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# Quantum theoretical approach to meson production in nuclear media via Cherenkov mechanisms

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In this paper a quantum theoretical approach to single scalar and pseudoscalar meson production via mesonic Cherenkov mechanisms in homogeneous nuclear media is presented. The quantum mesonic Cherenkov coherence condition as well as the quantum rate of emission for Cherenkov mesons are derived. Some numerical estimates of these characteristic quantities as well as their semiclassical limits are discussed.

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## I. INTRODUCTION

The availability of relativistic hadron accelerators has revived the interest in the theoretical description of the production of photons and pions during the passage of relativistic hadrons through nuclear media via Cherenkov mechanisms [1–3]. Recent work [1] about Cherenkov $\pi$ production is based on a field-theoretical calculation of the $\pi$ propagator in nuclear matter. Our own recent work concerns Cherenkov $\gamma$ production [2] and a semiclassical treatment of Cherenkov $\pi$ production [3]. We refer the reader to the reference lists in our papers [2–4] for a collection of earlier work on the subject of $\pi$ production in nuclear media through Cherenkov processes.

Our semiclassical approach to the Cherenkov $\pi$ production [3] was based on a modified Klein-Gordon equation with a classical pointlike source term. This semiclassical treatment cannot account for the pseudoscalar or pseudovector character of the $\pi$-nucleon coupling since intrinsic parity is in fact a quantum concept; it has, however, the advantage that absorption effects can be taken into account in a simple and consistent way. In the present paper we present a quantum approach of the spontaneous single pseudoscalar meson production via the mesonic Cherenkov effect for nonabsorptive nuclear media. Then the single pseudoscalar meson emission, including both pseudoscalar and pseudovector couplings, is investigated. The absorption of pions is taken into account properly by the inclusion of a semiclassical absorption factor.

## II. THE REFRACTIVE INDEX OF MESONS IN A NUCLEAR MEDIUM

The scattering of a meson $M$ with rest mass $m_M$ and energy $\omega$ in a nuclear medium composed of nucleons can be described in terms of a refractive index $n(\omega)$ which relates the meson wave number $k$ in the nuclear medium to the free wave number $(\omega^2-m_M^2)^{1/2}$ by the relation $n(\omega)=k/(\omega^2-m_M^2)^{1/2}$. We apply here the Foldy-Lax multiple-scattering theory [5,6] the essential result of which is that the mesonic coherent scalar (or pseudoscalar) wave equation in a medium can be written as

$$[\nabla^2+(\omega^2-m_M^2)+4\pi\rho C\bar{f}(MN\leftarrow MN)]\varphi(\mathbf{r})=0 .$$

Here $\bar{f}(MN\leftarrow MN)$ is the configurational average of the elastic meson-nucleon scattering amplitude in the forward direction (even inelastic scattering might be present); $\varphi(\mathbf{r})$ is the coherent stationary mesonic wave in the medium. The factor $C$ is defined as the ratio between of the effective and the coherent mesonic field. If the scatterers are distributed completely at random, this factor $C$ will be unity. Hence, from the Foldy-Lax approach the following dispersion relation for the meson in a medium is obtained,

$$k^2=\omega^2-m_M^2+4\pi\rho C\bar{f}(MN\leftarrow MN) , \tag{1}$$

or, equivalently the following energy-momentum relation

$$k=n(\omega)(\omega^2-m_M^2)^{1/2} , \tag{2}$$

with

$$n^2(\omega)=1+\frac{4\pi\rho}{\omega^2-m_M^2}C\bar{f}^{MN}(\omega) . \tag{2a}$$

Here (since we have chosen $\hbar=c_\hbar=1$) $\omega$, $k$, and $m_M$ are the energy, momentum, and the rest mass of the physical meson inside the nuclear medium, respectively.

