Systematics of triaxial deformation in Xe, Ba, and Ce nuclei

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The (β,γ) deformation parameters of even-even Xe, Ba, and Ce nuclei have been calculated by using the triaxial rotor model. Deformation parameters calculated, on one hand, from decay properties and, on the other hand, from energies are in good agreement. The smooth dependence of the deformation parameters on Z and N is discussed. The results are compared with those extracted from properties of odd-A nuclei.

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I. INTRODUCTION

The experimental information accumulated in recent years has provided compelling evidence of dynamic triaxial deformation of transitional nuclei in the mass region $A = 120 - 140$, in particular of Xe and Ba nuclides [1,2]. The most striking feature is the presence of low lying 0^+ and 2^+ excited states. A simple description of a nucleus whose shape deviates from axial symmetry is provided by the rigid triaxial rotor model (RTRM) of Davydov and Filippov [3]. In this model the nuclear shape is characterized by constant values of the deformation parameters β and γ [4]. On the contrary , the interacting boson model (IBM) [5] and the dynamic collective model [6] treat β and γ as dynamical variables. The experimental features of even-even Xe and Ba nuclei have been quite well described by the IBM using a Hamiltonian close to the O(6) symmetry of the model [5]. In the limit of infinite boson number, this symmetry corresponds to the γ-unstable model of Willets and Jean [7]. Even though the O(6) symmetry provides only an approximate description of the nuclei investigated, it constitutes a good reason to consider these nuclei as being gamma soft. The RTRM constitutes an approximation in which the vibrational variables β and γ are replaced by fixed effective values [8,9]. As a matter of fact, a careful analysis of the properties of side bands has shown that the gamma-soft model provides a better description of transitional nuclei in this mass region than a γ rigid model [18].

Nevertheless most properties of the ground and quasigamma bands can be described by the RTRM. The relationship between the IBM and the RTRM have been investigated in Ref. [11]. Moreover, calculations of odd-A nuclei in the same region by means of the rigid triaxial rotor plus particle model have been quite successful [10]. Roughly speaking, it seems that the value of γ which has to be used in the RTRM correspronds to the average value $\langle\gamma\rangle$ of the wave function of the IBM. The main aim of the present work is to see whether we can get a consistent set of (β,γ) deformation parameters for even-even Xe, Ba, Ce, and Nd nuclei by using the RTRM. For this purpose we will use the energies and decay properties of the ground bands and quasigamma bands as described by the RTRM.

II. THE RIGID TRIAXIAL ROTOR MODEL

The Hamiltonian of the RTRM can be written as

$$H = \frac{\hbar^2}{2}\sum \frac{I_i^2}{\Theta_i}, \tag{1}$$

where I_i are the projections of the angular momentum on the intrinsic axes. The moments of inertia of the Davydov-Fillipov model are given by the hydrodynamical formula

$$\Theta_\kappa = \frac{4}{3}\Theta_0 \sin^2\left(\gamma - \frac{2\pi}{3}\kappa\right), \tag{2}$$

where Θ_0 depends only on β. With the exception of $\gamma = 0°$, $30°$, or $60°$, analytical expressions for energies and $E2$ transition probabilities must be deduced separately for each spin [3].

One of the interesting features of the RTRM is the natural way in which the quasigamma band appears. There is no gamma vibration in this model. The states of the quasigamma band are also rotational states. The number of states with angular momentum I allowed by the symmetry of the triaxial rotor is $I = (0, 2^2, 3, 4^3, 5^2, \ldots)$ [4]. The energies of the $I = 2$ states are given by

$$E_{2^+_{1,2}} = \left(\frac{6\hbar^2}{2\Theta_0}\right)\frac{9-(-1)^{\sigma_{1,2}}\sqrt{81-72\sin^2(3\gamma)}}{4\sin^2(3\gamma)}, \tag{3}$$

