since K^+-hypernuclear scattering cannot be studied directly at this time, one could resort to K^+ production reactions. For obvious reasons, the cleanest of such reactions would be (γ,K^+) or a reaction involving the strangeness-changing weak-interaction mechanism such as $e^-p\to\Lambda+\nu_e$ [36]. The useful interpretation of such processes will not be free of ambiguities, however [37], and such small effects, although theoretically interesting, cannot be successfully measured at present.

Concluding our discussion of hypernuclei we still believe that the most promising scenario for probing the effect of the strong MFT potentials is via hypernuclear magnetic-moments measurements in a closed-shell plus Λ system [7,22,34]. Such measurements are experimentally difficult, however. Other effects we have looked into prove to be too small to be experimentally detected.

2. *Meson-nucleus (especially K^+) scattering*

In this subsection we deal briefly with meson elastic and inelastic scattering off nuclei, evaluating its role in our study of relativistic nuclear response. Much of the following discussion is true for all mesonic probes currently used in nuclear studies, namely, the pion, K^+, and K^-. However, the K^+ meson holds the promise of providing the cleanest mesonic probe for nuclear physics with very small theoretical uncertainties [38]. Consequently we shall mainly discuss here applications involving K^+ mesons.

The Lorentz-invariant meson-nucleon T matrix, which is a Dirac matrix (dimension 4×4) in the spinor space can be decomposed as in Chew, Goldberger, Low, and Nambu [39]:

$$T=A(s,t)+\tfrac{1}{2}[(K_i+K_f)\gamma]B(s,t) . \tag{83}$$

The functions A and B depend on the Mandelstam variables s and t; K_i and K_f are the initial and final kaon momenta. This form, Eq. (83), is valid for on-shell scattering or partly off-shell where the nucleon is on-mass-shell but the meson is not.

Elastic meson-nucleus scattering can now be studied in the impulse approximation and within the relativistic nuclear MFT. For a closed-shell nucleus (where data exist), only the scalar $[A(s,t)]$ and fourth (timelike) component of the vector part $[\propto \gamma^0 B(s,t)]$ contribute to the elastic transition matrix element. Thus, no large effects of the enhanced small components in the Dirac single-particle spinors are present. We note, however, that the elastic scattering amplitude will depend on both the *vector* $(\langle \psi N^{\dagger}\psi_N \rangle)$ *and scalar* $(\langle \bar{\psi}_N \psi_N \rangle)$ densities in this description. This is at variance with the nonrelativistic calculation, where no difference occurs between ρ_v and ρ_s. Since the latter is smaller by as much as 7% than the former [12], this may prove to be an interesting possibility to explore [40]. Furthermore, Wallace [40] has looked into the vacuum polarization corrections for the scattering amplitude. Such corrections affect differently the scalar and vector densities [41], thereby contributing to further modifications in the transition matrix. The overall effect turned out to be small relative to present discrepancy between theory and experiment [40]. (This is a closed-shell, ground-state relativistic effect and does not involve any core response. We note that in order to get an appreciable effect it would be necessary to find a situation where the scalar-density contribution is dominant. This does not seem to happen for threshold K^+ scattering. Moreover, the K^+-N Lorentz-invariant amplitudes do not have large scalar and vector components with opposite signs such that changing the difference $\rho_v - \rho_s$ has a large effect.) Note that [38] K^+-nucleus elastic scattering calculations are in much better agreement with the experimental data for ^{40}Ca than for ^{12}C, but a systematic A dependence has not been established yet.

The total K^+-nucleus scattering cross section poses an interesting puzzle [38]: the ratio of the total cross section of K^+-carbon to K^+-deuteron scattering shows a discrepancy between theory and experiment, with the calculated cross section roughly 10–15 % below the experimental one. The total cross section is evaluated from the forward scattering amplitude through the optical theorem, thus a similar effect from the difference between the scalar and vector densities could also be of interest. Although not expected to be large, this is certainly an interesting correction to look at.

