PHYSICAL REVIEW C VOLUME 48, NUMBER 3 SEPTEMBER 1993

Hot and dense asymmetric nuclear matter

C. Das
Physics Department, Banki College, Banki, Cuttack, India

R. Sahu
Physics Department, Berhampur University, Berhampur 760 007, Orissa, India

R. K. Tripathi
13 Natalie Drive, Hampton, Virginia 23666
(Received 8 March 1993)

The temperature dependence of different thermodynamic quantities such as the free energy, chemical potential, symmetry energy, single particle potential, equation of state, etc., is studied for asymmetric nuclear matter within a fully self-consistent model with an effective interaction. The equation of state is found to be quite soft in agreement with supernova calculations. The critical temperature for the occurrence of a liquid gas phase transition is found to decrease with the proton-to-neutron ratio γ and finally vanishes for pure neutron matter. The possibility of π^- condensation has also been studied.
PACS number(s): 21.65.+f, 21.30.+y, 25.75.+r, 97.60.Jd

I. INTRODUCTION

In the last few years, the study of hot and dense nuclear matter has attracted considerable interest in nuclear physics and astrophysics. In particular, the investigation of the nuclear matter equation of state under unusual conditions of temperature and pressure plays an important role in the understanding of supernova explosions and neutron stars. Recently Baron, Cooperstein, and Kahana [1] found a softening of the equation of state of neutron-rich nuclear matter at high density. Within a hydrodynamical model, they have shown that the prompt bounce-shock mechanism in type-II supernovas can be understood if the compressibility is taken to be considerably lower than the values given by most sophisticated microscopic nuclear matter theories.

In the supernova implosion-explosion stage, neutrons outnumber protons at the core center by approximately two to one because of photodisintegration and electron capture. Hence, for the understanding of such supernova explosions, the properties of asymmetric nuclear matter are quite important. In the last few years, we [2, 3] have been developing a fully self-consistent model which is a generalization of the Brueckner theory to finite temperatures in which scattering to intermediate states is taken into account and the degeneracy and the single particle potential are calculated self-consistently. This model [2] has been found to be quite successful in describing the experimental data on entropy production in heavy-ion collisions. We [3] have also used this model to calculate the temperature dependence of different thermodynamic quantities for symmetric nuclear matter and neutron matter. The results are in reasonably good agreement with the predictions of other thermodynamic models [4–6]. The success of our model for symmetric nuclear matter and pure neutron matter has encouraged us to extend it to asymmetric nuclear matter.

Up to now, only a few theoretical calculations have been reported for asymmetric nuclear matter. ter Haar and Malfliet [7] have performed a relativistic Dirac-Brueckner calculation, based on a one-boson-exchange interaction. However, their calculation is restricted to zero temperature. Again, they observe a stiff equation of state in contrast with the outcome of supernova calculations. Wiringa, Fiks, and Fabrocini [8] have reported microscopic calculations of the equation of state of dense nuclear and neutron matter within the variational approach. They have extended the calculation to asymmetric nuclear matter by interpolation. Bombaci and Lombardo [9] have performed a systematic calculation of asymmetric nuclear matter within the framework of the Brueckner-Bethe-Goldstone approach. However, all these calculations are restricted to zero temperature.

In Sec. II, we give a brief discussion of the model. The results are analyzed in Sec. III. Section IV contains the summary and the conclusions of the present study.

II. THE MODEL

We have already discussed the details of our model in our earlier publications [2, 3, 10–12]. We give here a few important steps for completeness. In our formalism, we have extended the Brueckner theory by including the temperature effects. In the finite temperature case, we start by calculating the grand thermodynamic potential per unit volume:

$$\Omega = -P = -T \ln \operatorname{tr} \exp\left[-(H - \mu_n)/T\right], \qquad (1)$$

where H, P, T, μ, and n are the Hamiltonian, pres-

0556-2813/93/48(3)/1056(6)/$06.00

sure, temperature, chemical potential, and number density, respectively. The main reason for taking the grand thermodynamic potential lies in the fact that it can be expressed as a linked cluster expansion analogous to zero temperature Brueckner-Goldstone expansion, i.e.,

