Dynamic response function and large-amplitude dissipative collective motion

Xizhen Wu,[1] Fumihiko Sakata,[2] Yizhong Zhuo,[1,3] and Zhuxia Li[1]
[1]*China Institute of Atomic Energy, P.O. Box 275(18), Beijing, People's Republic of China*
[2]*Institute for Nuclear Study, University of Tokyo, Tokyo 188, Japan*
[3]*Institute of Theoretical Physics, Academia Sinica, Beijing, People's Republic of China*
(Received 23 October 1992; revised manuscript received 17 May 1993)

Aiming at exploring microscopic dynamics responsible for the dissipative large-amplitude collective motion, the dynamic response and correlation functions are introduced within the general theory of nuclear coupled-master equations. The theory is based on the microscopic theory of nuclear collective dynamics which has been developed within the time-dependent Hartree-Fock (TDHF) theory for disclosing the complex structure of the TDHF manifold. A systematic numerical method for calculating the dynamic response and correlation functions is proposed. By performing numerical calculation for a simple model Hamiltonian, it is pointed out that the dynamic response function gives important information in understanding the large-amplitude dissipative collective motion which is described by an ensemble of trajectories within the TDHF manifold.

PACS number(s): 21.60.Ev, 24.10.Cn, 24.60.Lz

I. INTRODUCTION

The large-amplitude nuclear collective motion, such as the low-energy fission process and deep inelastic heavy-ion collision, have been quite successfully described phenomenologically with Fokker-Planck or Langevin type equations [1–4]. In these descriptions, one usually introduces some assumption; the reaction process is assumed to be described by a few shape parameters put in by hand, and some kind of statistical hypothesis like an existence of thermal bath, random matrix, etc., are employed.

The basic theoretical problems underlying this kind of research are twofold. How does the self-sustained and self-organizing system, the nuclei, determine its evolution path? And, why does the nucleus governed by the deterministic equation exhibit some probabilistic and stochastic aspects? Since we are interested in the dissipative large-amplitude collective motion whose characteristic energy per nucleon is much smaller than the Fermi energy, the above basic problems ought to be studied within the time-dependent Hartree-Fock (TDHF) theory. Concerning the first problem, it has been studied under the adiabatic assumption [5] how to dynamically extract the collective path described by a few relevant degrees of freedom. Recently, a general method capable of extracting the collective submanifold out of the TDHF manifold, and capable of disclosing an exceedingly rich structure of the TDHF manifold far from the stable mean field has been developed [6] without introducing any adiabatic assumption. Based on recent enormous progress in the general theory of nonlinear dynamical system [7], the first basic problem is achieved by defining the most natural coordinate system for a given collective trajectory whose characteristic property in the small-amplitude region near a certain stable mean field is only known in advance. According to the self-consistent collective coordinate (SCC) method [8], it is always possible to introduce an optimal coordinate system called the dynamical canonical coordinate (DCC) system for a given trajectory, where the whole system is optimally divided into the relevant (collective) and irrelevant (intrinsic) degrees of freedom. This separation in the degrees of freedom is very important not only for introducing some coarse graining procedure [9] to the irrelevant degrees of freedom, but also for exploring the nonlinear dynamics between the collective and single-particle modes of motion.

To investigate the second basic problem, one has to study the dynamical evolution displayed by an ensemble of TDHF trajectories. Here it should be noticed that an understanding on the time evolution of an ensemble of trajectories is not necessarily obtained from the topological structure of the TDHF manifold, which is studied in terms of many individual trajectories with different initial conditions. Namely, an appearance of the chaotic sea in the TDHF manifold does not always justify an introduction of some statistical treatments. Aiming at clarifying what is happening during the time evolution of an ensemble of trajectories, especially at clarifying the validity of statistical ansatz employed in the conventional nuclear transport theories, a general theory of nuclear coupled-master equation for the relevant and irrelevant partial distribution functions was proposed [10], by exploiting the full knowledge obtained by the first basic problem. By performing the numerical calculations of the coupled-master equations for a simple soluble model [11], it was pointed out that the dynamical fluctuation of the coupling between relevant and irrelevant degrees of freedom plays an important role for the diffusive property of the distribution function, and for a dissipation of the large-amplitude collective motion. At this stage, there arises a more fundamental question: How do the relevant and irrelevant subsystems interact and respond with each other through the dynamical fluctuations?

In order to carefully examine the above question, in the present paper, a new approach of using a *dynamic response function* is proposed. The linear response func-

 ©1993 The American Physical Society

tion describes various behaviors of the system which slightly deviates from a state of equilibrium under the influence of external force [12], and has been applied to the nuclear dissipative motion [9,13]. However, it is less known (even not known) how to define and evaluate the response function in a region far from the state of equilibrium and how it behaves. Starting with the general coupled-master equation defined in the DCC system, we introduce an *instantaneous* dynamical response and correlation functions without introducing any statistical hypothesis. In Sec. II, the general definition of dynamic response and correlation functions is presented. In Sec. III, a systematic method for calculating the dynamical response function is proposed. In Sec. IV, a feasibility of the theory is shown by applying it to a simple soluble model Hamiltonian.

II. DEFINITION OF DYNAMIC RESPONSE FUNCTION

A. Coupled-master equation

Aiming at introducing various notations used in this paper, we briefly recapitulate the theory of nuclear coupled-master equation [10]. We start with the TDHF theory, because we are interested in the low-energy dissipative large-amplitude collective motion. The basic equation of the TDHF theory is known to be formally equivalent to the classical canonical equations of motion with K degrees of freedom [14],

$$i\dot{C}_j=\frac{\partial H}{\partial C_j^*}\ ,\ i\dot{C}_j^*=-\frac{\partial H}{\partial C_j};\ \ j=1,\ldots,K \qquad \left[C_j=\frac{1}{\sqrt{2}}(q_j-ip_j)\right]\ , \tag{2.1}$$

where K stands for a total number of particle-hole pairs. With the aid of distribution function $\rho(t)$, a time evolution of ensemble of TDHF trajectories is described by the Liouville equation

$$\dot{\rho}(t)=-i\mathcal{L}\rho(t)\ , \tag{2.2}$$

where $\mathcal{L}$ denotes the TDHF Liouville operator defined by

$$\mathcal{L}\equiv-i\sum_j\left[\frac{\partial H}{\partial C_j}\frac{\partial}{\partial C_j^*}-\frac{\partial H}{\partial C_j^*}\frac{\partial}{\partial C_j}\right]\ . \tag{2.3}$$

In order to understand the microscopic dynamics displayed by the distribution function $\rho(t)$ in Eq. (2.2), it is decisive to introduce the most natural coordinate system for a given collective motion. With the aid of the SCC method, it is possible to introduce the DCC system where the whole system is divided into the small number of optimal relevant $(\eta_a,\eta_a^*;a=1,\ldots,L\ll K)$ and the large number of irrelevant $(\xi_\alpha,\xi_\alpha^*,\alpha=1,\ldots,K-L)$ degrees of freedom for a given collective motion. A detailed discussion on choosing the most natural coordinate system is found elsewhere [6,8]. The DCC system is defined in such a dynamical way that the collective submanifold described by the relevant variables (η_a,η_a^*) alone satisfies a stationary condition with respect to small variations toward irrelevant variables. In the DCC system, the Liouville equation is expressed as

$$\begin{aligned}\dot{\rho}(\eta,\eta^*;\xi,\xi^*:t)&=-i\mathcal{L}\rho(\eta,\eta^*;\xi,\xi^*:t)\\&\equiv\{H,\rho(\eta,\eta^*;\xi,\xi^*:t)\}_{\rm PB}\ ,\end{aligned} \tag{2.4}$$

where the symbol $\{A,B\}_{\rm PB}$ stands for a Poisson bracket

$$\begin{aligned}\{A,B\}_{\rm PB}\equiv&-i\sum_a\left[\frac{\partial A}{\partial\eta_a}\frac{\partial B}{\partial\eta_a^*}-\frac{\partial A}{\partial\eta_a^*}\frac{\partial B}{\partial\eta_a}\right]\\&-i\sum_\alpha\left[\frac{\partial A}{\partial\xi_\alpha}\frac{\partial B}{\partial\xi_\alpha^*}-\frac{\partial A}{\partial\xi_\alpha^*}\frac{\partial B}{\partial\xi_\alpha}\right]\ .\end{aligned} \tag{2.5}$$