Returning to the main subject of the present work, we now discuss the core-response corrections in inelastic meson-nucleus scattering. In view of the Lorentz structure of the T matrix [see Eq. (83)], the situation is similar to inelastic electron scattering [42]. We shall be interested in a relatively simple model of one-particle–one-hole excitations of closed-shell nuclei, where the reaction is studied within the impulse approximation (this is an approximate tool for studying medium-energy K^+-nucleus interactions). The term in T involving γ gives rise to scattering amplitudes which are linear in the lower components of the Dirac spinors. Using closed-shell spinor solutions for the description of the p-h states would enhance the cross section for such excitations relative to the equivalent nonrelativistic results (by approximately M_N/M_N^*). An example is [42] the 1^+, $T=0$ excitation in ^{12}C; we can only deal with isoscalar excitations within the present model, as already explained. The 1^+, $T=1$ excitation [43] in ^{12}C is, therefore, not appropriate for studies within the σ-ω model.

In the spirit of our preceding discussion, a consistent relativistic treatment must include the core response as well. Even if we model the excitation as a pure p-h state neglecting conventional particle-hole interactions, the core-response effects cannot be neglected in a relativistic nuclear-model calculation. Thus, the core response shall significantly affect the enhanced isoscalar p-h transition currents, just as it suppresses the nuclear ground-state p-h convection currents in this framework (in the spirit of Sec. II B). This calculation is *not* identical with those presented in Sec. II, since it involves transition matrix elements rather than the "diagonal" bulk nuclear properties and nonzero momentum transfers, $Q\neq 0$. However, the pertinent core response is a slowly varying function of four-momentum for low-lying excitations, so the underlying physics and much of the current ideas can be directly applied. A detailed specific application is the subject of a different work.

3. π^0 and K^+ photoproduction; πNN and $KN\Lambda$ nuclear vertices

In this subsection we introduce the motivation for and discuss relativistic MFT studies of meson photoproduction reactions. We start with a discussion of threshold (γ,π^0) and then turn to (γ,K^+). The effect of the nuclear many-body medium on the πNN and $KN\Lambda$ vertices is also discussed, and the relevance of the relativistic core response is examined.

Dealing with pion photoproduction we shall only be interested in the threshold region, in order to avoid the necessity of dealing with the Δ-isobar contributions (which are dominant away from threshold but are not yet included in the relativistic σ-ω model). Furthermore, charged-pion photoproduction processes are purely isovector and, consequently (as already discussed), are not well understood in the pertinent model. We shall thus be interested in neutral pion photoproduction where the π^0 kinetic energy is of the order of 10 MeV.

Under these conditions the free reaction is described (in an effective Lagrangian approach) [44] by tree-level diagrams with exchanges of a nucleon in the s and u channels, and an ω meson in the t channel. The nucleon-exchange diagrams provide equal isovector and isoscalar contributions to the transition operator. The π^0 production cross section is much smaller (by about 2 orders of magnitude at threshold) than the corresponding $\pi^{\pm}$ cross section since the dominant Kroll-Ruderman (catastrophic) term is missing in the former case. This is also the reason one expects the (γ,π^0) reaction to be sensitive to the scalar mean field (or M_N^*).

Of special interest to researchers working on this topic is coherent photoproduction [45,46]. In contrast with charged pions, neutral pions can be produced coherently from nuclei on neutrons as well as protons while the target nucleus remains in its ground state. Coherent neutral pion photoproduction occurs over the whole nuclear volume. In the impulse approximation the amplitude for the process is the sum of the elementary amplitudes for π^0 production on single nucleons. Thus, this is a "bulk" nuclear process. As a result of the small (γ,π^0) cross section as well as difficulties in detecting in coincidence the photons following the π^0 decay, only a small amount of work has been done since the pioneering studies of Schrack, Leiss, and Penner [45]. This situation is changing rapidly, however [45].