$$\Omega = \Omega_0 + \Omega_1 + \Omega_2 + \cdots , \tag{2}$$

where $\Omega_0, \Omega_1, \Omega_2, \ldots$ are the contributions to the thermodynamic potential due to the unperturbed part, one body part (single particle potential), and two body part (binary collision) of the Hamiltonian. Since our formalism is based on a linked cluster expansion, it is expected to be reliable up to very high densities. Our formalism is limited up to Ω_2. In this formalism, we have used the Brueckner reaction matrix instead of the bare NN force. The number density n is given by

$$n = \sum_\tau n_\tau , \tag{3}$$

where n_τ is the number density of nucleons with isospin τ (+ for protons and − for neutrons). The single particle energy is given by

$$\epsilon_\tau = \frac{\hbar^2 k^2}{2m_\tau} + U_\tau(k), \tag{4}$$

where k is the momentum and m_τ is the nucleon mass. The single particle potential $U_\tau(k)$ is defined by

$$U_+(k_1) = \frac{1}{2\pi^2}\int_0^\infty dk_2[\, n_+(k_2) g_{++}(E_s, k_1, k_2) + n_-(k_2) g_{-+}(E_s, k_1, k_2)], \tag{5}$$

where $n_\tau(k)$ is the Fermi distribution function given by

$$n_\tau(k) = \frac{1}{1+\exp\{[\epsilon_\tau(k)-\mu_\tau]/T\}}. \tag{6}$$

In the above equation μ_τ is the chemical potential of the nucleons with isospin τ. We have put the Boltzmann constant equal to 1. The number density n_τ is obtained by integrating the distribution function $n_\tau(k)$ over all momenta and weighting it with spin degeneracy. The g's are the interaction matrices

$$g_{\tau\tau'}(E_s, k_1, k_2) = \frac{\arctan[\pi\rho_E Q_{\tau\tau'} K_{\tau\tau'}(E_s)]}{\pi\rho_E Q_{\tau\tau'}}, \tag{7}$$

where ρ_E is the single particle level density and the K matrix satisfies the integral equation

$$K_{\tau\tau'}(E_s) = V_{\tau\tau'} + V_{\tau\tau'}\frac{Q_{\tau\tau'}}{E_s - H_0} K_{\tau\tau'}. \tag{8}$$

Here $V_{\tau\tau'}$ is the realistic nuclear interaction, $Q_{\tau\tau'}$ is the Pauli operator given by

$$Q_{\tau\tau'} = [1 - n_\tau(k_1)][1 - n_{\tau'}(k_2)], \tag{9}$$

and E_s is the starting energy of the two particles and is given by

$$E_s = \frac{\hbar^2}{2m_\tau}k_1^2 + \frac{\hbar^2}{2m_{\tau'}}k_2^2 + U_\tau(k_1) + U_{\tau'}(k_2). \tag{10}$$

It may be noted that the single particle potential is needed in calculating $n_\tau(k)$ [Eq. (6)], which is in turn required to calculate the single particle potential itself. Hence the single particle potential is calculated by iteration. Because of this self-consistency of the single particle potential, the scattering to intermediate states is taken into account properly through the Pauli operator. The equation of state thus obtained is expected to be valid up to densities much higher than the normal nuclear matter densities. The chemical potential μ_τ is determined by the number density constraint [Eq. (3)]. It should be noticed that Eqs. (3) and (5) warrant double self-consistency which must be satisfied with respect to the single particle potential and chemical potential. The internal energy, pressure, and free energy are given by

$$\frac{u}{A} = \frac{1}{n}\sum_\tau \frac{2}{(2\pi)^3}\int d^3k\, n_\tau(k)\left(\frac{\hbar^2 k^2}{2m_\tau} + \frac{1}{2}U_\tau(k)\right), \tag{11}$$

$$P = \frac{1}{\pi^2}\sum_\tau \int_0^\infty dk\, k^2\, n_\tau(k)\left(\frac{1}{3}k\frac{d\epsilon_\tau}{dk} + \frac{1}{2}U_\tau(k)\right), \tag{12}$$