The Hamiltonian in the DCC system is divided into three parts; H_η depends on the relevant, H_ξ on the irrelevant, and $H_{\rm coupl}$ on both the relevant and irrelevant variables, i.e.,

$$H=H_\eta+H_\xi+H_{\rm coupl}\ . \tag{2.6}$$

The dynamical relation between the relevant and irrelevant degrees of freedom during the time evolution of an ensemble of trajectories is studied by introducing a pair of reduced distribution functions

$$\begin{aligned}&\rho_\eta(\eta,\eta^*:t)\equiv{\rm Tr}_\xi\rho(\eta,\eta^*;\xi,\xi^*:t)\ ,\\&\rho_\xi(\xi,\xi^*:t)\equiv{\rm Tr}_\eta\rho(\eta,\eta^*;\xi,\xi^*:t)\ ,\end{aligned} \tag{2.7}$$

when the total distribution function is properly normalized,

$${\rm Tr}\rho(\eta,\eta^*;\xi,\xi^*:t)=1\ , \tag{2.8}$$

with

$$\begin{aligned}&{\rm Tr}\equiv{\rm Tr}_\eta{\rm Tr}_\xi\ ,\ {\rm Tr}_\eta\equiv\prod_{a=1}^{L}\int\int d\eta_a d\eta_a^*\ ,\\&{\rm Tr}_\xi\equiv\prod_{\alpha=1}^{K-L}\int\int d\xi_\alpha d\xi_\alpha^*\ .\end{aligned} \tag{2.9}$$

Using the partial distribution functions in Eq. (2.7), the coupling term $H_{\rm coupl}$ in Eq. (2.6) is further divided into three parts, the average term, fluctuation term, and constant term as

$$\begin{aligned}&H_{\rm coupl}=H_{\rm aver}(t)+H_\Delta(t)-E_0(t)\ ,\\&H_{\rm aver}(t)=H_\eta(t)+H_\xi(t)\ ,\\&H_\eta(t)={\rm Tr}_\xi H_{\rm coupl}\rho_\xi(t)\ ,\ H_\xi(t)={\rm Tr}_\eta H_{\rm coupl}\rho_\eta(t)\ ,\\&H_\Delta(t)=H_{\rm coupl}-H_{\rm aver}(t)+E_0(t)\ ,\\&E_0(t)={\rm Tr}H_{\rm coupl}\rho_\eta(t)\rho_\xi(t)\ .\end{aligned} \tag{2.10}$$

By exploiting the time-dependent projection operator

$$P(t)\equiv\rho_\eta(t){\rm Tr}_\eta+\rho_\xi(t){\rm Tr}_\xi-\rho_\eta(t)\rho_\xi(t){\rm Tr}_\eta{\rm Tr}_\xi\ , \tag{2.11}$$

which was first proposed in Ref. [15] and was used in Ref. [16], one may introduce separable and correlated parts of the distribution functions as

$$\rho_s(t)\equiv\rho_\eta(t)\rho_\xi(t)=P(t)\rho(t)\ ,\ \rho_c(t)\equiv\{1-P(t)\}\rho(t)\ , \tag{2.12}$$

respectively. After integrating by parts, the Liouville equation (2.2) is reduced to the master equation for $\rho_s(t)$ given by

$$\dot{\rho}_s(t) \equiv -iP(t)\mathcal{L}\rho_s(t) - \int_{t_0}^{t} dt' P(t)\mathcal{L}g(t,t') \times \{1-P(t')\}\mathcal{L}\rho_s(t') - iP(t)\mathcal{L}g(t,t_0)\rho_c(t_0) , \tag{2.13}$$

where $g(t,t')$ represents a propagator,

$$g(t,t') \equiv T \exp\left\{-i\int_{t_0}^{t'}[1-P(\tau)]\mathcal{L}d\tau\right\} , \tag{2.14}$$

with T being the Dyson time-ordering operator. As an initial condition at $t=t_0$, one may choose a *stationary bundle* of trajectories satisfying

$$\rho(t_0)=\rho_\eta(t_0)\rho_\xi(t_0) , \ \rho_c(t_0)=0 , \tag{2.15}$$

which states that there is no correlation between the relevant and irrelevant degrees of freedom at the initial time t_0. Here it should be noticed that the initial choice in Eq. (2.15) is only possible under our proper definition of the DCC system [11].

From Eq. (2.13), one gets a coupled-master equation for the partial relevant and partial irrelevant distribution functions given by

$$\begin{aligned}
\dot{\rho}_\eta(t) &= -i[\mathcal{L}_\eta+\mathcal{L}_\eta(t)]\rho_\eta(t) - \int_{t_I}^{t} d\tau \mathrm{Tr}_\xi \mathcal{L}_\Delta(t)g(t,\tau)\mathcal{L}_\Delta(\tau)\rho_\eta(\tau)\rho_\xi(\tau) - i\mathrm{Tr}_\xi P(t)\mathcal{L}g(t,t_I)\rho_c(t_I) , \\
\dot{\rho}_\xi(t) &= -i[\mathcal{L}_\xi+\mathcal{L}_\xi(t)]\rho_\xi(t) - \int_{t_I}^{t} d\tau \mathrm{Tr}_\eta \mathcal{L}_\Delta(t)g(t,\tau)\mathcal{L}_\Delta(\tau)\rho_\eta(\tau)\rho_\xi(\tau) - i\mathrm{Tr}_\eta P(t)\mathcal{L}g(t,t_I)\rho_c(t_I) ,
\end{aligned} \tag{2.16}$$

where t_I takes any value between the initial time t_0 and the present time t. If one chooses $t_I=t_0$ and applies the initial condition in Eq. (2.15), the last term in the right-hand side (rhs) of Eq. (2.16) disappears. The Liouville operators $\mathcal{L}_\eta$, $\mathcal{L}_\xi$, $\mathcal{L}_\eta(t)$, $\mathcal{L}_\xi(t)$, and $\mathcal{L}_\Delta(t)$ appearing in Eq. (2.16) are defined by using the quantities in Eqs. (2.6) and (2.10) as

$$\begin{aligned}
&\mathcal{L}_\eta * = i\{H_\eta, * \}_{\mathrm{PB}} , \ \mathcal{L}_\xi * = i\{H_\xi, * \}_{\mathrm{PB}} , \\
&\mathcal{L}_\eta(t) * = i\{H_\eta(t) , * \}_{\mathrm{PB}} , \ \mathcal{L}_\xi(t) * = i\{H_\xi(t) , * \}_{\mathrm{PB}} , \\
&\mathcal{L}_\Delta(t) * = i\{H_\Delta(t) , * \}_{\mathrm{PB}} , \ \mathcal{L}_{\mathrm{coupl}}(t) * = i\{H_{\mathrm{coupl}}, * \}_{\mathrm{PB}} .
\end{aligned} \tag{2.17}$$

In the rhs of the first equation in Eq. (2.16), the first term expresses the *mean-field effects,* i.e., the effects coming from H_η as well as the average effects of the coupling $H_\eta(t)$. The third term depends on a correlated part of the distribution function $\rho_c(t_I)$ at a separation time t_I when the total system is divided into the relevant and irrelevant subsystems described by $\rho_\eta(t_I)$ and $\rho_\xi(t_I)$. On the other hand, the second term represents the dynamic fluctuation effects $H_\Delta(t)$ which start to act at t_I. The corresponding terms in the second equation of Eq. (2.16) are also understood in a similar way.