Coherent photoproduction of neutral pions provides, therefore, a considerable enhancement of the cross section for the ground-state transition. Unfortunately, the process is not appropriate for a study of relativistic nuclear currents and core-response effects: while all nucleons can contribute coherently in spin-independent production, the spin-flip contributions are nonexistent for elastic coherent production from closed-shell nuclei, or at most only a few nucleons can contribute coherently in spin-flip production from closed-shell nuclei. Thus, the spin-independent term of the production operator will be the major contributor for all nuclei, and the only large contribution for closed-shell nuclei, at least in the impulse approximation.

What is required for the present work, then, are p-h excitation cross sections. Noncoherent photoproduction has been measured by Arends *et al.* [45], who provide incoherent and total cross sections in addition to the coherent one. However, the incoherent (i.e., the total minus coherent) cross section is extremely small at threshold for the reason discussed above, and more sensitive measurements will be required for a meaningful comparison between theory and experiment.

Suzuko and Koch [47] have shown that the elementary single-particle (γ,π^0) operator at threshold (the spin-flip term) is indeed enhanced by a factor of M_N/M_N^* when embedded in the nucleus. These authors discuss the ground-state (g.s.) to g.s. transition $^{27}\mathrm{Al}(\gamma,\pi^0)^{27}\mathrm{Al}$. Relativistic nuclear core-response calculations, similar to those of isoscalar magnetic moments for closed core+1 systems, are indeed immediately applicable for g.s. to g.s. (γ,π^0) calculations on such nuclei. One should then compare the calculations with averaged experimental-phenomenological amplitudes for pairs or mirror nuclei (namely, a closed core plus one proton or one neutron), as was the case for isoscalar magnetic moments [5]. Fortunately, in practice the threshold (γ,π^0) reaction cross section on the extra neutron is expected to be some 2 orders of magnitude smaller than the pertinent extra-proton quantity [48], and the closed-core+proton cross section should suffice. Unfortunately, no threshold experimental data exist. It is clear, however, that the pertinent cross sections are very small; the coherent production (which is not sensitive to the relativistic model and core-response effects) will be the dominant mechanism, and the experiments will be very difficult. Note that, since the π^0 photoproduction amplitude is so small near threshold, multistep processes are expected to compete strongly with the impulse, direct π^0 production from a single nucleon target. Koch and Woloshyn [44], and later Bosted and Laget [49], have shown that the two-nucleon rescattering mechanism is as important as the direct π^0 production process. For a meaningful comparison between theory and experiment it would be necessary to include all such processes in the relativistic-model calculations. Furthermore, it is worthy of note that the elementary $p(\gamma,\pi^0)p$ reaction at threshold is not completely understood, as recent experiments as Saclay and Mainz have shown [50].

As in other processes involving pions and nucleons, it is possible to formulate a theory based on either a pseudoscalar (PS) or a pseudovector (PV) πNN coupling. Large differences occur between PS and PV πNN vertices in the nucleus as a result of the large scalar mean field [51]. Denoting the pion wave function by $\phi(\mathbf{x})$, and the nucleon wave functions by $\psi_{N_f}(\mathbf{x})$ and $\psi_{N_i}(\mathbf{x})$ for the final and initial states, respectively, the matrix element of the πNN vertex operator (with an outgoing pion of momentum t) is

$$H_{fi}=\int \psi^\dagger_{N_f}(\mathbf{x})\Gamma_{\pi NN}\phi(\mathbf{x})\psi_{N_i}(\mathbf{x})d^3x \ . \tag{84}$$

In the PS case $\Gamma_{\pi NN}\equiv\Gamma^{\mathrm{PS}}_{\pi NN}=g\gamma^0\gamma_5$, while in the PV case $\Gamma_{\pi NN}\equiv\Gamma^{\mathrm{PV}}_{\pi NN}=(g/2M_N)\gamma^0\gamma_5\not{t}$, where g is the πNN coupling strength. The relativistic nuclear many-body equivalence breaking term is a result of the strong scalar potential $(S=-g_s^N\phi_0)$ [51]:

$$H^{\mathrm{PV}}_{fi}=H^{\mathrm{PS}}_{fi}+g\int \psi^\dagger_{N_f}(\mathbf{x})\gamma^0\gamma_5\frac{S}{M_N}\phi(\mathbf{x})\psi_{N_i}(\mathbf{x})d^3x \ . \tag{85}$$

The extra piece is a "seagull" diagram [51,14].