$$F = u - TS. \tag{13}$$

We had used Sussex interaction in Ref. [2]. However, this interaction does not saturate correctly in nuclear matter. It gives insufficient binding by about 3 MeV per nucleon. Hence following the prescription of Tripathi, Elliot, and Sanderson [14] we have added a density-dependent term to the original Sussex interaction. The parameters of the density-dependent term are determined empirically so as to correctly reproduce the binding energies and the densities of nuclear matter and ^{16}O. Such a prescription has also been followed in Ref. [3]. If $V_{\rm SME}$ denotes the original tabulated Sussex interaction matrix elements, an effective interaction $V_{\rm eff}$ is defined as

$$V_{\rm eff} = V_{\rm SME} + W, \tag{14}$$

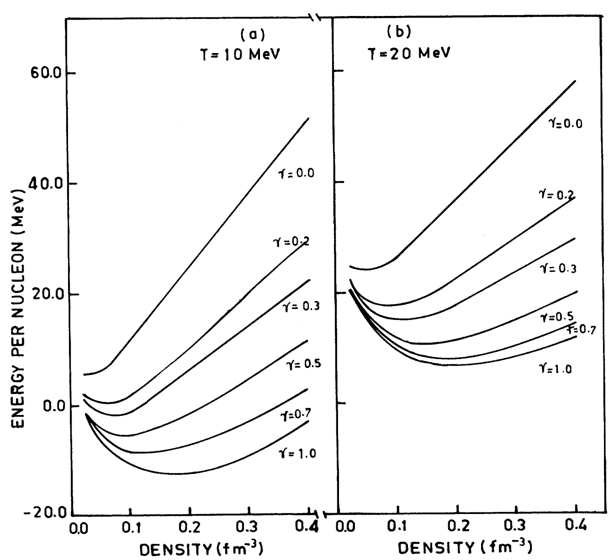

FIG. 1. Energy per nucleon versus density for different values of γ: (a) $T = 10$ MeV and (b) $T = 20$ MeV.

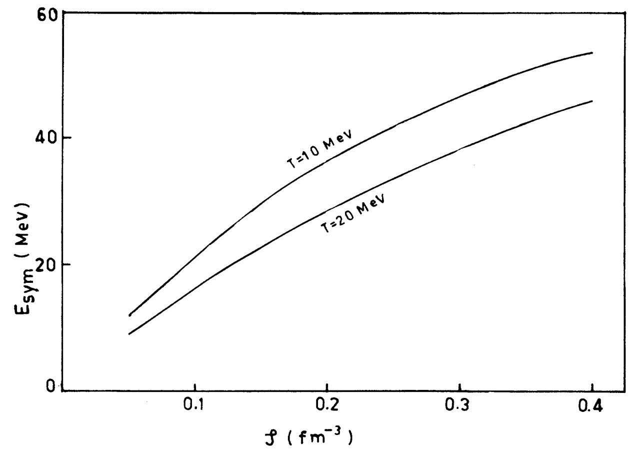

FIG. 2. Symmetry energy versus density at $T = 10$ MeV and 20 MeV.

where W is a simple density-dependent addition given by

$$W = \sum_{i<j} A\,\delta(\mathbf{r})\rho^{\alpha}(\mathbf{R}) + \tfrac{1}{2}[B(1+P_{ij}) + C(1-P_{ij})]\,\exp(-r^2/a^2), \qquad (15)$$

where $\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$, $\mathbf{R} = \frac{1}{2}(\mathbf{r}_i + \mathbf{r}_j)$, ρ is the density, and P_{ij} the space exchange operator. The values of the different parameters of this equation are taken from Ref. [14] and are $A = 568$ MeV fm$^{3+3\alpha}$, $B = -101.7$ MeV, $C = 90.6$ MeV, and $\alpha = 1/3$.