B. Dynamic Response and Correlation Functions

The coupled-master equation (2.16) is still equivalent to the original Liouville equation (2.2). To discuss whether the total system is reasonably divided into two subsystems, and whether some coarse graining procedure is safely introduced or not, one has to start with asking what is happening in the ensemble of trajectories during its time evolution process. To this aim, we retain the third term in the rhs of Eq. (2.16) with explicit t_I dependence, and will exploit it in studying dynamical change of the distribution function. As is clearly seen from Eq. (2.16), the instantaneous dynamical property of partial distribution functions at the separation time t_I ought to be studied by evaluating an influence of the fluctuation term just after t_I.

Let us introduce the *mean-field* propagator

$$\begin{aligned}
&g_{\mathrm{mf}}(t,t') \equiv T \exp\left\{-i\int_{t'}^{t} d\tau [1-P(\tau)]\mathcal{L}_{\mathrm{mf}}(\tau)\right\} , \\
&\mathcal{L}_{\mathrm{mf}}(t) \equiv \mathcal{L}_\eta+\mathcal{L}_\xi+\mathcal{L}_\eta(t)+\mathcal{L}_\xi(t) .
\end{aligned} \tag{2.18}$$

The propagator in Eq. (2.14) is then expressed as

$$g(t,t') = g_{\mathrm{mf}}(t,t') + O(\mathcal{L}_\Delta(t)) . \tag{2.19}$$

By further introducing the following propagator

$$G_{\mathrm{mf}}(t,t') \equiv T \exp\left\{-i\int_{t'}^{t} d\tau \mathcal{L}_{\mathrm{mf}}(\tau)\right\} = G_\eta(t,t')G_\xi(t,t') , \tag{2.20}$$

with

$$\begin{aligned}
G_\eta(t,t') &\equiv T \exp\left\{-i\int_{t'}^{t} d\tau [\mathcal{L}_\eta+\mathcal{L}_\eta(\tau)]\right\} , \\
G_\xi(t,t') &\equiv T \exp\left\{-i\int_{t'}^{t} d\tau [\mathcal{L}_\xi+\mathcal{L}_\xi(\tau)]\right\} ,
\end{aligned} \tag{2.21}$$

one may prove that there holds a relation

$$g_{\mathrm{mf}}(t,t')\mathcal{L}_\Delta(t')\rho_\eta(t')\rho_\xi(t') = G_{\mathrm{mf}}(t,t')\mathcal{L}_\Delta(t')\rho_\eta(t')\rho_\xi(t') . \tag{2.22}$$

Since the propagators $G_\eta(t,t')$ and $G_\xi(t,t')$ in Eq. (2.21) satisfy

$$\begin{aligned}
G_\eta(t_a,t_b) &= G_\eta(t_a,t)G_\eta(t,t_b) , \\
G_\xi(t_a,t_b) &= G_\xi(t_a,t)G_\xi(t,t_b) ,
\end{aligned} \tag{2.23}$$

and since

$$\mathrm{Tr}_\xi[\mathcal{L}_\xi+\mathcal{L}_\xi(t)]=0 , \ \mathrm{Tr}_\eta[\mathcal{L}_\eta+\mathcal{L}_\eta(t)]=0 , \tag{2.24}$$

Eq. (2.16) is then rewritten as

$$\dot{\rho}_\eta(t)=-i[\mathcal{L}_\eta+\mathcal{L}_\eta(t)]\rho_\eta(t)-i\mathrm{Tr}_\xi P(t)\mathcal{L}g(t,t_I)\rho_c(t_I)$$
$$-\mathrm{Tr}_\xi\int_{t_I}^{t}d\tau[G_\xi(\tau,t)\mathcal{L}_{\mathrm{coupl}}]$$
$$\times[G_\eta(t,\tau)\mathcal{L}_\Delta(\tau)\rho_\eta(\tau)\rho_\xi(\tau)]\ , \qquad (2.25a)$$

$$\dot{\rho}_\xi(t)=-i[\mathcal{L}_\xi+\mathcal{L}_\xi(t)]\rho_\xi(t)-i\mathrm{Tr}_\eta P(t)\mathcal{L}g(t,t_I)\rho_c(t_I)$$
$$-\mathrm{Tr}_\eta\int_{t_I}^{t}d\tau[G_\eta(\tau,t)\mathcal{L}_{\mathrm{coupl}}]$$
$$\times[G_\xi(t,\tau)\mathcal{L}_\Delta(\tau)\rho_\eta(\tau)\rho_\xi(\tau)]$$
$$(t\geq t_I\geq t_0)\ , \qquad (2.25b)$$

where the fluctuation effects are retained up to the second order in $\mathcal{L}_\Delta(t)$.

Without any loss of generality, H_{coupl} in the DCC system is expressed as

$$H_{\mathrm{coupl}}=\sum_l A^l(\eta,\eta^*)B^l(\xi,\xi^*)\ , \qquad (2.26)$$

where $A^l(\eta,\eta^*)$ and $B^l(\xi,\xi^*)$ are functions of relevant and irrelevant coordinates, respectively. The fluctuation term is then given as

$$H_\Delta(t)=\sum_l[A^l(\eta,\eta^*)-\langle A^l\rangle_t][B^l(\xi,\xi^*)-\langle B^l\rangle_t]\ , \qquad (2.27)$$

where

$$\langle A^l\rangle_t\equiv\mathrm{Tr}_\eta A^l(\eta,\eta^*)\rho_\eta(t)\ ,\ \langle B^l\rangle_t\equiv\mathrm{Tr}_\xi B^l(\xi,\xi^*)\rho_\xi(t)\ . \qquad (2.28)$$

Using Eq. (2.28), the average term in Eq. (2.10) is expressed as

$$H_\eta(t)=\sum_l A^l(\eta,\eta^*)\langle B^l\rangle_t\ ,$$
$$H_\xi(t)=\sum_a\langle A^l\rangle_t B^l(\xi,\xi^*)\ . \qquad (2.29)$$