According to Suzuki and Koch [47], the matrix element of the PS photoproduction operator evaluated between Hartree closed-shell single-particle spinors is enhanced by an extra factor of M_N/M_N^* over and above the corresponding enhancement in the PV case. While the relativistic core response is expected to cancel the M_N/M_N^* enhancement associated with the nuclear electromagnetic current [cf. Eq. (67)], it is unlikely to affect the difference between the PS and PV results. The detailed evaluation of the core-response effect for the nuclear (γ,π^0) reaction is an interesting issue which should be addressed in the future.

Of great interest are relativistic studies of kaon photoproduction [(γ,K^+), resulting in hypernuclear final states]. This subject has been studied [14,52] and reviewed [36] recently, and we shall mainly concentrate here on issues directly related to relativistic core response and hypernuclear physics, as well as important points ignored in the published literature.

Virtually all relativistic-model studies for kaon photoproduction have been performed within the impulse approximation, and the core response to the hypernuclear Λ-h excitations has not been evaluated. It is expected that including this mechanism should significantly affect the reported results. This study again requires the evaluation of the core-response effect on a nondiagonal, transition matrix element [see Eq. (80)], and will also be the subject of future research (as already discussed above). There are, however, interesting relativistic many-body effects that only arise in the hypernuclear case. Thus, in the kaon photoproduction case the final spinor is that of the Λ, and S/M_N in Eq. (85) is replaced by the ratio of the sums $-(g_s^N+g_s^\Lambda)\phi_0/(M_N+M_\Lambda)$. Moreover, in addition to the *scalar*-potential equivalence-breaking term we also find another term, proportional to the difference of the nucleon and hyperon *vector* potentials, $(g_v^N-g_v^\Lambda)V_0$. This term vanishes for $g_v^N=g_v^\Lambda$ which is the case analyzed by Friar [51], but in dealing with strange particles in nuclei it introduces new many-body effects these effects are large since $(g_v^N-g_v^\Lambda)V_0\simeq 0.25M_N$; cf. Eq. (53). Relativistic core-response corrections to this picture remain to be studied.

4. *Photonuclear knockout and capture reactions: (γ,p), $(e,e'p)$, and (p,γ)*

A satisfactory theoretical description of the (γ,N), $(e,e'N)$, and (p,γ) reactions involve a large number of nuclear processes and is currently not thoroughly understood. Theoretical and experimental aspects of the $(e,e'p)$ reaction have been recently reviewed by Frullani and Mougey [42]. Here we shall naturally mention primarily issues related to our main interest in the present work, namely, relativistic studies and core response.

The physical quantity of central interest in a relativistic calculation of electron-nucleus scattering is the matrix element of the nuclear electromagnetic current operator. The transition amplitude for the (γ,p) reaction can be written as

$$M_{fi}^{(\lambda)}z=\int\langle\psi_f\gamma^0|J_\mu(\mathbf{x})|\psi_i\rangle A_\lambda^\mu(\mathbf{x})d^3x \ , \tag{86}$$

where $|\psi_i\rangle$ is the initial target wave function (for A nucleons), $|\psi_f\rangle$ is the final-state wave function, $A_\lambda{}^\mu$ represents the photon with helicity λ, and J^μ is the nuclear effective electromagnetic current operator. In the one-photon exchange model for the $(e,e'p)$ reaction, $A_\lambda{}^\mu$ is replaced by a product of the lepton current and photon propagator [53]; equivalently, the Møller potential can be used for $A_\lambda{}^\mu$ [54].