III. RESULTS

The calculations are performed taking the effective interaction given by Eq. (14). We have calculated the binding energy per nucleon at different densities and temperatures using Eq. (11). In Fig. 1, we plot these at

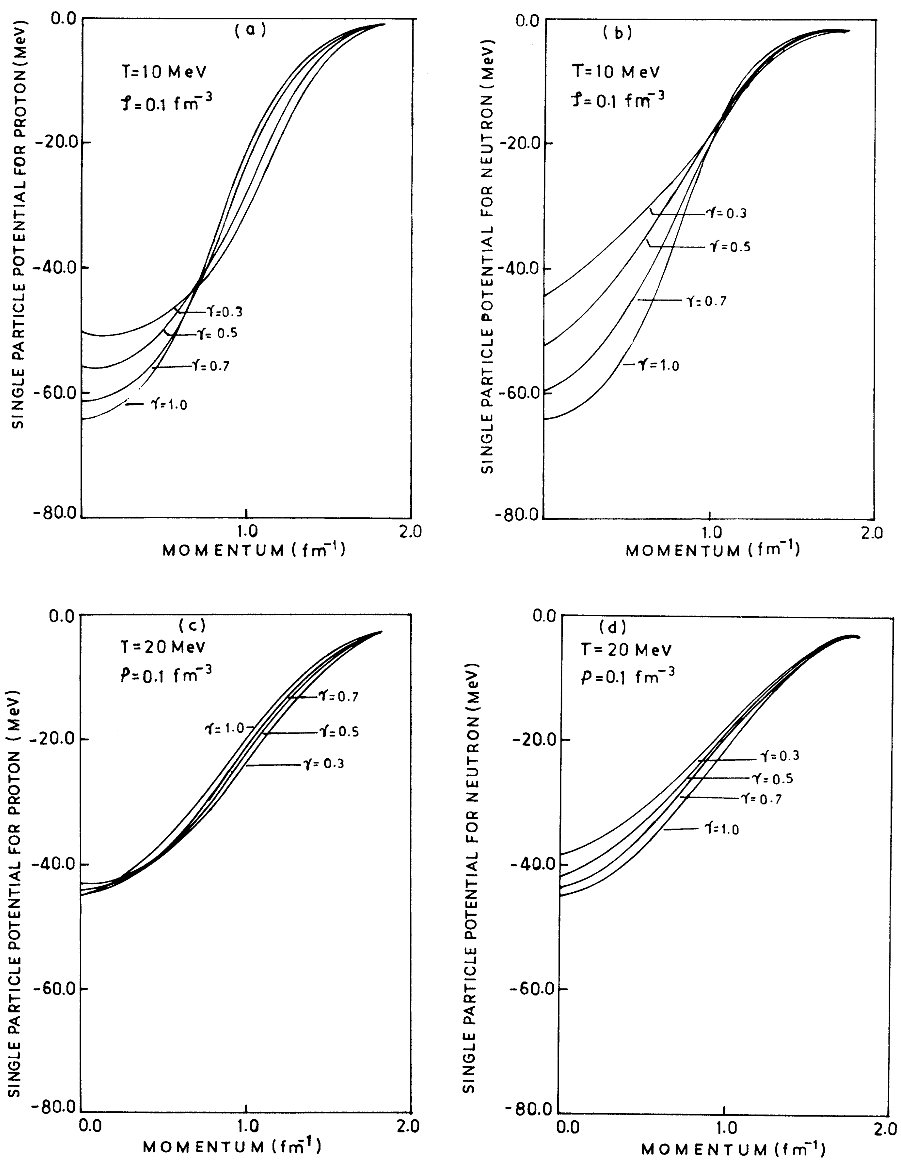

FIG. 3. Single particle potential versus momentum at $\rho = 0.1$ fm^{-3} (a) for protons at $T = 10$ MeV, (b) for neutrons at $T = 10$ MeV, (c) for protons at $T = 20$ MeV, and (d) for neutrons at $T = 20$ MeV.