With the aid of Eqs. (2.26)–(2.29), the coupled master equation (2.25a) is finally expressed as

$$\dot{\rho}_\eta(t)=-i[\mathcal{L}_\eta+\mathcal{L}_\eta(t)]\rho_\eta(t)-i\mathrm{Tr}_\xi P(t)\mathcal{L}g(t,t_I)\rho_c(t_I)$$
$$-\int_0^{t-t_I}d\tau\sum_{lm}\chi_{lm}(t,t-\tau)\{A^l,[G_\eta(t,t-\tau)(A^m-\langle A^m\rangle_{t-\tau})\rho_\eta(t-\tau)]\}_{\mathrm{PB}}$$
$$-\int_0^{t-t_I}d\tau\sum_{lm}\psi_{lm}(t,t-\tau)\{A^l,[G_\eta(t,t-\tau)\{A^m,\rho_\eta(t-\tau)\}_{\mathrm{PB}}]\}_{\mathrm{PB}}\ , \qquad (2.30)$$

where the dynamic response and correlation functions are defined by

$$\chi_{lm}(t,t-\tau)\equiv\mathrm{Tr}_\xi\{G_\xi(t-\tau,t)B^l,B^m\}_{\mathrm{PB}}\rho_\xi(t-\tau)\ , \qquad (2.31a)$$

$$\psi_{lm}(t,t-\tau)\equiv\mathrm{Tr}_\xi[G_\xi(t-\tau,t)B^l](B^m-\langle B^m\rangle_{t-\tau})\rho_\xi(t-\tau)\ . \qquad (2.31b)$$

In the same way as Eq. (2.30), Eq. (2.25b) is expressed as

$$\dot{\rho}_\xi(t)=-i[\mathcal{L}_\xi+\mathcal{L}_\xi(t)]\rho_\xi(t)-i\mathrm{Tr}_\eta P(t)\mathcal{L}g(t,t_I)\rho_c(t_I)$$
$$-\int_0^{t-t_I}d\tau\sum_{lm}X_{lm}(t,t-\tau)\{B^l,[G_\xi(t,t-\tau)(B^m-\langle B^m\rangle_{t-\tau})\rho_\xi(t-\tau)]\}_{\mathrm{PB}}$$
$$-\int_0^{t-t_I}d\tau\sum_{lm}\Psi_{lm}(t,t-\tau)\{B^l,[G_\xi(t,t-\tau)\{B^m,\rho_\xi(t-\tau)\}_{\mathrm{PB}}]\}_{\mathrm{PB}}\ , \qquad (2.32)$$

where

$$X_{lm}(t,t-\tau)\equiv\mathrm{Tr}_\eta\{G_\eta(t-\tau,t)A^l,A^m\}_{\mathrm{PB}}\rho_\eta(t-\tau)\ , \qquad (2.33a)$$

$$\Psi_{lm}(t,t-\tau)\equiv\mathrm{Tr}_\eta[G_\eta(t-\tau,t)A^l]$$
$$\times(A^m-\langle A^m\rangle_{t-\tau})\rho_\eta(t-\tau)\ . \qquad (2.33b)$$

Here, it should be noticed that the usual response function is defined for the system which is not far from the state of equilibrium. In this case, the response function tells us how the system slightly deviated from the state of equilibrium evolves, under an influence of external force. Thus, the response function depends on the time-independent property of the system.

On the other hand, the dynamic response functions in Eqs. (2.31) and (2.33) have instantaneous information on the relevant and irrelevant subsystems at the separation time t_I, whose time evolution has been governed by the original Liouville equation (2.2) up to t_I. Namely, they describe how the subsystems respond to the "external force" $H_\Delta(t)$ which starts to act at t_I. Since there remains the second term in the rhs of Eqs. (2.30) and (2.32), and since the higher-order fluctuation effects have been neglected in deriving the dynamic response and correlation functions, the dynamic response functions may have a sense in a short period just after t_I. However, they still have a lot of information on the subsystems, because the remaining effects are evaluated by the t_I dependence of the dynamic response and correlation functions.

Consequently, important information on the instantaneous dynamical structure of distribution function, on an effect coming from complex topological structure of the TDHF manifold, and on the applicability of separating the total system into the relevant and irrelevant subsystems ought to be studied through the t_I-dependent dynamical response and correlation functions.

III. HOW TO EVALUATE THE DYNAMIC RESPONSE FUNCTION

Let us rewrite the dynamic response function in Eq. (2.31) as follows:

$$\chi_{lm}(t,t-\tau)=\mathrm{Tr}_\xi\{F_\xi(t,t-\tau)B^l,B^m\}_{\mathrm{PB}}\rho_\xi(t-\tau)\ , \tag{3.1}$$

$$F_\xi(t,t')\equiv T\exp\left[i\int_{t'}^{t}d\tau[\mathcal{L}_\xi+\mathcal{L}_\xi(\tau)]\right]\ . \tag{3.2}$$

By introducing $t_I=t-\tau$, one obtains

$$\chi_{lm}(t_I+\tau,t_I)=\mathrm{Tr}_\xi\{F_\xi(t_I+\tau,t_I)B^l,B^m\}_{\mathrm{PB}}\rho_\xi(t_I)\ . \tag{3.3}$$

Here, t_I takes any value between the initial time t_0 and the present time t, and τ runs from 0 to $t-t_I$. The t_I dependence of $\chi_m(t_I+t,t_I)$ strongly reflects the dynamical evolution process of $\rho_\xi(t)$ which is not considered in the usual linear response function. In order to evaluate the dynamic response function in the DCC system, one has to calculate

$$B^l_\tau=B^l_\tau(\xi_\tau,\xi^*_\tau)\equiv F_\xi(t_I+\tau,t_I)B^l(\xi,\xi^*)\ , \tag{3.4}$$

where the phase space point denoted by $(\xi_\tau=\xi(t_I+\tau),\xi^*_\tau=\xi^*(t_I+\tau))$ is the image of the phase space point (ξ,ξ^*) at t_I. Thus the Poisson bracket in Eq. (3.3) is expressed as

$$\{B^l_\tau,B^m\}_{\mathrm{PB}}=-i\sum_\alpha\left[\frac{\partial B^l_\tau}{\partial\xi_\alpha}\frac{\partial B^m}{\partial\xi^*_\alpha}-\frac{\partial B^l_\tau}{\partial\xi^*_\alpha}\frac{\partial B^m}{\partial\xi_\alpha}\right]\ . \tag{3.5}$$

It should be noticed that the quantities ξ_α,ξ^*_α, and B^m are defined at time t_I, whereas B^l_τ is defined at $t_I+\tau$. In order to evaluate the Poisson bracket of two quantities B^l_τ and B^m with different time arguments, one has to consider the following term:

$$\frac{\partial B^l_\tau}{\partial\xi_\alpha}=\sum_\beta\left\{\frac{\partial B^l_\tau(\xi_\tau,\xi^*_\tau)}{\partial\xi_\beta(t_I+\tau)}\frac{\partial\xi_\beta(t_I+\tau)}{\partial\xi_\alpha}+\frac{\partial B^l_\tau(\xi_\tau,\xi^*_\tau)}{\partial\xi^*_\beta(t_I+\tau)}\frac{\partial\xi^*_\beta(t_I+\tau)}{\partial\xi_\alpha}\right\}\ . \tag{3.6}$$

Since $\xi_\alpha(t_I+\tau)$ and $\xi^*_\alpha(t_I+\tau)$ are generated by the propagator $F_\xi(t_I+\tau,t_I)$ in Eq. (3.2), they are described by the Hamilton's equations of motion,

$$i\dot\xi_\alpha=\frac{\partial[H_\xi+H_\xi(t)]}{\partial\xi^*_\alpha}\ , \tag{3.7}$$

which is organized by the mean-field Hamiltonian $H_\xi+H_\xi(t)$. From Eq. (3.7), one has

$$\xi_\alpha(t_I+\tau)=\xi_\alpha(t_I)+\int_{t_I}^{t_I+\tau}\dot\xi_\alpha(t')dt'\ . \tag{3.8}$$

