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Extended quasiparticle approximation and Brueckner theory

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The time-dependent Green's function method provides a basic theory for nuclear dynamics and heavy-ion collisions. The spectral function plays an important role in this theory containing information on correlations in the medium. These are usually neglected by using a quasiparticle approximation. In order to evaluate this particular approach it is important to establish in the static limit a link between this method and other traditional nuclear many-body theories such as the Brueckner approach. Using a self-consistent T-matrix approximation for the self-energy in the Green's-function approach, treating particles and holes on equal footing, the self-energy can be obtained and compared with the Brueckner expression. In the quasiparticle limit it contains terms up to and analogous to the Brueckner second order rearrangement energy. In an *extended* quasiparticle approximation (EQP) which does not violate the sum rule for the spectral function the Brueckner third-order rearrangement energy can also be reproduced. The latter approximation constitutes a bridge between the Brueckner and Green's-function methods. It assumes small absorption (and energy dependence) of the mean field. Some numerical estimates of the different approximation schemes that we discuss are shown. An iteration scheme for applying the EQP approximation is suggested.

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I. INTRODUCTION

The theoretical analysis of heavy-ion (H.I.) collisions has brought a renewed interest in microscopic theories of nuclear matter. These, in general, describe the collisional process in terms of a mean field and a two-nucleon collision term, that mainly relate to the static and the dynamic (nonequilibrium) properties of nuclear matter, respectively. A most interesting goal of the analysis is to derive an equation of state for nuclear matter from the experimental data, but the mean-field and two-nucleon collisional effects are never fully separated in the short time of the collisional process. Thus to obtain the equation of state (equilibrium property), the dynamics (nonequilibrium property) has also to be understood.

Most H.I. calculations and discussions of the equation of state and possible pionic effects, etc. have been based on the semiclassical Boltzmann-Uehling-Uhlenbeck (BUU), Vlasoz-Uehling-Uhlenbeck (VUU), etc. models. From a microscopic point of view these models seem too simplistic to justify a too close comparison with experiments. For a better treatment of both mean-field and collisional effects, close attention to temperature (excitation) dependence, quantal, and relativistic effects as well as particle production has to be included. It may also be necessary to go beyond the semiclassical approximation.

The only microscopic theory which is capable of describing both the statics and dynamics in a comprehensive way is the Green's-function approach first initiated by Martin and Schwinger [1] and later developed by Schwinger [2], by Keldysh [3], and by Kadanoff and Baym [4], and with specific application to nuclei by Danielwicz [5] and by Botermans and Malfliet [6]. The latter authors have also extended their work to include relativistic effects [7]. The static equilibrium limit of these dynamic nonequilibrium theories is also important both formally and for computational purposes. In the initial stage of a H.I. collision the nuclei are (normally) in their ground (temperature $T=0$) states. The transport theory has to be capable of treating these limits correctly.

Microscopic descriptions of finite nuclei are based on Brueckner theory. It is therefore important to make not only the formal connection but also the connection between Brueckner and Green's-function theory in the specific form that the Brueckner theory has been applied for finite nucleus calculations. Although Brueckner theory applies only to ground-state nuclei and is not a dynamic theory, there is also a connection with transport theories through the relation between the collision term of, for example, Uehling-Uhlenbeck form and the absorptive potential calculated by Brueckner theory [8]. It is our plan to apply the path-ordered Green's-function methods as discussed by Botermans and Malfliet [6] to the study of correlations in nuclei and its effect on nuclear dynamics. The purpose of this paper is to clarify some of the quantitative relations between the two methods especially as regards mean field (self-energy), correlations, and occupation numbers.

The Brueckner many-body theory of nuclei defines leading-order contributions to the mean field; in first order it is determined by the Brueckner reaction matrix. Second- and third-order contributions are also defined and are usually referred to as rearrangement terms because they are obtained from changes in the reaction matrix (rearrangements in the correlations) as a particle is removed or added. The second-order contribution results from a change in the Pauli operator while the third-order contribution results from changes in the energy denominator.