Some remarks have to be added. First, we emphasize that the scattering amplitude in Eqs. (1)–(2a) must be evaluated at the wave number $k$ inside the medium rather than at the free wave number $(\omega^2-m_M^2)^{1/2}$. Second, more detailed considerations show that some conditions have to be fulfilled such that the scattering of a meson from one of the constituents of the nuclear medium can be treated as a two-body rather than a many-body problem. Those problems as well as the conditions for the adiabatic approximation for the motion of the nuclear scatterers have been systematically discussed by Fesh-

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bach [7] in connection with the justification of the nuclear optical potential. The reader should consult the literature quoted above and also the references Goldberger and Watson [8], Johnson and Bethe [9], and Ericson and Weise [10].

## III. QUANTUM THEORETICAL APPROACH TO THE MESONIC CHERENKOV MECHANISM AND SEMICLASSICAL LIMITS

The quantum theory of the usual electromagnetic Cherenkov radiation in a dielectric medium was originally developed by Ginsburg [11]. Later, other authors also treated this problem (see references quoted in the textbook of Zrelov [12]). We use in the following an analogous procedure for developing a quantum theoretical approach of the *nuclear mesonic Cherenkov radiation* (NMCR) from spin-$\frac{1}{2}$ particles moving as sources in a nuclear medium.

### A. Quantum NMCR conditions

First, we investigate the radiation condition. In Fig. 1 we display schematically the Cherenkov emission process of a meson $M$ (with energy $\omega$ and momentum $\mathbf{k}$) that is radiated from an incident baryon $B_1$ (with energy $E_1$ and momentum $\mathbf{p}_1$) that itself goes over into a final baryon $B_2$ (with energy $E_2$ and momentum $\mathbf{p}_2$). The energy-momentum relation in the nuclear medium requires

$$E_1=E_2+\omega, \quad \mathbf{p}_1=\mathbf{p}_2+\mathbf{k} \ . \tag{3}$$

For initial and final baryons $B_1$ and $B_2$ we assume that the usual mass-shell relations

$$E_i^2-|\mathbf{p}_i^2|=M_i^2, \quad i=1,2 \ , \tag{4}$$

are also valid inside the nuclear medium, while for the meson $M$ we take the energy-momentum relation (2). We are interested in the angles $\theta_{1k}$ and $\theta_{12}$, for which one obtains ($|\mathbf{p}_i|=p_i$, $|\mathbf{k}|=k$)

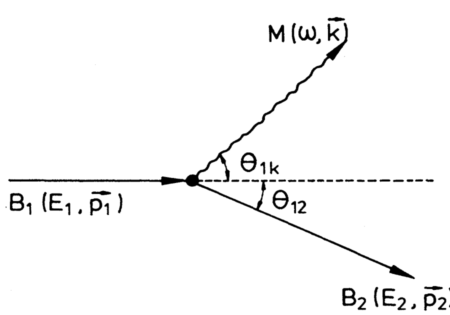


FIG. 1. Diagram for single meson production ($B_1 \rightarrow B_2 M$) via Cherenkov mechanism.

$$\cos\theta_{1k}=\frac{v_{\text{ph}}^M(\omega)}{v_1}+\frac{k^2-\omega^2+M_2^2-M_1^2}{2p_1k} =\frac{v_{\text{ph}}^M(\omega)}{v_1}+O\left[\frac{1}{p_1}\right] , \tag{5a}$$

$$\cos\theta_{12}=\frac{p_1^2+p_2^2-k^2}{2p_1p_2}\simeq 1-O\left[\frac{1}{p_1}\right] . \tag{5b}$$

$v_{\text{ph}}^M(\omega)$ is the mesonic phase velocity defined as $v_{\text{ph}}^M=\omega/k$. Therefore, the interacting system (nuclear medium plus initial hadron plus physical meson) can in general make real transitions from the initial state $|A,\mathbf{p}_1;\bar{0}-\bar{0}-\bar{0}\rangle$ (the state without physical mesons) to other states, e.g., a final state $|A,\mathbf{p}_2;\bar{0}-\cdots\bar{1}_k\cdots\bar{1}_k\cdots-\bar{0}\rangle$ with some physical mesons present in the nuclear medium. Our interest is in the emission of a single real meson by the initial hadron without the excitation of the medium,

$$|A,\mathbf{p}_1;\bar{0}-\bar{0}-\bar{0}\rangle \rightarrow |A,\mathbf{p}_2;\bar{0}-\bar{1}_k-\bar{0}\rangle \ . \tag{6}$$