The time integral in Eq. (3.8) is evaluated numerically by introducing a discretized small time step Δt. The simplest way to evaluate Eq. (3.8) is given by the following formula:

$$\xi_\alpha(t_I+\tau)=\xi_\alpha(t_I)+\sum_{n=0}^{N}\dot\xi_\alpha(t_I+n\Delta t)\Delta t\ ,\ \Delta t\equiv\frac{\tau}{N+1}\ , \tag{3.9}$$

which is valid in the first order of Δt. If the above expression does not give a sufficient accuracy, one may use the first-order result as preevaluated value for the formula which is valid in $(\Delta t)^2$. The expression in Eq. (3.9) is calculated iteratively. The nth step equation is given by

$$\xi_\alpha(t_I+n\Delta t)=\xi_\alpha(t_I+(n-1)\Delta t)+\dot\xi_\alpha(t_I+(n-1)\Delta t)\Delta t\ , \tag{3.10}$$

where the functional form of $\xi_\alpha(t_I+(n-1)\Delta t)$ with respect to $\xi_\alpha(t_I)$ and $\xi^*_\alpha(t_I)$ is supposed to be known by the preceding $(n-1)$st step equation, whereas the second term in the rhs of Eq. (3.10) is also expressed as the functions of the quantities $\xi_\alpha(t_I)$ and $\xi^*_\alpha(t_I)$ through

$$\dot\xi_\alpha(t_I+(n-1)\Delta t)=-i\frac{\partial[H_\xi+H_\xi(t)]}{\partial\xi^*_\alpha}\bigg|_{t=t_I+(n-1)\Delta t,\xi=\xi(t_I+(n-1)\Delta t)}\ . \tag{3.11}$$

In this way, the quantities $\xi_\alpha(t_I+\tau)$ are expressed as functions of the quantities at $t=t_I$. By using the resultant functional forms, one calculates the rhs of Eq. (3.6) so as to get an explicit expression of the Poisson bracket of two quantities with different time arguments.

The dynamic response function in Eq. (3.3) is calculated numerically by using the pseudoparticle method where the distribution function is simulated by a set of representative pseudoparticles as

$$\rho(t)=\frac{1}{N_p}\sum_{n=1}^{N_p}\delta(\xi-\xi_n(t))\delta(\xi^*-\xi^*_n(t))\times\delta(\eta-\eta_n(t))\delta(\eta^*-\eta^*_n(t))\ . \tag{3.12}$$

In Eq. (3.12), the coordinates $\eta_n(t)$, $\eta^*_n(t)$, $\xi_n(t)$, and $\xi^*_n(t)$ for the nth particle are determined by the Hamilton's equation of motion given by

$$i\dot\eta=\frac{\partial H}{\partial\eta^*}\ ,\ i\dot\xi=\frac{\partial H}{\partial\xi^*}\ . \tag{3.13}$$

The resultant distribution function $\rho(t)$ in Eq. (3.12) simulates the solution of the original Liouville equation (2.2). In order to calculate the dynamical response function $\chi_{lm}(t_I+t,t_I)$ for a specific time t_I, one has to calculate $\rho(t)$ up to a time $t=t_I$.

IV. DYNAMIC RESPONSE FUNCTION FOR A SIMPLE SOLUBLE MODEL

A. Model Hamiltonian

The adopted model is a modification of the SU(3) Hamiltonian introduced by Li, Klein, and Dreizler [17], whose property in the TDHF manifold has been well studied. The model Hamiltonian is given by

$$\hat{H}=\sum_{i=0}^{2}\xi_i K_{ii}+\tfrac{1}{2}\sum_{i=1,2}V_i(K_{i0}K_{i0}+\text{H.c.})\ . \tag{4.1}$$

There are three levels with energies $\varepsilon_0<\varepsilon_1<\varepsilon_2$ and each level has N-fold degeneracy. The fermion pair operators K_{ij} are defined as

$$K_{ij}=\sum_{m=1}^{N}C_{im}^{\dagger}C_{jm}\ , \tag{4.2}$$

where $C_{im}^{\dagger}$ and C_{im} represent the fermion creation and annihilation operators, respectively. In this paper, we consider a system with N particles, whose lowest energy state $|\phi_0\rangle$ in a case with $V_i=0$ is given by

$$|\phi_0\rangle=\prod_{m=1}^{N}C_{0m}^{\dagger}|0\rangle\quad(C_{im}|0\rangle=0)\ . \tag{4.3}$$

The TDHF equation is expressed as

$$\delta\langle\phi(t)|\{i\,\partial/\partial t-\hat{H}\}|\phi(t)\rangle=0\ , \tag{4.4}$$

where $|\phi(t)\rangle$ is the general time-dependent single Slater determinant given by

$$|\phi(t)\rangle=\exp\left[i\sum_{j=1,2}(f_jK_{j0}+\text{H.c.})\right]|\phi_0\rangle\ . \tag{4.5}$$

With the aid of the canonical variable representation of the TDHF manifold [14], the TDHF Eq. (4.4) is known to be expressed by the canonical equation of motion with two degrees of freedom which is organized by the following Hamiltonian:

$$\begin{aligned}
&H(q_1,p_1;q_2,p_2)=\langle\phi(t)|\hat{H}|\phi(t)\rangle=H_1+H_2+H_{\text{coupl}}\ ,\\
&H_i=\frac{1}{2}(\varepsilon_i-\varepsilon_0)(q_i^2+p_i^2)+\frac{1}{2}V_i(N-1)(q_i^2-p_i^2)\\
&\qquad-\frac{N-1}{N}\frac{V_i}{4}(q_i^4-p_i^4)\ ,\ i=1,2\ ,\\
&H_{\text{coupl}}=\sum_{i=1,2}A^iB^i\ ,\\
&A^i=\left[\frac{V_i(1-N)}{4N}\right]^{1/2}[q_1^2+(-1)^ip_1^2]\ ,\\
&B^i=\left[\frac{V_i(1-N)}{4N}\right]^{1/2}[q_2^2-(-1)^ip_2^2]\ .
\end{aligned} \tag{4.6}$$

Regarding (q_1,p_1) as the relevant degree of freedom and (q_2,p_2) as the irrelevant degrees of freedom, one notices that the coordinate system $(q_1,p_1;q_2,p_2)$ corresponds to the DCC system, because there exists a stationary condition called the maximal-decoupling condition [8],

$$\left.\frac{\partial H}{\partial q_2}\right|_{p_2=q_2=0}=\left.\frac{\partial H}{\partial p_2}\right|_{p_2=q_2=0}=0\ ; \tag{4.7}$$

i.e., there are no coupling terms which have linear dependence on irrelevant degrees of freedom. In other words, there exists a two-dimensional stationary collective submanifold $\Sigma^2:\{q_1,p_1\}$ which is embedded in the four-dimensional TDHF manifold $M^4:\{q_1,p_1;q_2,p_2\}$. With the aid of the maximal-decoupling condition (4.7), the trajectory starting from any point on Σ^2 is described by

$$\dot{q}_1=\frac{\partial H_1}{\partial p_1}\ ,\ \dot{p}_1=-\frac{\partial H_1}{\partial q_1}\ , \tag{4.8}$$

which is organized by H_1 alone and is specified by the energy E_{coll} of the trajectory.