In the impulse approximation (which may be used as a starting point), the nuclear current is a one-body operator given as a sum of one-nucleon effective electromagnetic currents. The on-shell nucleon current operator (sometimes called the Dirac-plus-Pauli form) is [cf. Eq. (67)]

$$\langle J^\mu\rangle=\bar{U}_N\left[\frac{e}{2}(F_1^s+\tau_3F_1^v)\gamma^\mu + \frac{i}{4M_N}(F_2^s+\tau_3F_2^v)\sigma^{\mu\nu}Q_\nu\right]U_N \ , \tag{87}$$

where F_1 and F_2 are the Dirac and Pauli form factors normalized such that at a vanishing four-momentum transfer $Q^2=0$, $F_1^s(0)=F_1^v(0)=1$, and $\kappa=\frac{1}{2}[F_2^s(0)+\tau_3F_2^v(0)]$ is the nucleon anomalous magnetic moment. In an impulse-approximation model of the (γ,N) reaction the photon (γ) transfers nearly all its energy to the nucleon (N), so the magnitude of the nuclear momentum transfer is large.

The final-state wave function can be approximately written as a product of the residual nucleus (with $A-1$ bound nucleons) and a relative proton-residual nucleus continuum wave function using a single-channel optical-model wave function. In an independent-particle model, the residual nucleus can be described in terms of a one-hole state relative to the initial nuclear state. It has been emphasized by Gari and Hebach [55] and by Noble [56], and more recently discussed by Lourie [57], that the single-particle knockout mechanism with a transition from a bound single-particle state to a continuum state may be inadequate for a description of the (γ,N) reaction. Usually, orthogonality of the initial- and final-state wave functions is not taken care of in impulse approximation treatments. Meson exchange currents and nucleon-nucleon correlations, which are relatively insignificant for differential cross sections, provide the dominant contributions to the total cross sections. Moreover, the impulse approximation violates nuclear many-body current conservation (gauge invariance) [55,58]. However, such considerations are outside the scope of our present treatment. (At the time of this writing we are unaware of any *relativistic* treatment addressing all these problems consistently. Such a treatment should involve, however, isovector mechanisms which are outside the scope of the σ-ω MFT framework.) For the purpose of a preliminary discussion of the relativistic core response one can follow, as a first step, the simpler (perhaps inadequate) procedure of Refs. [58–61].

Taking a closed-shell nucleus for the initial state, the final state has a hole in the closed core plus one particle in the continuum. In a relativistic Hartree calculation, the reaction is therefore sensitive to core-response effects. These remain to be evaluated in a future study along the lines of the p-h formalism in Sec. II, but appropriately modified for a treatment of the transition (nondiagonal) matrix element at $Q\neq 0$, as already emphasized above for other applications. Of interest are (γ,n) data [62]. There the Dirac current in Eq. (87) vanishes while the magnitudes and shapes in existing (γ,p) and (γ,n) cross sections are similar. [This may be an indication that a one-body mechanism does not adequately describe the (γ,N) reaction.]

Likewise, relativistic core response is likely to play an important role in the proton radiative capture reaction (p,γ) [63]. In a single-particle, direct-reaction model [63] an incident nucleon of momentum $\mathbf{p}$ interacts with a target nucleus taken as a closed core -1 in its ground state. The nucleon emits a photon and is captured into a single-particle state. The transition amplitude for the reaction is (in the notation of Ref. [63]) similar to Eq. (86), where now ψ_i is the initial wave function (composed of the incident proton in a continuum scattering state and the target nucleus in its ground state) and ψ_f is the final-state nuclear wave function (with the proton in a bound single-particle state).