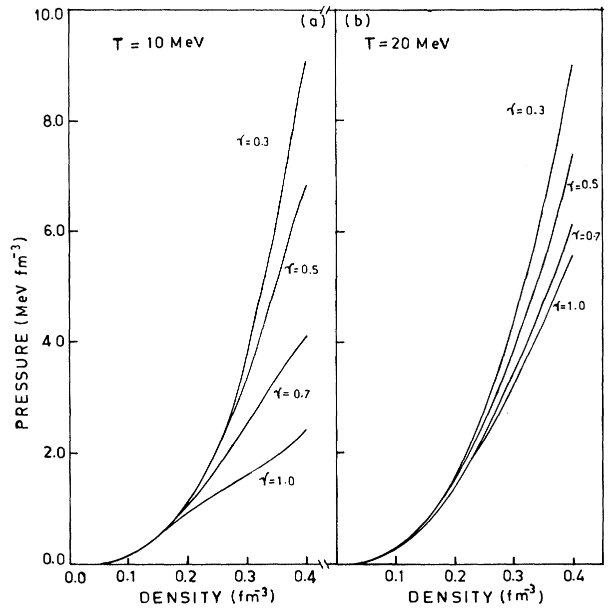

FIG. 4. Equation of state at different asymmetries γ: (a) $T = 10$ MeV and (b) $T = 20$ MeV.

different values of γ. As can be seen from the figure, for a decreasing proton-neutron ratio γ, the saturation density shifts to lower values in agreement with observations. Similar results were also obtained by ter Haar and Malfliet [7], Bombaci and Lombardo [9], and Wiringa, Fiks, and Frabrocini [8] at zero temperature. Bombaci and Lombardo have confirmed the empirical parabolic law satisfied by the binding energy per nucleon. They have shown that the quadratic dependence of the binding energy upon the asymmetry parameter $\beta = \frac{N-Z}{N+Z}$ is valid up to high values of β at zero temperature. Such a parabolic law for binding energy is also found to hold at $T = 10$ MeV and 20 MeV in our calculation. Under this approximation, the symmetry energy can be evaluated using the equation

$$\begin{aligned} E_{\rm sym} &= B(\rho, \beta = 1) - B(\rho, \beta = 0) \\ &= B(\rho, \gamma = 0) - B(\rho, \gamma = 1). \end{aligned} \tag{16}$$

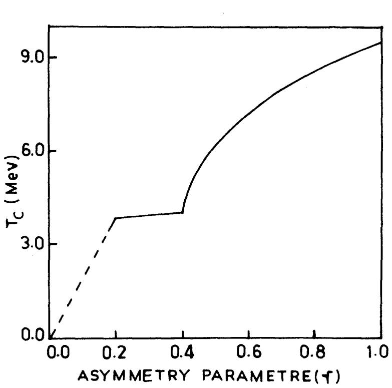

FIG. 5. Critical temperature versus asymmetry γ.

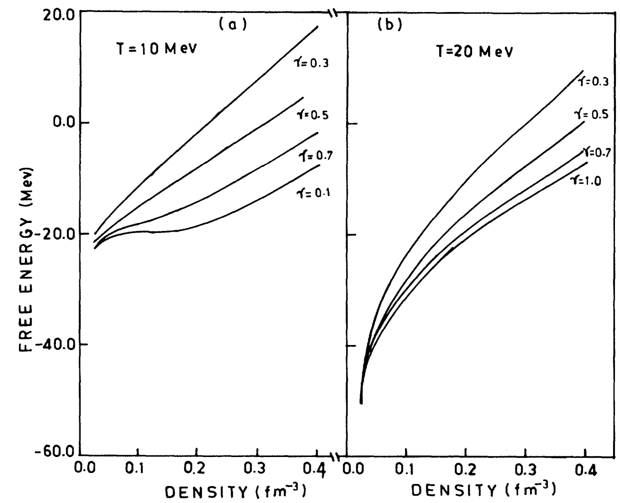

FIG. 6. Free energy versus density at $T = 10$ MeV and 20 MeV at different asymmetries γ.

We have plotted the symmetry energy versus density at $T = 10$ and 20 MeV in Fig. 2. Our results at $T = 10$ MeV are almost similar to the values reported by Bombaci and Lombardo [9] at zero temperature.

