Resonance structures through time-dependent potentials

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Electron-positron pair production in time-dependent potentials is considered. We find that the particle production distribution probability shows interesting resonance structures. Some numerical applications to heavy ion collisions are made.

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I. INTRODUCTION

The usual way to treat scattering problems is to consider potentials that vary in a given region of the space. For example, a flux of particles is incident over a region of localized potential, the target, and after the interaction, particles are scattered in all directions. In this paper we wish to concentrate instead on time-dependent potentials for which production of particles can occur. It is hoped that the calculations presented here may be useful for a better understanding of electron-positron production in situations (such as the scattering of ions) where the effective potentials are strongly time dependent.

The resonance pattern for pair production in a strong time-dependent electric field has been discussed in 1989 by Cornwall and Tiktopoulos [1] and later by Balantekin and Fricke [2]. These authors, using methods in field theory, showed that a time potential of a general "bell" shape (corresponding to a time-dependent electric field that changes its direction in a given interval of time) generates a resonance structure in the electron-positron pair production. Cornwall and Tiktopoulos have worked out an explicit calculation for a two δ-function electric field configuration (square-well potential). Balantekin and Fricke have generalized their results, considering a doubly pulsed electric field.

In this work we want to show how an elementary "quantum mechanics method" can give origin to a resonance pattern not only for a two δ-function electric field configuration with opposite directions, but also for a constant electric field, applied for a time period τ. The finite time period gives rise to resonant structures. In the limit of infinite τ, the model reduces to standard nonresonant results [3].

The Dirac equation for an electron in a time-dependent potential is

$$i\frac{\partial\psi}{\partial t}=H\psi \ , \tag{1}$$

with H given by

$$H=[\mathbf{p}-e\mathcal{A}(t)]\cdot\alpha+m\beta \ . \tag{2}$$

Since $\mathbf{p}$ here is a constant of motion, we may choose the momentum along the z axis, so that the Hamiltonian can be written as

$$H=[p-e\mathcal{A}(t)]\alpha_z+m\beta \ . \tag{3}$$

If we write ψ in terms of its components u_1, u_2, u_3, and u_4, the Dirac equation can be written as

$$i\dot{u}_1=mu_1+[p-e\mathcal{A}(t)]u_3 \ , \tag{4a}$$

$$i\dot{u}_2=mu_2-[p-e\mathcal{A}(t)]u_4 \ , \tag{4b}$$

$$i\dot{u}_3=[p-e\mathcal{A}(t)]u_1-mu_3 \ , \tag{4c}$$

$$i\dot{u}_4=-[p-e\mathcal{A}(t)]u_2-mu_4 \ . \tag{4d}$$

Equation (4a) is coupled to Eq. (4c) and Eq. (4b) to Eq. (4d), hence if we fix the spin we can simply consider, for example, Eqs. (4a) and 4(c); that is the same as to consider the two-dimensional Hamiltonian

$$H=[p-e\mathcal{A}(t)]\sigma_x+m\sigma_z \ . \tag{5}$$

In Sec. II we discuss the pair-production probability for the Hamiltonian given in Eq. (5) for two different time-dependent potentials (cited above). In Sec. III we apply our Hamiltonian for a qualitative understanding of pair production in the collision of heavy ions.

II. EXAMPLES OF TIME-DEPENDENT POTENTIALS

As the first example we consider the square-well potential

$$\mathcal{A}(t)=\begin{cases}\mathcal{A} & (0<t<\tau)\\ 0 & (t<0, t>\tau)\ .\end{cases} \tag{6}$$

For $t<0$ and $t>\tau$, Eq. (5) becomes

$$H=p\sigma_x-m\sigma_z \ . \tag{7a}$$

Equation (7a) has eigenvalues

$$E_\pm=\pm\sqrt{m^2+p^2} \ , \tag{7b}$$

and eigenvectors

$$|p,+\rangle=\frac{1}{\sqrt{(m+E)^2+p^2}}\begin{bmatrix}m+E\\ p\end{bmatrix} , \tag{7c}$$

$$|p,-\rangle=\frac{1}{\sqrt{(m-E)^2+p^2}}\begin{bmatrix}m-E\\ p\end{bmatrix} . \tag{7d}$$

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For $0<t<\tau$, Eq. (5) is given by

$$H_i=(p-e\mathcal{A})\sigma_x+m\sigma_z \ . \tag{8a}$$

It has eigenvalues

$$E_{i\pm}=\pm\sqrt{m^2+(p-e\mathcal{A})^2} \ , \tag{8b}$$

and eigenvectors

$$|p,+\rangle_i=\frac{1}{\sqrt{(m+E_i)^2+(p-e\mathcal{A})^2}}\begin{bmatrix} m+E_i \\ p-e\mathcal{A}\end{bmatrix} \ , \tag{8c}$$

$$|p,-\rangle_i=\frac{1}{\sqrt{(m-E_i)^2+(p-e\mathcal{A})^2}}\begin{bmatrix} m-E_i \\ p-e\mathcal{A}\end{bmatrix} \ . \tag{8d}$$

The probability that a pair is produced after the interaction can be calculated as follows [4].