Starting with a closed-shell -1 nucleus, transitions leading to final highly excited nuclear states or to the ground state of a closed-shell system are possible. The former are not satisfactorily understood, while the latter are accounted for reasonably well, in a direct reaction model [63]. In the impulse approximation one needs to evaluate the transition amplitude between an initial scattering state for the relative proton-nucleus wave function and a final single-particle shell model state. In a relativistic Hartree calculation, this reaction is therefore also sensitive to core-response effects (evaluated along the lines of the p-h formalism in Sec. II), appropriately modified (as already emphasized) for a treatment of the transition (nondiagonal) matrix element at $Q \neq 0$.

5. *Other nuclear processes*

A consistent relativistic study of the inelastic scattering of hadrons and leptons from nuclei also requires core-response corrections. As in Sec. III B 2 this calculation is not identical with those presented in Sec. II, since it involves transition matrix elements rather than the diagonal one, as well as nonzero momentum transfers $Q \neq 0$. However, the underlying physics is similar.

Inelastic proton-nucleus scattering (as well as elastic scattering on open-shell nuclei) relativistic Dirac formalism would require that the core-response corrections be included in order to be self-consistent. Such corrections are likely to arise mostly from terms such as $F_v \gamma(1) \cdot \gamma(2)$ in the scattering amplitude. Examples of calculations where such effects should be included and are expected to be important are Shepard, Rost, and Piekarewicz [64] [distorted wave impulse approximation (DWIA) Dirac formalism for inelastic nucleon-nucleus scattering], Rost and Shepard [64] (DWIA Dirac calculations of inelastic nucleon-nucleus scattering), and Piekarewicz, Amado, and Sparrow [64] [inelastic proton-nucleus scattering plane wave impulse approximation (PWDIA) Dirac formalism].

The situation is similar for inelastic electron scattering. Shepard *et al.* [42] found, in an early calculation, large relativistic effects resulting from the enhanced lower component of the bound Dirac nucleon wave function. These calculations [42] did not include core-response corrections, which are expected to be important since the origin of the large reported relativistic effects is similar to those discussed in this work for magnetic moments (and for elastic form factors).

Another case where core-response corrections are expected to be important is the electric dipole sum rule [65]. Interest in this quantity has been mostly due to the large discrepancy (by as much as a factor of 2) between the experimental value for the total integrated photoabsorption cross section [66], and the classical nonrelativistic TRK (Thomas-Reiche-Kuhn) sum rule [65]. While this enhancement has been explained in classical nonrelativistic nuclear physics as an effect of the tensor component of the nucleon-nucleon interaction and induced many-body correlations [67], Price and Walker [68] calculated the energy weighted electric dipole sum rule using the relativistic Dirac σ-ω mean field theory discussed in this work, for finite nuclei. They obtained enhancements over the nonrelativistic TRK result which were nearly large enough to bring the sum rule into agreement with experiment for a wide range of nuclear mass numbers. The enhancement was virtually entirely due to the inclusion of the large scalar mean field, namely, an effective mass (M_N^*) effect, as the sum-rule results are sensitive to the nuclear interior (where M_N^* is much smaller than M_N). These calculations [68] did not include core-response corrections, expected to be important in this case (where the basic physical process is the photoabsorption cross section).