The transition (6) will be identified as the lowest order process for the Cherenkov emission of a meson. This can be seen from Eqs. (5) since from the condition of the angle $\theta_{1k}$ to be real ($|\cos\theta_{1k}|\leq 1$) we get

$$v_{\text{ph}}^M(\omega)\leq v_1(1+O(1/p_1)) \ . \tag{7}$$

This is (except for the quantum correction which is very small for high energies of the incident hadron) just the *classical coherence condition for Cherenkov radiation.* More explicitly we have the *quantum coherence condition*

$$\left|\frac{v_{\text{ph}}^M(\omega)}{v_1}+\frac{n^2(\omega)(\omega^2-m_M^2)-\omega^2+M_2^2-M_1^2}{2p_1 n(\omega)(\omega^2-m_M^2)^{1/2}}\right|\leq 1 \ . \tag{8}$$

The second term can be seen to take into account recoil effects of the initial and final baryons with masses $M_1$ and $M_2$. It is straightforward to see that the conditions (7) and (8) cannot be fulfilled in the vacuum where $n(\omega)=1$.

Now, the threshold velocity $v_1^0$, i.e., the lowest projectile velocity for which the condition (8) is fulfilled, is given by

$$v_1^0=\frac{v_{\text{ph}}^M(\omega)}{1+F^2}+\frac{F}{(1+F^2)^{1/2}}\left[1-\frac{[v_{\text{ph}}^M(\omega)]^2}{1+F^2}\right]^{1/2} , \tag{8a}$$

where $F$ is given by

$$F\equiv\frac{k}{2M_1}\left[1-[v_{\text{ph}}^M(\omega)]^2+\frac{M_2^2-M_1^2}{k^2}\right] . \tag{8b}$$

(The second possibility for $v_1^0$ is unphysical.)

### B. Transition probability for NMCR

The transition probability (per unit time) from the initial state (6) to the final state (6) of the NMCR process is given in first order perturbation theory by the golden rule,

$$W_{fi}=2\pi|\tilde{H}_{fi}|^2\delta(E_1-E_2-\omega) \; , \tag{9}$$

where $\tilde{H}_{fi}$ are the (averaged) matrix elements of the Hamiltonian describing the interaction between the initial hadronic meson source and the effective quantized mesonic field. The number of mesons emitted per unit time by the Cherenkov process into the energy interval $(\omega,\omega+d\omega)$ is then given by

$$\frac{dN}{d\omega}=2\pi\int|\tilde{H}_{fi}|^2\frac{d\Omega}{4\pi}\rho(\omega)\delta(E_1-E_2-\omega) \; , \tag{10}$$

where $\rho(\omega)$ is the density of final physical meson states,

$$\rho(\omega)=\frac{1}{2\pi^2}k^2\frac{dk}{d\omega} \; , \tag{11}$$

and $d\Omega=d\varphi\sin\vartheta_{1k}\,d\vartheta_{1k}$ is the solid angle element into which the Cherenkov meson is emitted. From Eq. (10) and the relation [see Eq. (5a)]

$$\omega\simeq v_1 k(\omega)\cos\theta_{1k} \; , \tag{12}$$

we obtain after integration

$$\frac{dN}{d\omega}=\frac{1}{2\pi v_1}|\tilde{H}_{fi}|^2 k\frac{dk}{d\omega}\Theta(1-v_{\rm ph}^M(\omega)/v_1) \; , \tag{13}$$

where $\Theta$ is the Heavyside step function. The matrix element has to be taken at $\cos\theta_{1k}$ defined by Eq. (5a).