Following Eq. (2.10), the coupling term is divided into three parts,

$$\begin{aligned}
&H_{\text{coupl}}=H_{\text{aver}}(t)+H_{\Delta}(t)-E_0(t)\ ,\\
&H_{\text{aver}}(t)=H_{\eta}(t)+H_{\xi}(t)\ ,\\
&H_{\eta}(t)=\sum_{i=1,2}A^i\langle B^i\rangle_t\ ,\ H_{\xi}(t)=\sum_{i=1,2}\langle A^i\rangle_tB^i\ ,\\
&H_{\Delta}(t)=\sum_{i=1,2}(A^i-\langle A^i\rangle_t)(B^i-\langle B^i\rangle_t)\ ,\\
&E_0(t)=\sum_{i=1,2}\langle A^i\rangle_t\langle B^i\rangle_t\ .\\
&\langle A^i\rangle_t\equiv\text{Tr}_1A^i\rho_1(t)=\left[\frac{V_i(1-N)}{4N}\right]^{1/2}[\bar{q}_1^2+(-1)^i\bar{p}_1^2]\ ,\\
&\langle B^i\rangle_t\equiv\text{Tr}_2B^i\rho_2(t)=\left[\frac{V_i(1-N)}{4N}\right]^{1/2}[\bar{q}_2^2-(-1)^i\bar{p}_2^2]\ .
\end{aligned} \tag{4.9}$$

In Eq. (4.9), we use the notation

$$\begin{aligned}
&\bar{q}_1^2=\frac{1}{N_p}\sum_{n=1}^{N_p}q_{1,n}^2\ ,\ \bar{p}_1^2=\frac{1}{N_p}\sum_{i=n}^{N_p}p_{1,n}^2\ ,\\
&\bar{q}_2^2=\frac{1}{N_p}\sum_{n=1}^{N_p}q_{2,n}^2\ ,\ \bar{p}_2^2=\frac{1}{N_p}\sum_{n=1}^{N_p}p_{2,n}^2\ ,
\end{aligned} \tag{4.10}$$

where $q_{1,n}$, $p_{1,n}$, $q_{2,n}$, and $p_{2,n}$ denote a phase space point of the nth pseudoparticle described by Eq. (3.13) at time t.

The response function in Eq. (3.3) is expressed as

$$\chi_{ij}(t_I+\tau,t_1)=\text{Tr}_2\left[\frac{\partial B_\tau^i}{\partial q_2}\frac{\partial B^j}{\partial p_2}-\frac{\partial B_\tau^i}{\partial p_2}\frac{\partial B^j}{\partial q_2}\right]\rho_2(t_I)\ . \tag{4.11}$$

Here, B_τ^i are functions of $q_1(t_I+\tau)$, $p_1(t_I+\tau)$, $q_2(t_I+\tau)$, and $p_2(t_I+\tau)$ at time $t_I+\tau$, whereas B^i are functions of $q_1(t_I)$, $p_1(t_I)$, $q_2(t_I)$, and $p_2(t_I)$ at the separation time t_I when the fluctuation term $H_{\Delta}(t)$ is switched on. Since the distribution function $\rho_2(t_I)$ in Eq. (4.11) is simulated by a set of many representative pseudoparticles, it is expressed as

$$\rho_2(t)=\frac{1}{N_p}\sum_{n=1}^{N_p}\delta(p_2-p_{2,n}(t))\delta(q_2-q_{2,n}(t)) . \quad (4.12)$$

Using Eq. (4.12), the dynamic response function in Eq. (4.11) may be written as

$$\chi_{ij}(t_I+\tau,t_I)=\frac{1}{N_p}\sum_{n=1}^{N_p}\left[\frac{\partial B^i_\tau(n)}{\partial q_{2,n}}\frac{\partial B^j(n)}{\partial p_{2,n}}-\frac{\partial B^i_\tau(n)}{\partial p_{2,n}}\frac{\partial B^j(n)}{\partial q_{2,n}}\right], \quad (4.13)$$

where $B^i_\tau(n)$ is the functions of coordinates $q_{2,n}(t_I+\tau)$ and $p_{2,n}(t_I+\tau)$, whereas $B^i(n)$ depends on $q_{2,n}(t_I)$ and $p_{2,n}(t_I)$ of the nth pseudoparticle.

B. Numerical result

In order to study how the system evolves in time, one has to introduce a specific form for the initial distribution function in Eq. (2.15). Since we are interested in how the stationary bundle of trajectories accumulating around $\Sigma^2:\{q_1,p_1\}$ at the initial time t_0 evolves in time, we start with briefly summarizing a specific property of $\Sigma^2:\{q_1,p_1\}$. By using the parameters with $\varepsilon_0=0$, $\varepsilon_1=1$, $\varepsilon_2=2$, $N=10$, $V_1=-1/15$, and $V_2=-1/3$, it has been clarified that the collective submanifold $\Sigma^2:\{q_1,p_1\}$ is divided into three characteristic regions [18] by means of *collective trajectories* described by Eq. (4.8). The numerical calculation has shown that the trajectories with $E_{\rm coll}=8$, 5, and 1 are traveling in the *collective, dissipative,* and *stochastic* regions of Σ^2.

The initial distribution function $\rho(t_0)$ is chosen to be the stationary bundle,

$$\rho(t_0)=\rho_\eta(t_0)\rho_\xi(t_0) , \quad (4.14)$$

where an initial form of the relevant partial distribution function is given by

$$\rho_\eta(t_0)=N_\eta\delta(H_1(q_1,p_1)-E_{\rm coll}) . \quad (4.15)$$

Here δ represents a delta function and $E_{\rm coll}$ denotes a given collective energy carried by the relevant degrees of freedom at the initial time t_0. N_η is a normalization constant determined by the condition (2.8). Since there holds a relation

$$\dot\rho_\eta(t=t_0)=-i\mathcal{L}_\eta\rho_\eta(t=t_0)=0 , \quad (4.16)$$

which is justified by Eq. (4.7), the initial ensemble of trajectories described by Eqs. (4.14) and (4.15) represents a stationary bundle sticking around Σ^2. An initial form of the irrelevant partial distribution function $\rho_\xi(t=t_0)$ is assumed to be a uniform distribution in a region $(-0.5\le q_2<0.5,\ -0.5\le p_2<0.5)$.

In evaluating the dynamic response function, a time step in Eq. (3.9) is taken to be $\Delta t=\tau_{\rm coll}/400$, where $\tau_{\rm coll}$ denotes a characteristic periodic time of the trajectory described by Eq. (4.8). We also checked the accuracy of Eq. (3.9) by comparing the result with those obtained by using the formula which is valid in $(\Delta t)^2$. It turned out that the first-order approximation in Δt already gives a sufficient accuracy. In calculating the Poisson bracket of two quantities with different time arguments, we made the following approximation:

$$\left.\frac{\partial H_\xi(t)}{\partial\xi_\alpha}\right|_{t=t_I+n\Delta t,\xi=\xi(t=t_I+n\Delta t)}=\sum_a\langle A^a\rangle_{t=t_I+n\Delta t}\left.\frac{\partial B^a(\xi,\xi^*)}{\partial\xi_\alpha}\right|_{\xi=\xi(t=t_I+n\Delta t)}\approx\sum_a\langle A^a\rangle_{t=t_I}\left.\frac{\partial B^a(\xi,\xi^*)}{\partial\xi_\alpha}\right|_{\xi=\xi(t=t_I+n\Delta t)} . \quad (4.17)$$

The dynamic response function $\chi_{ij}(t_I+\tau,t_I)$ with $i=1$ and $j=2$ is calculated numerically. Here, i and j denote the superscripts B^i and B^j appearing in Eq. (4.6). In Fig. 1, the characteristic behavior of the dynamic response function for the collective case with $E_{\rm coll}=8$ is shown. Figures 1(a)–1(f) illustrate how the dynamic response function depends on the separation time t_I. From Figs. 1(a)–1(c), a slight growth of the dynamic response function with increasing t_I is observed, which may be caused by the initial sampling. Namely, the structure of the bundle changes in time at the initial period. After a short time, as is seen from Figs. 1(d) and 1(f), the dynamic response function shows the almost same shape irrespective of the time t_I, illustrating an establishment of stationary bundle. Namely, the irrelevant subsystem is not strongly affected by the fluctuation term, after the irrelevant distribution function reaches to the stationary structure. As was shown in Ref. 11, the mean field effects $\mathcal{L}_{\rm mf}$ play a dominant role, whereas the fluctuation term $\mathcal{L}_\Delta(t)$ has minor importance in a case with $E_{\rm coll}=8$.