Finally, we briefly discuss weak-interaction nuclear probes. Interesting studies exist in the literature [69], looking into the role of relativity when the nuclear system is studied by means of probes coupled to the (effective) charge-changing weak nuclear current (mediated by the charged vector meson W^+). Such currents have a vector–axial-vector ($V-A$) structure. The vector part (denoted by $J_\mu^{(\pm)}$) is similar in structure to the electromagnetic or baryon currents studied above. Of interest is the (effective) weak charge-changing axial-vector current ($J_{\mu 5}^{(\pm)}$). These currents affect semileptonic weak processes such as β decay (based on the $V-A$ model), muon capture (the free process would be $\mu^- p \to n \nu_\mu$, with a charge-changing weak current), radiative muon capture (where the free reaction is $\mu^- p \to n \nu_\mu \gamma$), or neutrino-induced reactions (e.g., inverse β decay) [69]. In particular, the considerable difference between the closed-core relativistic predictions based on the induced pseudovector and pseudoscalar axial-vector current is an interesting subject; of related interest is pion absorption and the difference between the pseudoscalar and pseudovector πNN vertex [Eqs. (84) and (85)]. These currents are, however, purely *isovector,* and therefore no large core-response corrections will be present; only a small core response mediated by the isovector ρ meson is found for an isovector baryon current in the extended MFT where this meson is added [5]. Furthermore, isovector currents are presently not well understood, as already discussed above, within the σ-ω model. Consequently, charge-changing weak nuclear processes are currently not a suitable probe of relativistic dynamics in general and core-response effects in particular. For a meaningful relativistic MFT study within the σ-ω model, it will therefore be necessary to identify reactions involving weak neutral currents (mediated by the neutral vector boson Z^0). The weak neutral current is given in the standard model by

$$J_\mu^{(0)} = J_\mu^{v_3} + J_{\mu_5}^{v_3} - 2\sin^2\theta_W \, J_\mu^{(\mathrm{em})} \ ;$$

$J_\mu^{(\mathrm{em})} = J_\mu^s + J_\mu^{v_3}$ is the electromagnetic current [Eq. (87)], which has isoscalar (s) and isovector (v_3) ($\alpha\tau_3$) components, and $\sin^2\theta_W$ of the Weinberg angle is a parameter determining the mixing of the electromagnetic and weak currents within the standard model. The ($J_\mu^{v_3} + J_{\mu_5}^{v_3}$) part is the third isovector component of the $V-A$ charge-changing weak nuclear current. Only the last term of $J_\mu^{(0)}$ will contribute for isoscalar transitions. Isoscalar weak neutral currents will therefore have similar nuclear characteristics to those of their electromagnetic counterparts, in particular with regard to the core response (which is our main interest here). Suitable reactions may be neutrino scattering, or scattering, with a parity-violation signal, or longitudinally polarized electrons off nuclei, resulting from the interference term between the weak neutral and electromagnetic currents. (The transition amplitude is first order in both the electromagnetic fine-structure constant α and the weak Fermi coupling constant G.) Elastic, inelastic, and inclusive isoscalar processes may be studied [70]. Unfortunately, weak neutral current processes have not been extensively studied in the literature mainly as a result of considerable experimental difficulties. Furthermore, other problems, such as the parity-violating nuclear interaction—resulting in an admixture of opposite-parity nuclear states, or parity-violating corrections to the nuclear electromagnetic current operators, may yet prove to be important. In such a case, a meaningful theoretical interpretation of future results would not be an easy task.

In conclusion, we have presented here a theoretical analysis of p-h states in a Dirac relativistic σ-ω mean field theory for nuclear matter, with a special emphasis on hypernuclear states. Applications to a number of other physical process of interest in nuclear and medium-energy physics have been discussed, however such applications should comprise the subject matter of separate studies. (Likewise an appropriate treatment of other many-body effects traditionally included in most nonrelativistic calculations remains an important open problem for the Dirac based theory.)

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[9] R. Chrien *et al.*, Nucl. Phys. **A478**, 705c (1988), have convincingly demonstrated this fact using the (π^+,K^+) reaction on a heavy nucleus (^{89}Y). Hypernuclei have been proposed as a clean means of distinguishing between hadronic and quark degrees of freedom in the many-body system. However, we are aware of any data that unequivocally demand a quark-level description of nuclear structure; and certainly the present state of knowledge precludes extreme possibilities such as total quark deconfinement.