Of these various contributions to the mean field, the first- and third-order terms have (at least in principle) been included in most self-consistent (alias *Hartree-Fock, Brueckner-Hartree-Fock,* or *renormalized Brueckner-Hartree-Fock*) microscopic calculations for finite nuclei. The second-order term has however been totally neglected in most such calculations although it has been calculated and discussed many times for nuclear matter [9–11]. In this case there is no direct need for a mean-field potential (the wave functions are plane waves) but it still enters indirectly by the variational search for the saturation density. The minimization of the energy with respect to density is equivalent to saying that the single particle energy (SPE) of a nucleon at the Fermi surface is equal to the binding energy per nucleon. This is, in fact, the Hugenholtz–Van Hove theorem. To satisfy this theorem the second-order term has to be included. Being strongly momentum dependent, changing by over 20 MeV from the bottom to the top of the Fermi sea at normal density and zero temperature [9–13], it is important in determining the effective mass, i.e., the nonlocality of the mean-field potential. Although the main importance of this Brueckner second-order rearrangement term for finite ground-state nuclei [14–16] is for SPE's, its importance for heavy-ion collisions has been accentuated by the BUU calculations with a momentum-dependent interaction done by Gale and co-workers [17]. The rearrangement terms are also important for calculation of occupation numbers and spectral functions. The imaginary part of the second-order rearrangement energy gives, for example, the width of hole states already calculated in Ref. [18].

This serves to show the important fact that the Brueckner rearrangement terms play an important role for the correct treatment of saturation and correlation properties of nuclei. It therefore seems important to find the corresponding terms in the Green's-function formulation. Particles and holes are here treated symmetrically. The effective interaction is usually defined not only with particle-particle, as in Brueckner theory, but also with hole-hole propagation in intermediate states. We shall find that as a result both formulations contain essentially the same "second-order rearrangement term."

An important ingredient of the Green's-function formalism is the spectral-function that contains information on the correlation among nucleons. A numerical solution requires an iterative procedure to include this function consistently. This leads in general to complicated energy integrations. In most works the spectral function is therefore approximated by a delta function of the energy peaked at the quasiparticle energy.

Such a simple approximation falls however short of including correlations that are routinely included, e.g., in Brueckner calculations. Iteration procedures using improved spectral functions are necessary. As a *first* iteration it is convenient to choose a quasiclassical approximation with all the strength at the quasi-particle peak. As a second iteration one may consider choosing the quasiparticle approximation including a strength function. This approximation suffers, however, from inconsistencies directly related to the fact that it does not obey the energy sum rule.

The extended quasiparticle approximation (EQP) was introduced in previous work [19]. It was shown to remedy some of the faults of the simpler approximations. The correlation structure was found to be similar to that obtained in Brueckner theory. In the present paper the EQP approximation is applied systematically to the path-ordered Green's-function formalism of Boltermans and Malfliet [6]. We shall find that it is necessary to go beyond the quasiparticle approximations in order to find the analog of Brueckner third-order rearrangements. Therefore, the EQP approximation is a considerable improvement.

The Brueckner formalism as well as applications are found in many publications using different approaches of presentation. The following section contains a concise presentation suitable for comparison with the Green's-function method which is presented in Sec. III. In Sec. IV three approximations of the spectral function, a main ingredient of the Green's-function method are discussed. The extended quasiparticle approximation proposed here has superior properties. This is demonstrated in Sec. V where the zero-temperature case is discussed in relation to Brueckner theory. Section VI contains a short discussion and summary.

II. BRUECKNER FORMALISM

Brueckner theory is strictly only applicable to zero-temperature nuclei. In its original form it only considers particle propagation in intermediate states by defining a two-body reaction matrix or "effective interaction" K with diagonal elements in nuclear matter given by

$$\langle \mathbf{p},\mathbf{p}'|K(\Omega)|\mathbf{p},\mathbf{p}'\rangle=\langle \mathbf{p},\mathbf{p}'|V|\mathbf{p},\mathbf{p}'\rangle+\sum_{\mathbf{p}'',\mathbf{p}'''}\langle \mathbf{p},\mathbf{p}'|V|\mathbf{p}'',\mathbf{p}'''\rangle\frac{[1-n(\mathbf{p}'')][1-n(\mathbf{p}''')]}{\Omega-e_{p''}-e_{p'''}+i\eta}\langle \mathbf{p}'',\mathbf{p}'''|K(\Omega)|\mathbf{p},\mathbf{p}'\rangle\ . \tag{1}$$