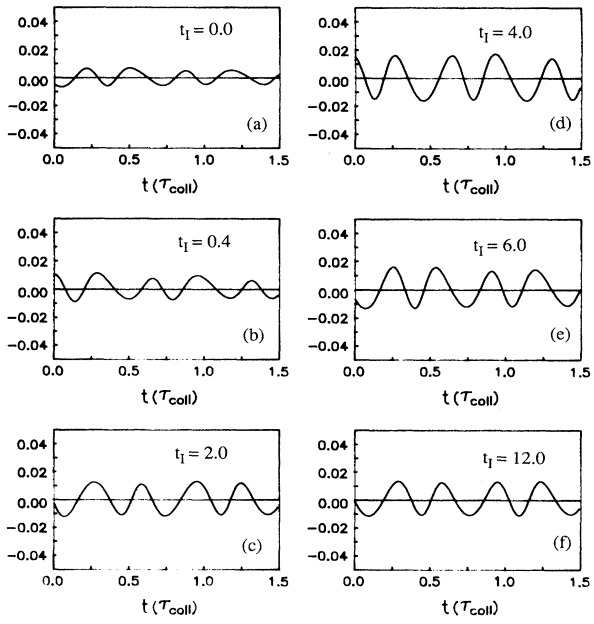

FIG. 1. Dynamic response function $\chi_{i=1,j=2}(t_I+t,t_I)$ with various separation time t_I in the regular case with $E_{\rm coll}=8$. The time t is measured in units of $\tau_{\rm coll}$.

Therefore one may draw the conclusion that the dynamical fluctuation may be neglected when the dynamic response function does not depend on the separation time t_I.

Figure 2 shows the dynamical response function in the dissipative case with $E_{\rm coll}=5$. Comparing Fig. 2 with Fig. 1, it is recognized that the magnitude of response is quite large, suggesting an important effect of $H_\Delta(t)$ in the dissipative case. As is clearly seen from Fig. 2, the characteristic behavior of the dynamic response function strongly depends on t_I. In the case of $t_I<0.4\tau_{\rm coll}$, the response of irrelevant subsystem to the fluctuation term becomes large as t_I increases. At time $t_I\approx 0.4\tau_{\rm coll}$, a large response appears. After $T_i=0.4\tau_{\rm coll}$, the response becomes smaller as the separation time t_I increases. The structure of the response function returns back at $t_I\approx 1.1\tau_{\rm coll}$, $2.0\tau_{\rm coll}$, $(2.5-2.6)\tau_{\rm coll}$, and so on to the almost same shape as in the case with $t_I=0.4\tau_{\rm coll}$, periodically. In Fig. 2, only a case with $t_I\approx(2.5-2.6)\tau_{\rm coll}$ is shown. From the above analysis, one may easily see that the dynamic response function for the dissipative region exhibits a clear structure depending on t_I.

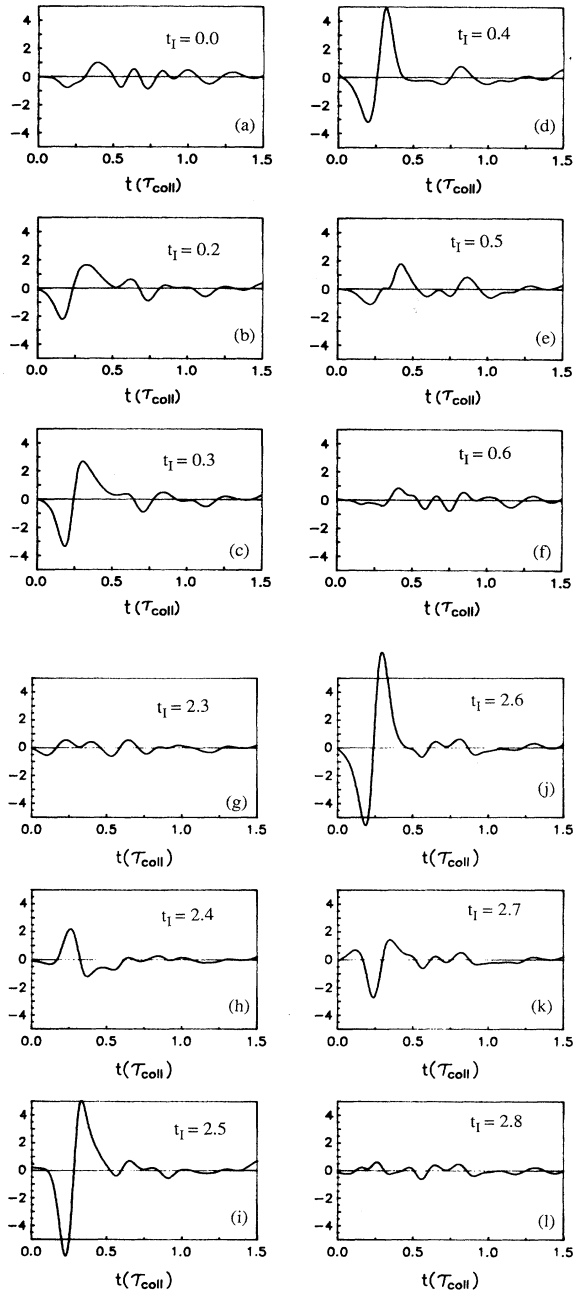

FIG. 2. Dynamic response function $\chi_{i=1,j=2}(t_I+t,t_I)$ with various separation time t_I in the dissipative case with $E_{\rm coll}=5$. The structure of response function at $t_I=(2.5-2.6)\tau_{\rm coll}$ has almost the same shape as in the case with $t_I=0.4\tau_{\rm coll}$. The time t is measured in units of $\tau_{\rm coll}$.