where

$$\sigma_{1,2} = 0, 1. \tag{4}$$

The energy ratio depends only on γ and not on Θ_0,

$$R_E(\gamma) = \frac{E_{2^+_2}}{E_{2^+_1}} = \frac{1+X}{1-X}, \tag{5}$$

where [29]

$$X = \sqrt{1-\frac{8}{9}\sin^2(3\gamma)}. \tag{6}$$

The reduced $E2$ transition probabilities from these states are given by

$$B(E2;2^+_{1,2}\rightarrow 0^+_1)=\frac{1}{16\pi^2}Q_0^2[1+(-1)^{\sigma_{1,2}}\chi(\gamma)], \tag{7}$$

where

$$\chi(\gamma)=\frac{3-2\sin^2(3\gamma)}{\sqrt{9-8\sin^2(3\gamma)}} \tag{8}$$

and Q_0 is defined as in the axially symmetric case by

$$Q_0=\frac{3}{\sqrt{5\pi}}ZR_0^2\beta \tag{9}$$

to first order in beta. The ratio of $B(E2)$ values from the 2^+_2 states depends only on γ and is independent of Q_0 and β. It is

$$R_B(\gamma)=\frac{B(E2;2^+_2\rightarrow 2^+_1)}{B(E2;2^+_2\rightarrow 0^+_1)}=\frac{20}{7}\frac{\sin^2(3\gamma)}{9-9\sin^2(3\gamma)-[3-2\sin^2(3\gamma)]\sqrt{9-8\sin^2(3\gamma)}}. \tag{10}$$

III. ESTIMATION OF DEFORMATION PARAMETERS

We will calculate first the (β,γ) deformation parameters from the decay properties. On the one hand, the ratio R_B can be deduced from the branching ratio in the decay of the 2^+_2 state, and this ratio depends only on γ. On the other hand, the absolute values of $B(E2;2^+_1\rightarrow 0^+_1)$ are known from lifetime experiments. Since γ is known from the branching ratios, we can extract Q_0 and the value of β. The $B(E2;2^+_2\rightarrow 2^+_1)$ value has been corrected for $M1$ contributions only in the cases of ^{126}Xe and ^{130}Ba. This small correction has been neglected in the other nuclei. Generally the $\delta(M1/E2)$ ratios are small [12].

The values of β and γ derived in this way are given in Table I under the labels β_B and γ_{R_B}, respectively,

TABLE I. γ and β values for even Xe, Ba, and Ce. Calculated values of γ and β using the indicated values of R_B, $B(E2,2^+_1\rightarrow 0^+_1)$, R_E, and the energies of the 2^+_1 state. The data are from the references cited. The γ values labeled with an asterisk are assumed to be near 30°.

		^{122}Xe	^{124}Xe	^{126}Xe	^{128}Xe	^{130}Xe	^{132}Xe
Ref.		[16,17]	[16,17]	[18,19]	[20,19]	[20,19]	[21,19]
R_B		17.8	25.6	67(18)	76.9	166.7	685(35)
$B(E2)$	($e^2\,b^2$)	0.265(20)	0.240(20)	0.154(5)	0.150(8)	0.130(10)	0.092(6)
γ_{R_B}	(deg)	24.7	25.5	27.2(4)	27.4	28.2	29.13(2)
β_B		0.261(20)	0.250(19)	0.191(6)	0.186(9)	0.170(13)	0.141(9)
R_E		2.55	2.39	2.26	2.19	2.09	1.94
$E_{2^+_1}$	(MeV)	0.331	0.354	0.389	0.443	0.536	0.668
γ_{R_E}	(deg)	24.2	25.1	26.0	26.6	27.7	30*
β_E		0.26	0.25	0.24	0.22	0.20	0.18