[10] It is a useful check, however, to consider the first-order correction to the single-particle energy $\epsilon_{B'}^{(+)}$ of a nucleon in a given state $|\lambda,\mathbf{k}\rangle_N$ or a Λ in the state $|\lambda,\mathbf{p}\rangle_\Lambda$. For a nucleon in the case of a N-h state

$$\delta\epsilon_N^{(+)}(\mathbf{k}) = {}_N\langle\lambda,\mathbf{k}|\delta h(B'=B=N)|\lambda\mathbf{k}\rangle_N$$

$$= -\frac{1}{\Omega}\left[\frac{g_s^{N2}}{m_s^2}\left[\frac{M_N^*}{E_p^{N*}} - \frac{M_N^*}{E_q^{N*}}\right]\frac{M_N^*}{E_k^{N*}} + \frac{g_v^{N2}}{m_v^2}\left[\frac{\mathbf{p}}{E_p^{N*}} - \frac{\mathbf{q}}{E_q^{N*}}\right]\cdot\frac{\mathbf{k}}{E_k^{N*}}\right]$$

and for the Λ in the case of a Λ-h excitation

$$\delta\epsilon_\Lambda^{(+)}(\mathbf{p}) = {}_\Lambda\langle\lambda,\mathbf{p}|\delta h(B'=B=\Lambda)|\lambda\mathbf{p}\rangle_\Lambda$$

$$= -\frac{1}{\Omega}\left[\frac{g_s^\Lambda}{m_s^2}\left[g_s^\Lambda\frac{M_\Lambda^*}{E_p^{\Lambda*}} - g_s^N\frac{M_N^*}{E_q^{\Lambda*}}\right]\frac{M_\Lambda^*}{E_p^{\Lambda*}} - \frac{g_v^\Lambda}{m_v^2}(g_v^\Lambda - g_v^N) + \frac{g_v^\Lambda}{m_v^2}\left[g_v^\Lambda\frac{\mathbf{p}}{E_p^{\Lambda*}} - g_v^N\frac{\mathbf{q}}{E_q^{N*}}\right]\cdot\frac{\mathbf{p}}{E_p^\Lambda}\right].$$

These corrections are in agreement with the first-order (in the fields) expansion of

$$\epsilon_{B'}^{(+)}(\mathbf{t}) = g_v^{B'}V_0 + [(\mathbf{t} - g_v^{B'}\mathbf{V})^2 + (M_B - g_s^{B'}\phi_0)^2]^{1/2}.$$

We note that in general $\delta\epsilon_{B'}^{(+)}(\mathbf{p})$ does not vanish even if the Λ occupies the lowest energy level, $\mathbf{p}=0$ (which is possible since the Λ is not Pauli blocked [9]). The correction $\delta\epsilon_{B'}^{(+)(}(\mathbf{t})$ is of order $1/\Omega$ [or $O(1/A)$ in a finite nucleus], and vanishes for $\Omega\rightarrow\infty$. In attempting a relativistic description of hypernuclei (e.g., along the lines of Ref. [7]) it is therefore appropriate to ignore such $O(1/A)$ corrections for the Λ single-particle energies, although the Λ hyperon *always* induces a core response. Single-particle energies are one example of intensive quantities which are not modified by the core response and are correctly described by the closed-core Hartree solutions. Other such cases are discussed in the main text.

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$$\rho_v = \frac{1}{\Omega}\sum_{\mathbf{k},\lambda}^{k_F} \bar{u}_N(\lambda,\mathbf{k})\gamma^0 u_N(\lambda,\mathbf{k}) = \frac{\eta}{(2\pi^3)}\int_0^{k_F} d^3k \ ,$$

while the scalar density is

$$\rho_s = \frac{1}{\Omega}\sum_{\mathbf{k},\lambda}^{k_F} \bar{u}_N(\lambda,\mathbf{k}) u_N(\lambda,\mathbf{k})$$

$$= \frac{\eta}{(2\pi)^3}\int_0^{k_F} \frac{M_N^*}{E_k^{N^*}} d^3k \ .$$

The scalar density involves $\bar{u}_N u_N$ and is smaller than the vector density due to Lorentz contraction.

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