(i) *Single pseudoscalar meson emission via NMCR with pseudoscalar coupling* $G_{MB_1B_2}$. Now, we calculate explicitly the matrix element $H_{fi}$ for the Cherenkov emission of a pseudoscalar meson from a spin-$\frac{1}{2}$ baryon that moves in a nuclear medium (see Fig. 1). In this case the matrix element $H_{fi}$ is given by

$$H_{fi}=iG_{MB_1B_2}\langle A,\mathbf{p}_2;\bar{0}-\bar{1}_k-\bar{0}|\gamma_5\varphi|A,\mathbf{p}_1,\bar{0}-\bar{0}-\bar{0}\rangle \; , \tag{14}$$

where $G_{MB_1B_2}$ is the coupling constant for the pseudoscalar coupling between meson $M$ and the baryons $B_1$ and $B_2$ in the nuclear medium, $\gamma_5$ is a Dirac matrix, and $\varphi$ is the coherent pseudoscalar mesonic field in the medium. The possibility of the nuclear medium changing its state was excluded in the NMCR model from the beginning.

Next, the matrix element (14) is evaluated by using the plane wave spinors $u_i(\mathbf{p}_i)\exp(-i\mathbf{p}\cdot\mathbf{r})$, $i=1,2$, for the initial and final baryon as well as the Fourier decomposition $\varphi=\sum(Q_k\varphi_k+Q_k^*\varphi_k^*)$ in the nuclear medium where $\varphi_k^*=n^{-1}(\omega)\exp(-i\mathbf{k}\cdot\mathbf{r})$ and $Q_k$ and $Q_k^*$ are respectively the annihilation and creation operators for a pseudoscalar meson $M$ with momentum $\mathbf{k}$ in the nuclear medium. Then, by introducing the standard normalized matrix element $\langle\bar{1}_k|Q_k^*|\bar{0}\rangle=(2\omega_k)^{-1/2}$ and $\langle\bar{1}_k|Q_k|\bar{0}\rangle=0$ we obtain after integration in Eq. (14)

$$H_{fi}=i\frac{G_{MB_1B_2}}{n(\omega)}\frac{1}{\sqrt{2\omega}}\bar{u}_2(\mathbf{p}_2)\gamma_5 u_1(\mathbf{p}_1) \; . \tag{15}$$

Then, summing over the final spin states of the baryon and averaging over the initial spin states, we get

$$|\tilde{H}_{fi}|^2=\frac{G^2_{MB_1B_2}}{n^2(\omega)}\frac{1}{2\omega}S(E_1,\omega) \; , \tag{16}$$

where

$$\begin{aligned}S(E_1,\omega)&=\frac{1}{2}\sum_f[\bar{u}_2(\mathbf{p}_2)\gamma_5u_1(\mathbf{p}_1)][\bar{u}_1(\mathbf{p}_1)\gamma_5\bar{u}_2(\mathbf{p}_2)]\\&=\frac{k^2-\omega^2+(M_1-M_2)^2}{2E_1(E_1-\omega)} \; ,\end{aligned} \tag{17}$$

where we used the Dirac spinors

$$u_i(\mathbf{p}_i)=N_i\begin{bmatrix}\chi_s\\ \dfrac{\sigma\cdot\mathbf{p}_i\chi_s}{E_i+M_i}\end{bmatrix} \; ,$$

$$N_i\equiv\left[\frac{E_i+M_i}{2E_i}\right]^{1/2} \; , \quad i=1,2 \; ,$$

to get the equivalent two-component matrix elements

$$\bar{u}_2(\mathbf{p}_2)\gamma_5u_1(\mathbf{p}_1)=N_1N_2\chi_s^+\left[\frac{\sigma\cdot\mathbf{p}_1}{E_1+M_1}-\frac{\sigma\cdot\mathbf{p}_2}{E_2+M_2}\right]\chi_s \; .$$

Here $\sigma\equiv(\sigma_1,\sigma_2,\sigma_3)$ are the usual Pauli matrices and $\chi_s\equiv\{\binom{1}{0},\binom{0}{1}\}$ are the two-component Pauli spinors.