Suppose that for $t<0$, the exact wave function is in the state

$$|\psi\rangle=|p,-\rangle, \quad t<0 \ ; \tag{9}$$

then the wave function for $0<t<\tau$ can be expanded in terms of $|p,\pm\rangle_i$ with constant coefficients

$$|\psi\rangle=a_-|p,-\rangle_i e^{iE_it}+a_+|p,+\rangle_i e^{-iE_it}, \quad 0<t<\tau \tag{10}$$

and for $t>\tau$, in terms of $|p,\pm\rangle$

$$|\psi\rangle=b_-|p,-\rangle e^{iEt}+b_+|p,+\rangle e^{-iEt}, \quad t>\tau \ . \tag{11}$$

The coefficients in Eqs. (10) and (11) are obtained by imposing the condition that the wave function be continuous at $t=0$ and $t=\tau$.

The pair-production probability is

$$P=|b_+|^2 \ ; \tag{12}$$

b_+ can be obtained by equating Eqs. (10) and (11) at $t=\tau$ and upon forming the scalar product

$$b_+=a_-\langle p,+|p,-\rangle_i e^{i(E_i+E)\tau} +a_+\langle p,+|p,+\rangle_i e^{-i(E_i-E)\tau} \ , \tag{13}$$

where

$$a_-={}_i\langle p,-|p,-\rangle, \quad a_+={}_i\langle p,+|p,-\rangle \tag{14}$$

are obtained by imposing the continuity at $t=0$.

The pair-production probability is given by

$$P=|b_+|^2=\frac{(mc^2)^2(e\mathcal{A})^2}{E_i^2E^2}\sin^2\left[\frac{E_i\tau}{\hbar}\right] \ . \tag{15}$$

If we express E_i as a function of E and $\mathcal{A}$ and write the probability in terms of the adimensional variables

$$x=\frac{E}{mc^2}, \quad y=\frac{e\mathcal{A}}{mc^2}, \quad T=\frac{\tau mc^2}{\hbar} \ , \tag{16}$$

we get for P

$$P=\frac{y^2}{x^2[x^2-2y\sqrt{x^2-1}+y^2]} \times\sin^2[(x^2-2y\sqrt{x^2-1}+y^2)^{1/2}T] \ . \tag{17}$$

In Figs. 1 and 2 we show graphs for P versus the kinetic energy of the electron (or the positron) for different values of the parameters y and T.

Equation (17) has the meaning of a pair-production distribution probability as function of the energy and clearly shows a resonance structure.

We notice that if we increase the value of the parameter y, $\mathcal{A}$ gets bigger. This means that if we supply more energy to the vacuum, the kinetic energy of the particles produced also increases. Similarly, if we increase the value of the parameter T, for a given value of y, we increase the time available for interaction and we produce more resonances.

We consider as the second example the case of a constant electric field $\mathcal{E}_0$.

The probability of electron-positron pair production in a constant electric field per unit time per unit volume is given by [3]

$$P\sim\exp\left[-\frac{\pi m^2}{e\mathcal{E}_0}\right] \ . \tag{18}$$

For a constant electric field, we compute below the probability that at $t=\tau$ a pair with a given energy is produced.