Integrations over continuum states are here and in the future for simplicity replaced by summations. Conservations of energy and momentum are implicit. Occupation numbers $n(\mathbf{p})$ refer to the "model," i.e., uncorrelated medium and are equal to 1 and 0 for occupied and unoccupied states, respectively. The $+i\eta$ term is included to obtain outgoing particle states in case of vanishing energy denominator and K is, in general, complex and all energies defined below in terms of K are therefore also complex. We assume that the energies e_p are, as is usual in Brueckner theory, defined by (neglecting the "off-energy-shell" effect in particle lines [9])

$$e_p = p^2/2m + \mathrm{Re}V_B^{(1)}(\mathbf{p},e_p) \ , \tag{2}$$

where subscript B stands for Brueckner and with the potential energy defined below. (Re and Im are used to indicate real and imaginary parts, respectively.) The total energy is given by

$$E_B = \sum_{\mathbf{p}} n(\mathbf{p})p^2/2m + \frac{1}{2}\sum_{\mathbf{p},\mathbf{p}'} \langle \mathbf{p},\mathbf{p}'|K(e_p+e_{p'})|\mathbf{p},\mathbf{p}'\rangle n(\mathbf{p})n(\mathbf{p}') \ . \tag{3}$$

Only the ground-state energy is real because of the Pauli blocking in Eq. (1). The exchange term is here and in the future omitted for shortness. We define SPE's $\epsilon(\mathbf{p})$ by (the Landau definition) [9]

$$\epsilon_B(\mathbf{p}) = \frac{dE_B}{dn(\mathbf{p})} \ , \tag{4}$$

which in our case are generally complex. By direct differentiation of the expression for the total energy in Eq. (3) one gets [10]

$$\epsilon_B(\mathbf{p}) = p^2/2m + V_B^{(1)}(\mathbf{p},e_p) + V_B^{(2)}(\mathbf{p},e_p) + V_B^{(3)}(\mathbf{p},e_p) \ , \tag{5}$$

where the superscript on V indicates the order of K. The first order is obtained by differentiation only with respect to the $n(\mathbf{p})$ explicit in Eq. (3) to get

$$V_B^{(1)}(\mathbf{p},e_p) = \sum_{\mathbf{p}'} \langle \mathbf{p},\mathbf{p}'|K(e_p+e_{p'})|\mathbf{p},\mathbf{p}'\rangle n(\mathbf{p}') \ . \tag{6}$$

The matrix K also depends implicitly on occupation numbers. The differentiation with respect to these give the so-called Brueckner second- and third-order (in K) rearrangement terms. The differentiation with respect to the occupation numbers in the Pauli operator, i.e., in the numerator of the second term of Eq. (1) leads to the second-order term

$$\begin{aligned} V_B^{(2)}(\mathbf{p},e_p) &= -\sum_{\mathbf{p}',\mathbf{p}'',\mathbf{p}'''} \frac{n(\mathbf{p}')n(\mathbf{p}'')[1-n(\mathbf{p}''')]}{e_{p'}+e_{p''}-e_{p'''}-e_p+i\eta} |\langle \mathbf{p}',\mathbf{p}''|K(e_{p'}+e_{p''})|\mathbf{p}''',\mathbf{p}\rangle|^2 \\ &= \sum_{\mathbf{p}',\mathbf{p}'',\mathbf{p}'''} \frac{n(\mathbf{p}'')n(\mathbf{p}''')[1-n(\mathbf{p}')]}{e_p+e_{p'}-e_{p''}-e_{p'''}-i\eta} |\langle \mathbf{p},\mathbf{p}'|K(e_{p''}+e_{p'''})|\mathbf{p}'',\mathbf{p}'''\rangle|^2 \ , \end{aligned} \tag{7}$$

where some relabeling was done in the last expression. Notice that the sign of $i\eta$ is negative which agrees with the chronological (causal) formalism [20] but is opposite to the retarded potential discussed later [see Eq. (60)].