Using now Eqs. (13), (16), and (17) it follows for the production rate, i.e., the number of mesons emitted per unit time into the energy interval $(\omega,\omega+d\omega)$,

$$\begin{aligned}\frac{dN^{PS(PS)}}{d\omega}&=\frac{|G_{MB_1B_2}|^2}{4\pi v_1 n^2(\omega)}\frac{dk}{d\omega}\frac{1}{v_{\rm ph}^M(\omega)}\\&\quad\times\frac{[n^2(\omega)-1]\omega^2-m_M^2n^2(\omega)+(M_1-M_2)^2}{4E_1(E_1-\omega)} \; .\end{aligned} \tag{18}$$

(ii) *Single pseudoscalar meson emission via NMCR and pseudovector coupling* $f_{MB_1B_2}$. The matrix element $H_{fi}$ in Eq. (13) for the Cherenkov emission of a pseudoscalar meson with the pseudoscalar coupling are given by

$$H_{fi}=i\frac{f_{MB_1B_2}}{m_M}\langle A,\mathbf{p}_2;\bar{1}_k|\gamma_\mu\gamma_5\partial_\mu\varphi|A,\mathbf{p}_1,\bar{0}\rangle \; ,$$

where $f_{MB_1B_2}$ is the pseudovector coupling constant for the meson $M$ and the initial and final baryons $B_1$ and $B_2$ (see again Fig. 1). $\gamma_\mu$ and $\gamma_5$ are the Dirac matrices, and $\varphi$ denotes the coherent mesonic field corresponding to the meson $M$ in the nuclear medium. Therefore, by a procedure similar to that carried out in the pseudoscalar coupling case we obtain the production rate

$$\begin{aligned}\frac{dN^{PS(PV)}}{d\omega}&=\frac{1}{4\pi v_1}\left[\frac{f_{MB_1B_2}}{m_M}\right]^2\frac{4M_1M_2}{n^2(\omega)}\frac{dk}{d\omega}\frac{1}{v_{\rm ph}^M(\omega)}\\&\quad\times\frac{[n^2(\omega)-1]\omega^2-m_M^2n^2(\omega)+(M_1-M_2)^2}{4E_1(E_1-\omega)} \; .\end{aligned} \tag{19}$$

From comparison of Eqs. (18) and (19) we see that the pseudoscalar and pseudovector couplings are equivalent if and only if $G_{MB_1B_2}=f_{MB_1B_2}2(M_1M_2)^{1/2}/m_M$, also in the nuclear medium.

(iii) *Single scalar meson emission via NMCR.* For comparison we present now the rate for the single scalar meson emission via a NMCR mechanism. Starting with the matrix element of the form

$$H_{fi}=iG_{MB_1B_2}\langle A,\mathbf{p}_2;\bar{0}-\bar{1}_k-\bar{0}|\varphi|A,\mathbf{p}_1,\bar{0}-\bar{0}-\bar{0}\rangle \ ,$$

and evaluating it as in the preceding coupling cases (i) and (ii) we obtain the following production rate:

$$\frac{dN^s}{d\omega}=\frac{1}{4\pi v_1}\frac{|G^s_{MB_1B_2}|^2}{n^2(\omega)}\frac{dk}{d\omega}\frac{1}{v^M_{\rm ph}(\omega)}\times\frac{[n^2(\omega)-1]\omega^2-m_M^2n^2(\omega)+(M_1+M_2)^2}{4E_1(E_1-\omega)} \ . \tag{20}$$

The essential difference between the expressions (18) and (20) for $dN^{ps}/d\omega$ and $dN^s/d\omega$ comes from the fact that in the last term of (18) the mass difference of the initial and final baryon occurs whereas (20) has the sum of these masses.