For this case, Eqs. (4a) and (4c) are changed to

$$i\dot{u}_1=mu_1+(p-e\mathcal{E}t)u_3 \ , \tag{19a}$$

$$i\dot{u}_3=(p-e\mathcal{E}t)u_1-mu_3 \ . \tag{19b}$$

Let

$$z=\frac{(p-e\mathcal{E}t)}{m}, \quad \gamma=m^2/e\mathcal{E} \ , \tag{19c}$$

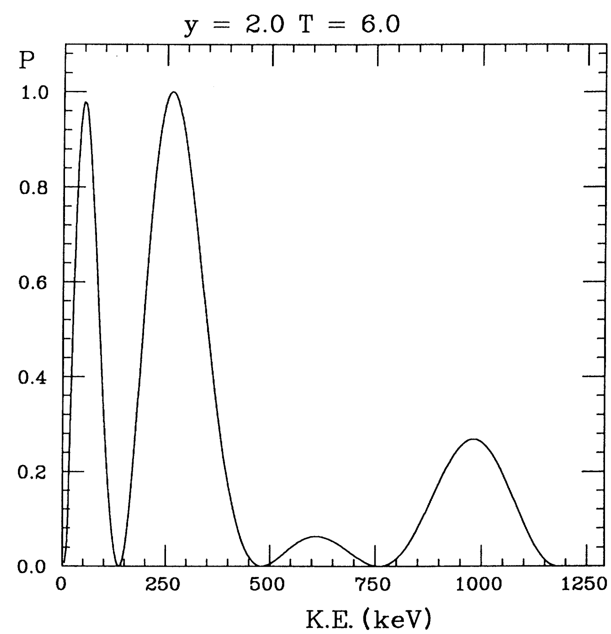

FIG. 1. Probability of pair production as a function of the electron kinetic energy in a square-well potential: $y=2.0$, $T=6.0$.

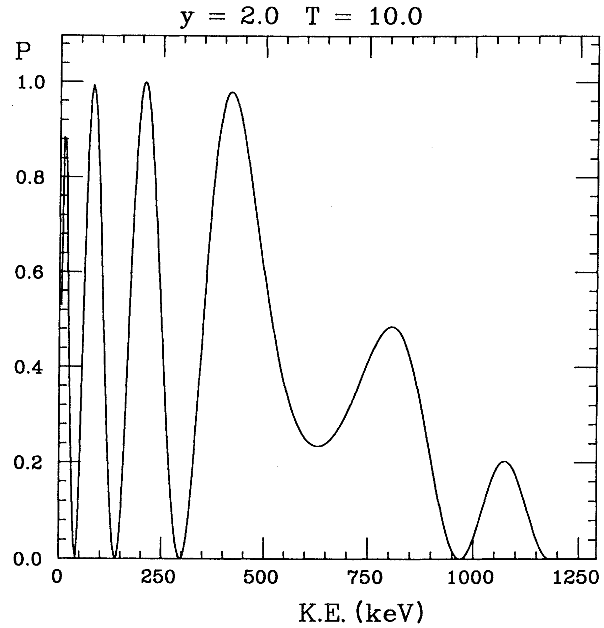

FIG. 2. Probability of pair production as a function of the electron kinetic energy in a square-well potential: $y=2.0$, $T=10.0$.

so that Eqs. (19) become

$$\dot{u}_1=i\gamma u_1+i\gamma z u_3 , \tag{20a}$$

$$\dot{u}_3=i\gamma z u_1-i\gamma u_3 . \tag{20b}$$

We can reduce the system of Eqs. (20) to a second order differential equation by making the following two separate transformations. First define

$$u_1+u_3=a_1 , \tag{21a}$$

$$u_1-u_3=b_1 , \tag{21b}$$

and then

$$a_1=\exp\left[i\gamma\frac{z^2}{2}\right]a , \tag{22a}$$

$$b_1=\exp\left[-i\gamma\frac{z^2}{2}\right]b . \tag{22b}$$

After eliminating b, we obtain a second order equation for a which reads

$$\ddot{a}+2\beta z\dot{a}-\beta^2 a=0 , \tag{23a}$$

with

$$\beta=i\frac{m^2}{e\mathcal{E}} . \tag{23b}$$

If we set

$$a=\chi(z)\phi(z) , \tag{24a}$$

with

$$\phi(z)=\exp\left[-\beta\frac{z^2}{2}\right] , \tag{24b}$$

and

$$\xi=\lambda z , \tag{24c}$$

Eq. (23a) can be further reduced:

$$\frac{d^2\chi}{d\xi^2}-\left[\epsilon+\frac{\xi^2}{4}\right]\chi=0 , \tag{25a}$$

with

$$\epsilon=\frac{1}{2}\left[1+i\frac{m^2}{e\mathcal{E}}\right] . \tag{25b}$$

Hence the general solution to the system of Eqs. (19) is [5]

$$u_1=\tfrac{1}{2}e^{i\gamma z^2/2}\{A[M_1+2z(\epsilon-\tfrac{1}{2})M_2]+B[e^{i\pi/4}z\sqrt{2\gamma}M_3+e^{-i\pi/4}\sqrt{2/\gamma}M_4]\} , \tag{26a}$$

$$u_3=\tfrac{1}{2}e^{i\gamma z^2/2}\{A[M_1-2z(\epsilon-\tfrac{1}{2})M_2]+B[e^{i\pi/4}z\sqrt{2\gamma}M_3-e^{-i\pi/4}\sqrt{2/\gamma}M_4]\} , \tag{26b}$$