		^{124}Ba	^{126}Ba	^{128}Ba	^{130}Ba	^{132}Ba	^{134}Ba
Ref.		[22,17]	[23,19]	[23,17]	[24,17]	[25,19]	[16,19]
R_B		5.8	7.6(11)	8.8(3)	16.1(18)	40.5(1.6)	178.8
$B(E2)$	($e^2\,b^2$)	0.401(9)	0.380(4)	0.276(17)	≤ 0.23	0.172(12)	0.136(32)
γ_{R_B}	(deg)	20.3	21.8(7)	22.3(1)	24.4(3)	26.4(1)	28.3
β_B		0.295(6)	0.284(3)	0.240(14)	≤ 0.217	0.190(13)	0.164(38)
R_E		3.79	3.41	3.11	2.54	2.22	1.93
$E_{2^+_1}$	(MeV)	0.230	0.256	0.284	0.357	0.465	0.605
γ_{R_E}	(deg)	20.0	20.9	21.8	24.3	26.3	30*
β_E		0.30	0.28	0.26	0.24	0.21	0.18

		^{128}Ce	^{130}Ce	^{132}Ce	^{134}Ce	^{136}Ce
Ref.		[26,19]	[26,19]	[26,19]	[27,19]	[28]
R_B			7.9(12)	16.7(8)	12.9(42)	100.0
$B(E2)$	($e^2\,b^2$)	0.430(36)	0.346(18)	0.354(28)	0.206(18)	
γ_{R_B}	(deg)		21.9(7)	24.5(1)	23.7(11)	27.7
β_B		0.289(24)	0.274(14)	0.269(21)	0.205(17)	
R_E			3.29	2.53	2.36	1.98
$E_{2^+_1}$	(MeV)	0.207	0.254	0.325	0.409	0.552
γ_{R_E}	(deg)		21.3	24.3	25.3	30*
β_E		0.26	0.30	0.28	0.26	0.22

together with the ratio R_B and $B(E2; 2_1^+ \rightarrow 0_1^+)$.

Since the energy ratio R_E, Eq. (5), depends only on γ, it also allows the calculation of this deformation parameter. However, there is no simple theoretical relation between the moment of inertia Θ_0 and β. In order to avoid this difficulty we make use of the semiempirical relation established by Grodzins [13] on the assumption of axial symmetry:

$$E_{2^+} B(E2; 2_1^+ \rightarrow 0^+) = 2.5 \times 10^{-3} Z^2 A^{-1}. \qquad (11)$$

This relation allows us to relate the parameter Θ_0 [14] to an effective value of β:

$$\frac{\hbar^2}{2\Theta_0} \cong \frac{204}{\beta_G^2 A^{7/3}}. \qquad (12)$$

If the γ deformation is taken into account, the corrected β deformation parameter is given by [3]

$$\beta = \beta_G \frac{9 - \sqrt{81 - 72\sin^2(3\gamma)}}{4\sin^2(3\gamma)}. \qquad (13)$$

The values of β and γ obtained in this way are labeled in Table I as γ_{R_E} and β_E.

IV. DISCUSSION

The deformation parameters extracted from experiment by using the RTRM vary smoothly with Z and N. In particular the deformation β increases with Z and decreases with N, at least in the investigated mass range. This trend is to be expected since the deformation should have a maximum at midshell. The reasons for the (Z, N) dependence of γ are less obvious. This can be explained by the different occupancies of the low K Nilsson orbitals which try to drive the nucleus towards a prolate shape and of the high K, oblate driving orbitals. Nuclei with a larger β value (for the same value of N) generally have smaller values of γ. The theoretical (Z, N) dependence of the deformation parameters has been thoroughly investigated by Wyss *et. al.* [15] by total Routhian calculations. Although the values of β and γ which correspond to minima of the total Routhian surfaces are somewhat different from those given in Table I, the trends are identical.

As for the the internal consistency of the deformation parameters extracted by using the RTRM, one notices that γ_{R_E} and γ_{R_B} are closer than β_E and β_B. This can be explained by assuming that the γ dependence of the moments of inertia is correctly reproduced by the hydrodynamical formula 2. At the same time we know that the β dependence of the moment of inertia given by the hydrodynamical model is not completely correct [4].