### C. Semiclassical limits of NMCR

Now, it is interesting to present here Eqs. (18)–(20) in the semiclassical limit: $v_1\simeq v_2\simeq v$, $M_1=M_2=M$. We get

$$\frac{dN^{PS(PS)}}{d\omega}\simeq\frac{1-v^2}{v}\frac{G^2_{MBB}}{4\pi}\eta(\omega) \ , \tag{21}$$

$$\frac{dN^{PS(PV)}}{d\omega}\simeq\frac{1-v^2}{v}\frac{f^2_{MBB}}{4\pi}\left[\frac{2M}{m_M}\right]^2\eta(\omega) \ , \tag{22}$$

$$\frac{dN^s}{d\omega}\simeq\frac{1-v^2}{v}\frac{[G^s_{MBB}]^2}{4\pi}\left[\eta(\omega)+\frac{1}{n^2(\omega)}\frac{[dk/d\omega]}{v^M_{\rm ph}(\omega)}\right] \ , \tag{23}$$

where

$$\eta(\omega)\equiv\frac{\omega^2-m_M^2}{4M^2}\sin^2\theta_c^{\max}\frac{[dk/d\omega]}{v^M_{\rm ph}(\omega)} \ , \tag{24a}$$

$$\sin^2\theta_c^{\max}=\sin^2\theta_c(v=1)\simeq1-[v^M_{\rm ph}(\omega)]^2 \ . \tag{24b}$$

Now, the results (21)–(23) can be compared with the semiclassical result [3] on NMCR [see also Eq. (I.52) in Ref. [4] or Eq. (3.6) in Ref. [13] for $a=0$]

$$\frac{dN}{d\omega}=\frac{(1-v^2)}{v}G^2_{\rm eff}\frac{1}{|\gamma(\omega)|^2} \ , \tag{25}$$

for the nonabsorptive nuclear medium case. Then, we see that the energy behavior, which is essentially given by the factor $(1-v^2)$, is the same for all types of couplings considered in Eqs. (21)–(23) and (25). Moreover, we see that the semiclassical limits (21)–(23) of the mesonic Cherenkov quantum spectra are in agreement with the semiclassical result (25) (for $a=0$) if and only if

$$\gamma(\omega)=\eta^{-1/2}(\omega), \quad G^2_{\rm eff}\equiv\frac{G^2_{MBB}}{4\pi} \tag{26a}$$

for the single pseudoscalar meson emission as NMCR with the pseudoscalar coupling,

$$\gamma(\omega)=\eta^{-1/2}(\omega), \quad G^2_{\rm eff}\equiv\left[\frac{2M}{m_M}\right]^2\frac{f^2_{MBB}}{4\pi} \tag{26b}$$

for the single pseudoscalar meson production as NMCR with the pseudovector coupling, and

$$\gamma(\omega)=\left[\eta(\omega)+n^{-2}(\omega)\frac{k}{\omega}\frac{dk}{d\omega}\right]^{-1/2}, \quad G^2_{\rm eff}\equiv\frac{|G^s_{MBB}|^2}{4\pi} \ , \tag{26c}$$

for the scalar meson emission as NMCR.

Now, from Eqs. (25)–(26) it is easy to see that the quantum approach where the pseudoscalarity of pions can be taken into account explicitly in fact modifies the semiclassical spectrum $dN/d\omega$ obtained in Ref. [3] for Cherenkov pions by a "quantum factor" $\eta(\omega)(1-\omega/E_1)^{-1}$ [cf. Eqs. (18) and (24a)].

## IV. NUMERICAL ESTIMATION

As an example, let us now consider the $\pi^0$ production as NMCR in the nuclear reactions

$$p+{}^{120}\text{Sn}\to\pi^0+p+{}^{120}\text{Sn} \ , \tag{27a}$$

$$p+{}^{208}\text{Pb}\to\pi^0+p+{}^{208}\text{Pb} \tag{27b}$$

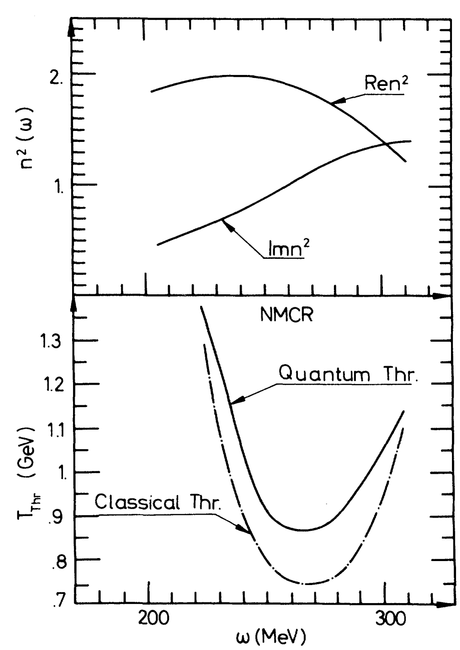


FIG. 2. The refractive index $n^2(\omega)$ and the classical and quantum mesonic Cherenkov thresholds $T_{\rm thr}(\omega)$ calculated for pions from Eqs. (2a), (8a), (8b), and (28) by using the experimental data of Pedroni *et al.* [14] (see text).