In order to understand the specific feature of the dynamical response function in Figs. 1 and 2, and to understand what happens in the time evolution of the ensemble of trajectories, the time-dependent structure of variance $\langle p_2^2-\langle p_2\rangle_t^2\rangle_t$ evaluated by using the original distribution function $\rho(t)$ organized by Eq. (2.2) are illustrated in Fig. 3 for both the collective and dissipative cases. As is recognized from Eq. (4.9), this quantity is directly related with the fluctuation term $H_\Delta(t)$. Its magnitude is very small in the collective case with $E_{\rm coll}=8$, which is consistent with the negligibly small effect of the fluctuation term shown in Fig. 1. On the other hand, it oscillates and takes a very large value in the dissipative case; its magnitude becomes large continuously and seems to reach some saturated value after a long period. The oscillation with nearly constant periodicity (it takes maximum values at $t\approx 0.4\tau_{\rm coll}$, $1.1\tau_{\rm coll}$, $2.0\tau_{\rm coll}$, and $2.6\tau_{\rm coll}$, etc.) shows characteristic feature of $\rho\xi(t)$, indicating a new microscopic dissipation mechanism of a finite many-fermion system. Reflecting this dissipation mechanism which is embraced in the t dependence of $\rho(t)$, the dynamic response function also shows a periodic

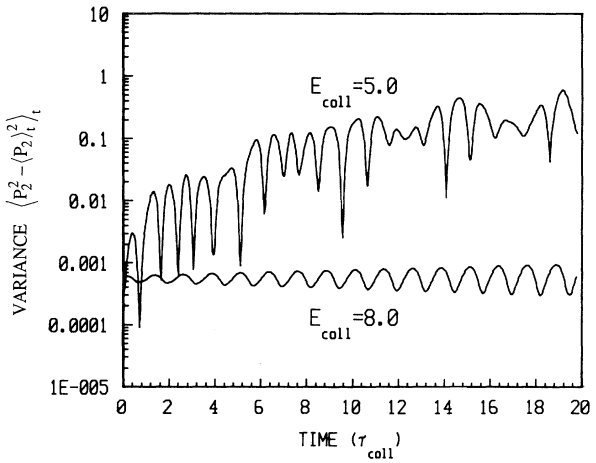

FIG. 3. Time dependence of variance $\langle p_2^2-\langle p_2\rangle_t^2\rangle_t$. The periodicity of the variance is clearly seen. It reflects on a periodic property of $\Delta H(t)$ as well as that of the dynamic response function. The time t is measured in units of $\tau_{\rm coll}$.

structure depending on t_I. Here it is worthwhile to mention that a characteristic structure of dynamic response function periodically observed at $t_I = 0.4\tau_{\text{coll}}$, $1.1\tau_{\text{coll}}$, $2.0\tau_{\text{coll}}$, and $(2.5-2.6)\tau_{\text{coll}}$, looks like that of the usual linear response function. From the above numerical result, it is clearly stated that the energy dissipation of collective motion in a finite system such as the nucleus is a periodic process characterized by both the short oscillating period and long saturating period.

V. DISCUSSION

On the basis of the general coupled-master equation in the DCC representation, the dynamic response and correlation functions are introduced, and a systematic numerical method for calculating these quantities is proposed. In this paper, we mainly focus our attention on analyzing what is happening in the time evolution of the ensemble of trajectories $\rho(t)$ by means of instantaneous behavior of the dynamic response function. With the aid of a simple soluble model Hamiltonian, it is illustrated that the dynamic response function gives important information in understanding the dynamical evolution process of $\rho(t)$ organized by Eq. (2.2), and important information on the ensemble of trajectories whether it spreads out over some region in the TDHF manifold and reaches to some time-independent object, or it still contains some time-dependent structure. Indeed, we have already shown some interesting and even surprising features, such as the period structure in the dynamic response function for the dissipative collective motion.

Recently, a new systematic method [6] has been proposed capable of exploring an exceedingly rich structure of the TDHF manifold, i.e., an infinite series of island structure representing a regular motion and many stochastic sea in between various islands. These complex structures of the TDHF manifold certainly reflect the complex nonlinear relation between the collective and single-particle modes of motion, which has been discussed in terms of a dynamical role of the adiabatic vs diabatic single-particle states, that of the adiabatic vs non-adiabatic collective motion, the level slippage dynamics, the level crossing, and pseudo-level-crossing dynamics, etc. [19–21]. It seems quite promising that the dynamic response function may give us important information on the microscopic origin of the dissipation mechanism, and on the role of the single-particle dynamics, e.g., the level-crossing dynamics of the TDHF manifold where the ensemble of TDHF trajectories exists. In order to explore these interesting subjects, the present calculations are too simple and too coarse. A more detailed analysis is undertaken by using more fine-tuned initial conditions for ensemble of trajectories, by calculating the Fourier decomposition of the dynamic response function and the higher-order moments of the distribution function $\langle p_i^m - \langle p_i \rangle_t^m \rangle_t$ with $m \geq 3$, and by relating them with the energy dissipation mechanism.

ACKNOWLEDGMENTS

This work is supported in part by the National Natural Science Foundation of China for international cooperation.

[1] Wu Xizhen and Zhuo Yizhong, Chin. Phys. (New York) **1**, 671 (1981); Wu Xizhen *et al.*, Com. Theor. Phys. (Beijing) **1**, 769 (1982); Feng Renfa, Wu Xizhen, and Zhuo Yizhong, Chin. Phys. (New York) **9**, 774 and 850 (1989).

[2] P. Grange, J. I. Li, and H. A. Weidenmüller, Phys. Rev. C **27**, 2063 (1983).

[3] A. J. Sierk, H. Hofmann, F. Scheuter, and D. Vautherin, Nucl. Phys. **A424**, 239 (1984); F. Scheuter and H. Hofmann, *ibid.* **A394**, 477 (1983); G. D. Adeev and V. V. Pashkevich, *ibid.* **A502**, 405c (1989).

[4] Y. Abe, C. Gregoire, and H. Delagrange, J. Phys. **47**, c4-329 (1986); P. Fröbrich, Nucl. Phys. **A545**, 87c (1992).

[5] M. Baranger and M. Véneroni, Ann. Phys. **114**, 123 (1978); K. Göke and P.-G. Reinhard, Ann. Phys. **112**, 328 (1978).

[6] F. Sakata and T. Marumori, *Direction in Chaos* (World Scientific, Singapore, 1992), Vol. 4.

[7] See for example, A. J. Lichtenberg and M. A. Lieberman, Appl. Math. Sci. **38** (1983).

[8] T. Marumori, T. Maskawa, F. Sakata, and A. Kuriyama, Prog. Theor. Phys. **64**, 1294 (1980).

[9] R. Balian and M. Véneroni, Ann. Phys. **164**, 334 (1985).

[10] F. Sakata, M. Matsuo, T. Marumori, and Y. Zhuo, Ann. Phys. (N.Y.) **194**, 30 (1989).

[11] M. Matsuo, F. Sakata, and T. Marumori, Prog. Theor. Phys. **82**, 1084 (1989).

[12] See for example, R. Kubo, M. Toda, and N. Hashitsume, *Statistical Phys. II* (Springer, New York, 1991).

[13] H. Hofmann, *Proceedings of the Phase Space Approach to Nuclear Dynamics,* Trieste, 1985 (World Scientific, Singapore, 1986), p. 306, and references therein.

[14] F. Sakata, T. Marumori, Y. Hashimoto, and T. Une, Prog. Theor Phys. **70**, 424 (1983); P. Kramer and M. Saraceno, *Lecture Notes in Physics No. 140* (Springer, New York, 1981); A. Kuriyama and M. Yamamura, Prog. Theor. Phys. **82**, 1084 (1989).

[15] C. R. Willis and R. H. Picard, Phys. Rev. A **9**, 1343 (1974).

[16] Y. Zhuo and X. Wu, Chin. Phys. **1**, 969 (1981).

[17] S. Y. Li, A. Klein, and R. M. Dreizler, J. Math. Phys. **11**, 975 (1970).

[18] K. Muramatsu, F. Sakata, Y. Yamamoto, and T. Marumori, Prog. Theor. Phys. **77**, 347 (1987).

[19] D. L. Hill and J. A. Wheeler, Phys. Rev. **89**, 1102 (1953).

[20] W. J. Swiatecki, Nucl. Phys. **A488**, 375c (1988).

[21] W. Nörenberg, Phys. Lett. **104B**, 107 (1981).