In Fig. 3, we plot the single particle potential versus momentum at density $\rho = 0.1$ for different asymmetries and at temperatures $T = 10$ and 20 MeV. The variation of the single particle potential with the density for symmetric nuclear matter and neutron matter was discussed in our earlier publication also [3]. Here we want to study how the single particle potential varies with the asymmetry. For symmetric nuclear matter, our results agree quite well with those of Lejeune *et al.* [4]. This gives us confidence to extend our model to asymmetric nuclear matter. We find the depth of the proton and neutron single particle potential to decrease with asymmetry at $T = 10$ MeV. This is because as the proton ratio decreases, the $T = 0$ part of the nuclear force becomes smaller and hence the depth of the potential decreases.

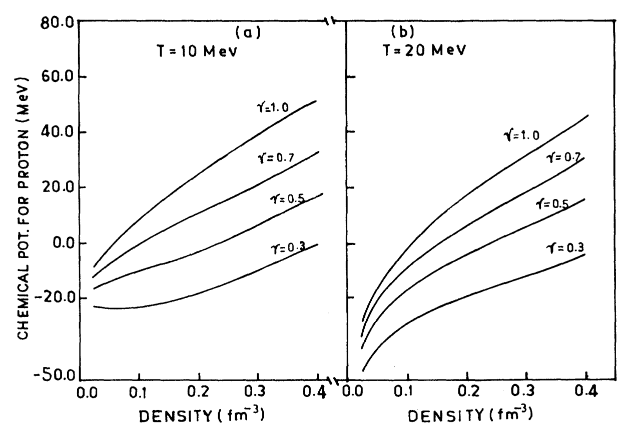

FIG. 7. Chemical potential for proton at $T = 10$ MeV and 20 MeV at different asymmetries γ.

However, as we go to $T = 20$ MeV, the proton single particle potential almost becomes independent of asymmetry. This is probably due to the dominance of the thermal energy. Bombaci and Lombardo [9] have also studied the variation of single particle potential with asymmetry at zero temperature. The depth of the potential in our case is smaller since our calculation is at finite temperature.

In Fig. 4, we plot pressure versus density at different asymmetries and temperatures. We [3] find from our calculation that the critical temperature for the occurrence of a liquid gas phase transition in symmetric nuclear matter is 9.5 MeV. Most of the other calculations [5,6] predict a much higher value for the critical temperature. However, ter Haar and Malfliet [13] have performed a fully self-consistent Dirac-Bruckner calculation and found the critical temperature to be below 10 MeV. This critical temperature decreases with the decrease of the asymmetry parameter γ as shown in Fig. 5. However, this decrease in the critical temperature is not uniform. From $\gamma = 1$ to $\gamma = 0.4$, the decrease in the critical temperature is almost uniform. However, the critical temperature is almost constant for γ lying between 0.4 and 0.2. Then finally for $\gamma = 0$ (neutron matter), we have $T_c = 0$, indicating the absence of a phase transition. The equation of state for all values of the asymmetry parameter γ lies in a narrow band both at $T = 10$ MeV and $T = 20$ MeV. A similar trend was also observed by ter Haar and Malfliet at zero temperature. However, their equation of state is quite stiff. As has been pointed out, Baron, Cooperstein, and Kahana [1] have tried to understand the iron-core collapse of a type-II supernova within a hydrodynamic model. They have found that one can obtain sustained shocks and prompt explosions if the equation of state is taken to be soft. On comparison, we also find our equation of state similar to Baron, Cooperstein, and Kahana [1]. Our calculated compressibility also points to a soft equation of state. For example, at $T = 10$ MeV, we find the value of $K = 84$ at $\gamma = 0.4$ compared to the value of $K = 158$ for symmetric nuclear matter. But this value drops to $K = 77$ at $\gamma = 0.2$.

In Fig. 6, we plot the free energy versus density at different asymmetries at $T = 10$ and 20 MeV. Our results for symmetric nuclear matter and neutron matter agree well with those of Friedmann and Pandharipandy [6], Lejeune *et al.* [4], and Baldo *et al.* [5]. We find that the free energy increases with a decrease in the asymmetry parameter γ.