The differentiation with respect to starting energies e_p ($e_{p'}$) in the denominator of K, which depend on the occupation numbers through Eq. (6) , leads to third-order terms

$$\begin{aligned} V_B^{(3a)}(\mathbf{p},e_p) = -\sum_{\mathbf{p}',\mathbf{p}'',\mathbf{p}''',\mathbf{p}''''} & \frac{n(\mathbf{p}')n(\mathbf{p}'')[1-n(\mathbf{p}''')][1-n(\mathbf{p}'''')]}{(e_{p'}+e_{p''}-e_{p'''}-e_{p''''}+i\eta)^2} \\ & \times \langle \mathbf{p},\mathbf{p}'|K(e_p+e_{p'})|\mathbf{p},\mathbf{p}'\rangle |\langle \mathbf{p}',\mathbf{p}''|K(e_{p'}+e_{p''})|\mathbf{p}''',\mathbf{p}''''\rangle|^2 \ . \end{aligned} \tag{8}$$

The differentiation with respect to intermediate energies $e_{p''}$ ($e_{p'''}$) leads similarly to third-order terms

$$\begin{aligned} V_B^{(3b)}(\mathbf{p},e_p) = \sum_{\mathbf{p}',\mathbf{p}'',\mathbf{p}''',\mathbf{p}''''} & \frac{n(\mathbf{p}')n(\mathbf{p}'')[1-n(\mathbf{p}''')][1-n(\mathbf{p}'''')]}{(e_{p'}+e_{p''}-e_{p'''}+i\eta)^2} \\ & \times \langle \mathbf{p},\mathbf{p}'''|K(e_p+e_{p'''})|\mathbf{p},\mathbf{p}'''\rangle |\langle \mathbf{p}',\mathbf{p}''|K(e_{p'}+e_{p''})|\mathbf{p}''',\mathbf{p}''''\rangle|^2 \ . \end{aligned} \tag{9}$$

It is quite practical to do the indicated differentiation numerically in Eq. (4) as done previously [21]. However, the explicit three equations (7), (8), and (9) which were used for preliminary evaluations [10] are also useful for discussing and evaluating occupation probabilities as shown below.

The sum $V_B^{(3a)}+V_B^{(3b)}$ can also be written as

$$\begin{aligned} V_B^{(3)}(\mathbf{p},e_p) = &-\sum_{\mathbf{p}'} \langle \mathbf{p},\mathbf{p}'|K(e_p+e_{p'})|\mathbf{p},\mathbf{p}'\rangle n(\mathbf{p}')C_{p'} \\ &+ \sum_{\mathbf{p}'} \langle \mathbf{p},\mathbf{p}'|K(e_p+e_{p'})|\mathbf{p},\mathbf{p}'\rangle \\ &\times [1-n(\mathbf{p}')]C'_{p'} \ , \end{aligned} \tag{10}$$

where

$$\begin{aligned} C_p = -\sum_{\mathbf{p}',\mathbf{p}'',\mathbf{p}'''} & \frac{n(\mathbf{p}')[1-n(\mathbf{p}'')][1-n(\mathbf{p}''')]}{(e_p+e_{p'}-e_{p''}-\epsilon_{p'''}+i\eta)^2} \\ & \times |\langle \mathbf{p},\mathbf{p}'|K(e_p+e_{p'})|\mathbf{p}'',\mathbf{p}'''\rangle|^2 \end{aligned} \tag{11}$$

and

$$\begin{aligned} C'_p = \sum_{\mathbf{p}',\mathbf{p}'',\mathbf{p}'''} & \frac{n(\mathbf{p}')n(\mathbf{p}'')[1-n(\mathbf{p}'')]}{(e_{p'}+e_{p''}-e_{p'''}-e_p+i\eta)^2} \\ & \times |\langle \mathbf{p}',\mathbf{p}''|K(e_{p'}+e_{p''})|\mathbf{p}''',\mathbf{p}\rangle|^2 \ . \end{aligned} \tag{12}$$