Triaxial rotor plus particle calculations for odd-A Xe and Ba nuclei have been carried out recently. The β and γ values fitted to excitation energies, branching ratios and lifetimes are close to those of the present work (see Table II). It is difficult to decide whether these slight differences are due to simplifying assumptions contained in the model or to a core polarization effect. More in-

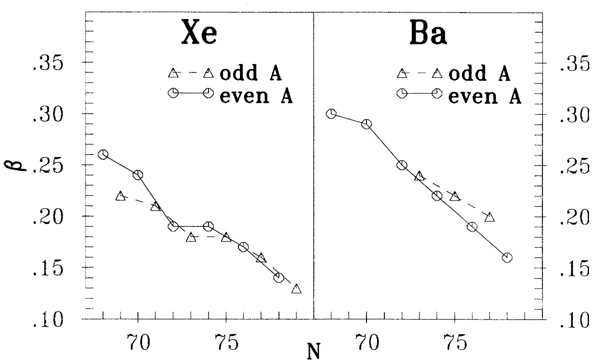

FIG. 1. Dependence of the deformation parameter β on the neutron number for odd and even Xe and Ba nuclei .

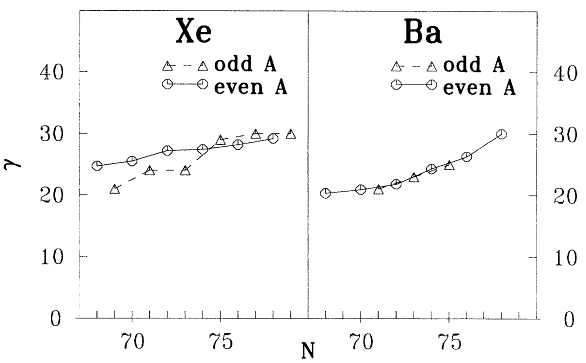

FIG. 2. Dependence of the deformation parameters γ on the neutron number for odd and even Xe and Ba nuclei .

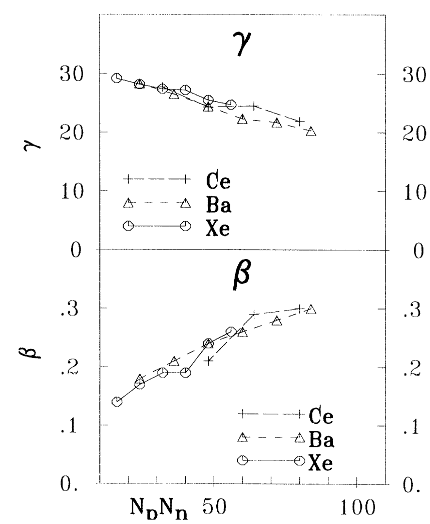

FIG. 3. Dependence of the deformation parameters β and γ on the product $N_p N_n$ [30] of the number of valence protons (particles) and neutrons (holes) for even Xe, Ba, and Ce nuclei.

TABLE II. β and γ for odd Xe and Ba. β and γ values from triaxial rotor plus particle calculations [10].

	^{123}Xe	^{125}Xe	^{127}Xe	^{129}Xe	^{131}Xe	^{133}Xe	^{127}Ba	^{129}Ba	^{131}Ba
β	0.22	0.21	0.18	0.18	0.16	0.13	0.24	0.22	0.20
γ (deg)	21	24	24	29	30	30	21	23	25

formation on proton odd nuclei could help in answering this question, because in this mass region the core should be polarized in opposite ways by a proton and neutron, respectively. The (Z, N) dependence of the deformation parameters, as well as a comparison with those fitted in Xe and Ba nuclei, are displayed in Figs. 1–3. Since we can assume that in reality the considered nuclei are gamma soft [18], the rigid triaxial rotor model represents only an approximate description. The quoted deformation parameters, in particular those of γ, must be considered only as effective parameters which could be model dependent. The closeness of γ_{eff} to the maximum triaxiality value $\gamma = 30°$ shows that a description which ignores the gamma degree of freedom would be incomplete.

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