at the proton kinetic energy $T_p = 3$ GeV.

The differential NMCR cross sections $d\sigma/d\omega$ for $\pi^0$ mesons can be obtained from the expression (18) by multiplying with the factor $V/v_1$, where $V$ is the "collision volume" $V = (4\pi r_0^3/3)(1+A_T^{1/3})^3$, $r_0 = 1.12$ fm, and $A_T$ is the number of target nucleons. Numerical values for the refractive index, used for the calculation of $d\sigma/d\omega$, are obtained from Eq. (2a) and the experimental data of Pedroni [14], and are displayed in Fig. 2. In Fig. 2 we also give the classical and quantum kinetic energy thresholds $T_{\text{thr}}$ calculated according to the relation

$$T_{\text{thr}} = m_P\{[1 - v_{\text{thr}}^2(\omega)]^{-1/2} - 1\} , \tag{28}$$

where $v_{\text{thr}} = v_{\text{ph}}^M(\omega)$ for the classical NMCR threshold, and $v_{\text{thr}} = v_1^0(\omega)$, given by Eq. (8a), for the quantum NMCR threshold. Numerical values for the $\pi^0$ emission as NMCR in $^{120}$Sn and $^{208}$Pb are displayed in Fig. 3 in both absorptive and nonabsorptive cases. The absorption was taken into account in a standard way. We used the relation

$$\left[\frac{d\sigma}{d\omega}\right]_{\text{abs}} = \left[\frac{d\sigma}{d\omega}\right]_{\text{nonabs}} \langle F_{\text{abs}} \rangle , \tag{29a}$$

where

$$\langle F_{\text{abs}} \rangle = \frac{1}{2R}\int_0^{2R} \exp(-Ax)dx = \frac{1-\exp(-2AR)}{2AR} ; \tag{29b}$$

the absorption coefficient is given by

$$A \equiv 2\,\text{Im}k(\omega) = 2(\omega^2 - m_M^2)^{1/2}\text{Im}n(\omega) = \frac{(\omega^2 - m_M^2)^{1/2}\text{Im}n^2(\omega)}{\text{Re}n(\omega)}$$

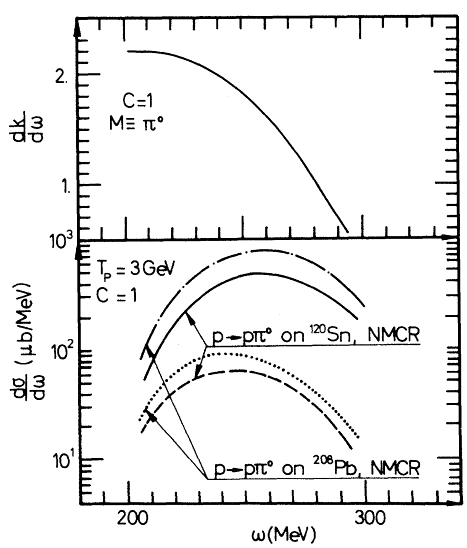


FIG. 3. The NMCR cross sections $d\sigma/d\omega$ for $\pi^0$ production on $^{120}$Sn (solid and dashed lines) and for $^{208}$Pb (dotted and dash-dotted lines) for both nonabsorptive and absorptive cases, respectively, and $dk/d\omega$ in upper part of figure (see text in Sec. IV).

$R$ is the radius of the target nucleus. The factor $dk/d\omega$ in Eq. (18) was also estimated numerically, and is presented in Fig. 3.