with

$$\gamma=\frac{m^2}{e\mathcal{E}},\quad \epsilon=\frac{1}{2}+i\frac{1}{2}\frac{m^2}{e\mathcal{E}},\quad z=\frac{(p-e\mathcal{E}t)}{m} , \tag{26c}$$

$$M_1=M(-\tfrac{1}{2}\epsilon+\tfrac{1}{4},\tfrac{1}{2},-i\gamma z^2) , \tag{26d}$$

$$M_2=M(-\tfrac{1}{2}\epsilon+\tfrac{5}{4},\tfrac{3}{2},-i\gamma z^2) , \tag{26e}$$

$$M_3=M(-\tfrac{1}{2}\epsilon+\tfrac{3}{4},\tfrac{3}{2},-i\gamma z^2) , \tag{26f}$$

$$M_4=M(-\tfrac{1}{2}\epsilon+\tfrac{3}{4},\tfrac{1}{2},-i\gamma z^2) . \tag{26g}$$

The constants A and B are obtained imposing the condition that at $t=0$ the wave function ψ is in the state of negative energy $|p,-\rangle$

$$|p,-\rangle=\begin{bmatrix}\psi_1(0)\\ \psi_2(0)\end{bmatrix} \tag{27a}$$

with

$$\psi_1(0)=\frac{1-\sqrt{1+w^2}}{[2(1+w^2-\sqrt{1+w^2})]^{1/2}} , \tag{27b}$$

$$\psi_2(0)=\frac{w}{[2(1+w^2-\sqrt{1+w^2})]^{1/2}} , \tag{27c}$$

where $w=p/m$.

The coefficient A is given by

$$A=\frac{\gamma_1 e^{-i\gamma w^2/2}}{\phi_1} , \tag{28a}$$

with

$$\gamma_1=\psi_1(0)+\psi_2(0)-\theta_1[\psi_1(0)-\psi_2(0)] , \tag{28b}$$

$$\theta_1 = \frac{e^{i\pi/2}\gamma w M_3}{M_4} , \tag{28c}$$

$$\phi_1 = M_1 - 2w(\epsilon - \tfrac{1}{2})\theta_1 M_2 . \tag{28d}$$

The coefficient B is given by

$$B = \frac{\gamma_2 e^{-i\gamma w^2/2}}{\phi_2} , \tag{29a}$$

with

$$\gamma_2 = [\psi_1(0) + \psi_2(0)]\theta_2 - [\psi_1(0) - \psi_2(0)] , \tag{29b}$$

$$\theta_2 = \frac{2w(\epsilon - \frac{1}{2})M_2}{M_1} , \tag{29c}$$

$$\phi_2 = \theta_2 e^{i\pi/4}\sqrt{2\gamma} w M_3 - e^{-i\pi/4}\sqrt{2/\gamma} M_4 . \tag{29d}$$

To find the probability that at time $t=\tau$, an electron which was at time $t=0$ in a state of negative energy is found to be in the state of positive energy, we need

$$|p - e\mathcal{E}\tau, +\rangle = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} , \tag{30a}$$

with

$$d_1 = \frac{1+w_1}{w_2} , \tag{30b}$$

$$d_2 = \frac{w-v}{w_2} , \tag{30c}$$

$$w_1 = \sqrt{1+(w-v)^2}, \quad w_2 = \sqrt{(1+w_1)^2+(w-v)^2} , \tag{30d}$$

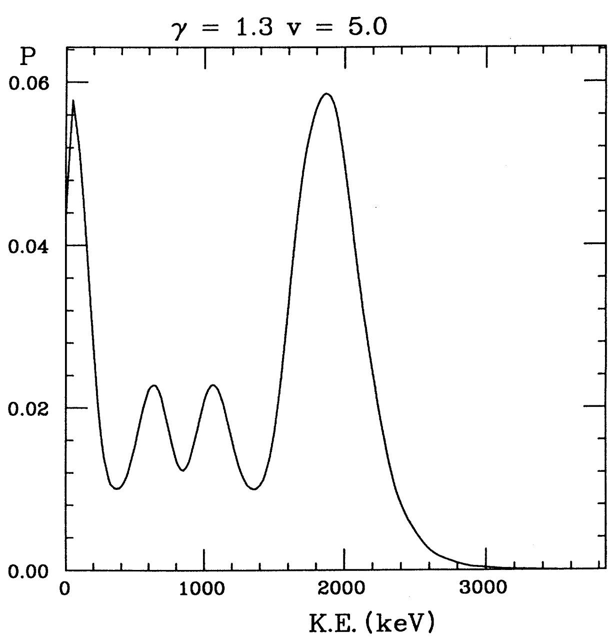

FIG. 3. Probability of pair production as a function of the electron kinetic energy for a constant electric field: $\gamma=1.3$, $v=5$.

and $w=p/m$, $v=e\mathcal{E}T/m$.