In Figs. 7(a) and 7(b), we plot the chemical potential for a proton at $T = 10$ MeV and 20 MeV, and in Figs. 8(a) and 8(b) the neutron chemical potential is plotted at these temperatures. In our calculation the chemical potential is calculated self-consistently. We find that on increasing the neutron concentration (decreasing γ), μ_p decreases, but μ_n increases, which is expected.

Whether pions appear in dense nuclear matter has been of considerable theoretical interest. It has been shown by Baym and Pethik [15] that π^- will form via $n \rightarrow p + \pi^-$ if the neutron proton chemical potential difference $\mu_n - \mu_p$ exceeds the π^- rest mass $m_{\pi^-} = 139.6$ MeV provided one neglects the interaction of pions with

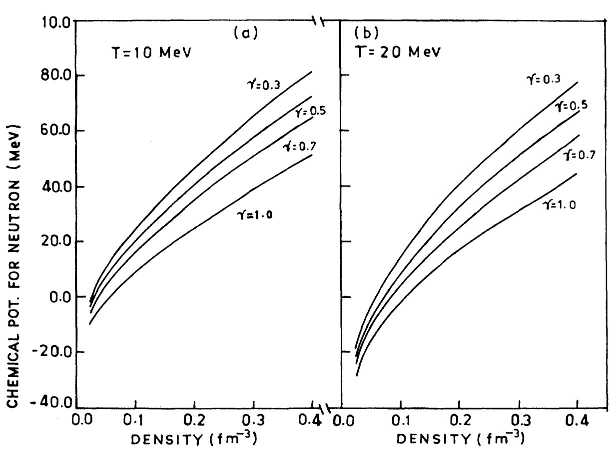

FIG. 8. Chemical potential for neutron at $T = 10$ MeV and 20 MeV at different asymmetries γ.

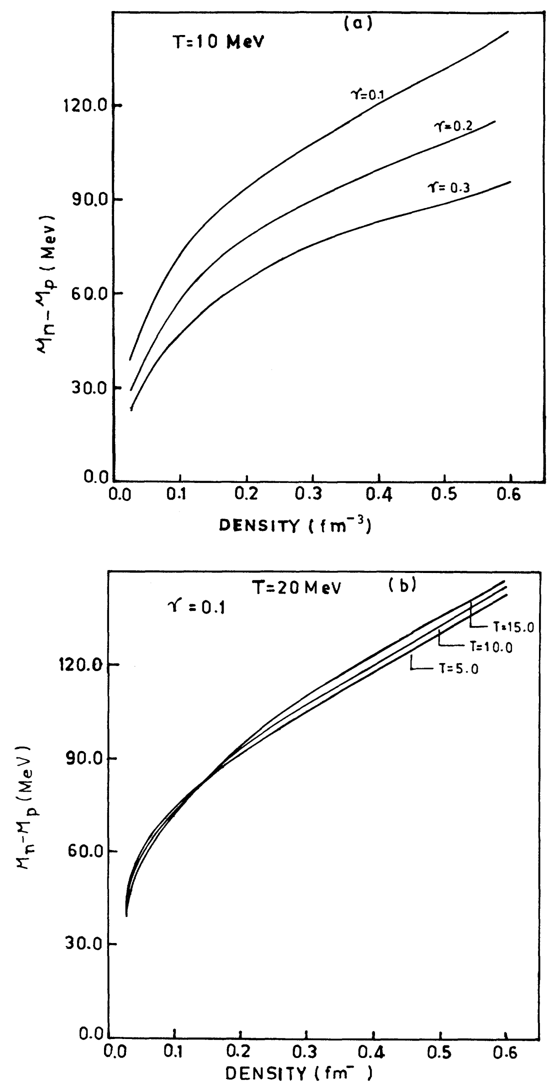

FIG. 9. $\mu_n - \mu_p$ versus density: (a) $T = 10$ MeV and (b) $T = 20$ MeV.

matter. We have calculated the chemical potential self-consistently. In Fig. 9, we plot $\mu_n - \mu_p$ versus density at $T = 10$ MeV and 20 MeV. It is found that $\mu_n - \mu_p$ is greater than the rest mass of the pion for $\rho \geq 3\rho_0$ for $\gamma \leq 0.1$, i.e., for neutron-rich matter at $T = 10$ MeV. However, for $T = 20$ MeV, we find $\mu_n - \mu_p > m_{\pi^-}$ for all values of γ for $\rho > 3\rho_0$. Thus we predict the occurrence of pion condensation at high density. The pion condensation is found to be favored at higher temperature.