Now, the results of Fig. 3 can be compared with the constant ($\omega$-independent) values of the semiclassical NMCR spectrum $(d\sigma/d\omega)_{\text{nonabs}} = 54.9$ mb/MeV and $(d\sigma/d\omega)_{\text{nonabs}} = 87.3$ mb/MeV for pions produced in $^{120}$Sn and $^{208}$Pb (see Ref. [3]), respectively, by protons with $T_p = 3$ GeV. Thus, we see that the quantum effects on NMCR for pion production are important and that the NMCR cross sections remain large enough to be experimentally accessible. However, a special experimental technique is necessary in order to extract the NMCR yield from the background produced by other mechanisms.

## V. DISCUSSION AND OUTLOOK

The main results and conclusions may be summarized as follows.

(i) The theory of NMCR as an important collective effect in a nuclear medium is described by a quantum approach for the emission of scalar and pseudoscalar mesons from spin-$\frac{1}{2}$ baryons.

(ii) Some numerical results for the $\pi^0$ emission as NMCR are presented in Sec. IV. They show that the NMCR spectrum estimated in the quantum approach is sufficiently large to be measurable. However, an experimental investigation of this effect needs special methods, such as coincidence measurements of $B_i$, $\pi$, $B_f$, for the separation of NMCR from the background produced by other mechanisms.

(iii) Our results can also be used as ingredients in different phenomenological models for the coherent meson production in inclusive or exclusive nuclear reactions. But, then many other ingredients such as form factors, distorted wave corrections, final state resonant interactions, etc., have to be added in a consistent way. Here we restricted ourselves to the estimation of NMCR as a nuclear medium effect that can be experimentally investigated.

It is important to note that in all considerations above the influence of the nuclear medium on the propagation properties of the Dirac particles $B_1$ and $B_2$ (see Fig. 1) was neglected. This seems to be justified only for high primary energy $E_1$ and $E_2 \simeq E_1$, when the refractive indices of the $B_1$ and $B_2$ particles can be approximated by unity. For the final baryon $B_2$ the medium modifications [introduced by the energy-momentum relation $p_2^2 = n_B^2(E_2)(E_2^2 - M_2^2)$, $n_B(E_2)$ the baryonic refractive index] can be very important even for the mesonic spectrum at very high mesonic energies $\omega$ ($\omega \simeq E_1 - M_2$), beyond the mesonic Cherenkov band, where a transition of type (6) in the nuclear medium can also take place. But in this case the transition can be identified with the process of single meson emission via *"baryonic" Cherenkov effects* [4]. This can be proved by using the first part of Eq. (5b) and the energy-momentum relations for final particles in the nuclear medium, when one obtains the quantum baryonic Cherenkov coherence condition

$$|\cos\theta_{1k}| = \left| \frac{v_{\rm ph}^B(E_2)}{v_1} + \frac{p_2^2 - E_2^2 + m_M^2 - M_1^2}{2p_1p_2} \right| \le 1 ,$$

which at high projectile energy goes practically over into the classical baryonic Cherenkov coherence condition $v_{\rm ph}^B(E_2) \le v_1$, where $v_{\rm ph}^B(E_2) = E_2/p_2$ is the phase velocity of the final baryon in the nuclear medium. Moreover, the quantum theory for the Cherenkov mechanisms must be developed in a more general way in order to take into account absorption effects as well as the finite size effects even in the definition of the density (11) of final states (see Ref. [15]). All these improvements of the quantum theory of the mesonic Cherenkov mechanism should be worked out in future investigations.

Finally, we note that all above investigations [e.g., Eq. (18)] can be extended to the estimation of the $\eta$ and $K$ meson production rates, via Cherenkov mechanisms, in the nuclear reactions as, e.g., $A(p,\eta p)A$, $A(p,K^+\Lambda^0(\Sigma^0))A$, $A(p,K^0\Sigma^+)A$ or $A(\bar{p},\overline{K}{}^0\overline{\Sigma}{}^+)A$, $A(\bar{p},K^-\overline{\Lambda}{}^0)$.

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