Finally the pair-production probability can be expressed as

$$P = |u_1 d_1 + u_3 d_2|^2 . \tag{31}$$

In Fig. 3 we show P versus the kinetic energy of the electron or positron for $\gamma=1.3$ and $v=5.0$. As we see in Eq. (18), γ must be of order one to have a significant pair production.

III. APPLICATIONS TO HEAVY-ION COLLISIONS

In the past decade heavy ion-atom collision experiments at energies close to the Coulomb barrier (5.7–6.2 MeV/nucleon) have revealed a narrow resonance structure at about 300 keV c.m. energy in the positron spectrum of various supercritical and subcritical collision systems [6–9]. These lines occur in all collision systems with combined charges $Z_u = Z_1 + Z_2$ from 164 up to 188 with similar energies. Moreover there is strong indication of more than one line per system and that the positron lines are in coincidence with electron lines of the same energy [10,11]. (General reviews on the subject may be found in Refs. [12–15].)

In a crude picture when one uranium ion and one uranium atom approach closer than a critical distance R_{cr}, a quasiatom is transiently formed and pair production becomes feasible. If we do not consider the pair-production process due to nuclear excitations, we can distinguish two principal pair-production mechanisms: (i) dynamically induced positrons, created by the vacancies in the negative energy continuum; (ii) spontaneous positron emission, a process that occurs when the preionized lowest electronic level enters the negative energy continuum and becomes a resonance that might eventually decay.

However, if we require Rutherford trajectories for the collision systems (the scattering is quasielastic), the dive time τ is of order 10^{-21} sec, whereas the spontaneous decay width corresponds to a time of the order 10^{-19} sec. Thus the measured spectrum is expected to be mainly dominated by the distribution of the dynamically induced positrons.

A variety of theoretical explanations have been suggested as a source of these lines. The most popular ones are the following: (i) spontaneous positron emission [16,17] (supercritical systems), (ii) decay of a neutral particle ("axion") [18–22], (iii) a new quantum electrodynamics phase [23–28], and (iv) interference effects among different amplitudes [29,30].

We want to point out here that also the dynamically induced positron process (specifically transitions between negative and positive continuous energy states) can give origin to a resonance structure and not only to a broad structureless background.

Obviously the potentials we have examined are only illustrative and perhaps too simple to be able to "explain" the e^+e^- peaks in the heavy atom collision experiments. However, it is interesting that the approximate numerical values using these simple potentials hint rather strongly that the observed resonances may indeed be due to the intense electromagnetic fields created by the colliding nuclei [1,31]. For example, when we solve the Dirac equation for the square-well potential, the values for the critical range R_{cr} and the time duration τ needed to obtain resonances at "correct" energies are consistent with other estimates.

R_{cr} is expected to be much less than 135 fm and the calculated values are in the range 30–50 fm in different approximations (Ref. [13], p. 310). The characteristic duration of a heavy-ion collision, i.e., the time τ the nuclei spend together closer than the critical distance, is found to lie between 10^{-20} and 10^{-21} sec for different models (Ref. [13], p. 330).

In our first example—square-well potential—we get a resonance structure in the energy interval between 0 and 1000 keV when $y=2.0$. If we assume

$$e\mathcal{A}=\frac{(Ze)^2}{AR_{cr}}\ , \tag{32}$$

where for uranium atoms $Z=92$ and $A=238$, we get $R_{cr}\sim 50$ fm, a number consistent with other estimates referred to above. Figure 1 is obtained for $T=6.0$ which corresponds to a duration time τ of the order 7.5×10^{-21} sec; Fig. 2 where $T=10$ gives a τ of order 1.25×10^{-20} sec, once again consistent with other estimates.

In our second example—constant electric field—we have very wide resonances in energy.

$\gamma=1.3$ gives an electric field $\mathcal{E}_0$ of the order 10^{16} V/cm, that substituted in the expression for $v=5$ gives a T of the order 7.9×10^{-21} sec. If we assume

$$e\mathcal{E}_0 cT=\frac{(Ze)^2}{AR_{cr}}\ , \tag{33}$$

we get for uranium $R_{cr}\sim 21$ fm, again not far from the other estimates.

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