IV. CONCLUSIONS

We have developed a self-consistent model to calculate different thermodynamic quantities at different temperatures. Our model is an extension of the Brueckner theory to finite temperature. We have used an effective interaction which consists of the Sussex interaction and a density-dependent part. The parameters of the density-dependent part are fixed so as to correctly reproduce the binding energies and the densities of nuclear matter and ^{16}O. We find from our calculation that the saturation density shifts to lower values with a decrease of $\gamma = Z/N$ as expected. Assuming the parabolic law for the binding energy, we have extracted the symmetry energy at $T = 10$ MeV and 20 MeV. The symmetry energy is found to decrease with temperature. We have calculated the single particle potential and chemical potential self-consistently and studied the dependence of these quantities on the asymmetry γ. In our calculation, we find the occurrence of a liquid gas phase transition at $T = 9.5$ MeV for symmetric nuclear matter. This value of the critical temperature is smaller than most other calculated results [5, 6]. However ter Haar and Malfliet [13] found the phase transition to occur below $T = 10$ MeV from their Dirac-Brueckner calculation. The critical temperature is found to decrease with asymmetry γ. However, for $\gamma = 0.4$–0.2, the critical temperature is almost constant. Finally for neutron matter ($\gamma = 0$), the critical temperature $T_c = 0$, indicating the absence of phase transition. The calculated free energy is found to increase with the decrease in the asymmetry parameter γ.

We predict the occurrence of pion condensation for $\rho > 3\rho_0$. The pion condensation is found to be favored at higher temperature.

We are thankful to Professor S. P. Misra for many useful discussions.

[1] E. Baron, J. Cooperstein, and S. Kahana, Phys. Rev. Lett. **55**, 126 (1985); Nucl. Phys. **A440**, 744 (1985).
[2] C. Das, R. K. Tripathi, and J. Cugnon, Phys. Rev. Lett. **56**, 1663 (1986).
[3] C. Das, R. K. Tripathi, and R. Sahu, Phys. Rev. C **45**, 2217 (1992).
[4] A. Lejeune, P. Grange, M. Martzolff, and J. Cugnon, Nucl. Phys. **A453**, 189 (1986).
[5] M. Baldo, I. Bombaci, L. S. Ferreira, G. Giansiracusa, and U. Lombardo, Phys. Lett. B **215**, 19 (1988).
[6] B. Friedman and V. R. Pandharipande, Nucl. Phys. **A361**, 502 (1981).
[7] B. ter Haar and R. Malfliet, Phys. Rev. Lett. **59**, 1652 (1987).
[8] R. B. Wiringa, V. Fiks, and A. Fabrocini, Phys. Rev. C **38**, 1010 (1988).
[9] I. Bombaci and U. Lombardo, Phys. Rev. C **44**, 1892 (1991).
[10] R. K. Tripathi, Phys. Rev. C **25**, 1114 (1982).
[11] R. K. Tripathi, Phys. Lett. **101B**, 369 (1981).
[12] C. Das and R. Sahu, Phys. Lett. B **289**, 217 (1992).
[13] B. ter Haar and R. Malfliet, Phys. Rep. **149**, 207 (1987).
[14] R. K. Tripathi, J. P. Elliot, and E. A. Sanderson, Nucl. Phys. **A380**, 483 (1982); N. I. Kasis, J. P. Elliot, and E. A. Sanderson, *ibid.* **A359**, 386 (1981).
[15] G. Baym and C. Pethik, Annu. Rev. Nucl. Sci. **25**, 27 (1975).