By direct inspection one also finds after slightly generalizing Eq. (6) by replacing e_p by ω

$$C_p=-\left[\frac{\partial\,\mathrm{Re}V_B^{(1)}(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=e_p}. \tag{13}$$

Adding $V_B^{(3)}$ to $V_B^{(1)}$ of Eq. (6) the mean field is therefore normalized by factors $1-C_p$ which typically are ~ 0.8 depending on the strength of the correlations. Likewise

$$C_p'=-\left[\frac{\partial\,\mathrm{Re}V_B^{(2)}(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=e_p}. \tag{14}$$

Equation (10) shows explicitly that the first sum with factors $n(\mathbf{p}')$ is a contribution from normally occupied states while the second with factors $[1-n(\mathbf{p}')]$ is from normally unoccupied states. The coefficient C_p defined by Eqs. (11) and (13) is conveniently interpreted as the correction to the first order (in K) SPE due to the depletion of states in the correlated medium and the coefficient C_p' defined by Eqs. (12) and (14) is the amplitude for finding a nucleon excited into state p because of these correlations. These coefficients therefore relate the occupation numbers $\rho(\mathbf{p})$ in the correlated medium to the occupation numbers $n(\mathbf{p})$ of the uncorrelated medium. The relations are

$$\rho(\mathbf{p})=n(\mathbf{p})(1-C_p) \tag{15}$$

for the hole states and

$$\rho(\mathbf{p})=[1-n(\mathbf{p})]C_p' \tag{16}$$

for particle states. Using the expressions for C and C' given in Eqs. (13) and (14) one obtains for the state with momentum $\mathbf{p}$

$$\rho(\mathbf{p})=n(\mathbf{p})\left[1+\left[\frac{\partial\,\mathrm{Re}V_B^{(1)}(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=\omega_0}\right]$$

$$+[1-n(\mathbf{p})]\left[\frac{\partial\,\mathrm{Re}V_B^{(2)}(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=\omega_0}. \tag{17}$$

This result agrees with Jeukenne, Lejeune, and Mahaux [22]. The discontinuity at the Fermi surface is given by the quasiparticle strength

$$Z_B(\mathbf{p})=1+\left[\frac{\partial\,\mathrm{Re}V_B(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=\omega_0}, \tag{18}$$

where $V_B=V_B^{(1)}+V_B^{(2)}$.

We also find that the third-order rearrangement energy can be simply included by calculating

$$V_B^{(1)}(\mathbf{p},e_p)+V_B^{(3)}(\mathbf{p},e_p)$$

$$=\sum_{\mathbf{p}'}\langle\mathbf{p},\mathbf{p}'|K(e_p+e_{p'})|\mathbf{p},\mathbf{p}'\rangle\rho(\mathbf{p}'), \tag{19}$$

which should be compared with Eq. (6). We shall use these results below.

The Brueckner formalism described above includes energy-diagrams shown in Figs. 1(a) and 1(b) while the SPE diagrams are shown in Fig. 2. If instead of the Brueckner choice for e_p given by Eq. (2) one chooses also to include $\mathrm{Re}V_B^{(3)}$, one can also include the energy diagrams shown in Fig. 1(c) but one has to be careful with overcounting. Second-order insertions can also be included, in principle, by including $\mathrm{Re}V_B^{(2)}$ but off-energy-shell effects will reduce the contribution of these considerably [9]. No serious attempt, however, has been made to calculate this.

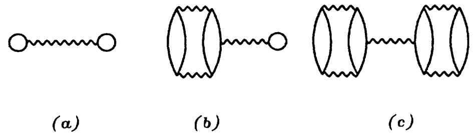

(a) (b) (c)

FIG. 1. Energy diagrams discussed in text. Horizontal lines are effective interactions.

III. GREEN'S-FUNCTION FORMALISM

The spectral function $S(\mathbf{p},\omega)$ is an important quantity for a correlated many-body system. With S given the total energy of the system is determined by

$$E=2\int_{-\infty}^{+\infty}f(\omega)d\omega\int_{-\infty}^{+\infty}\frac{d^3p}{(2\pi\hbar)^3}[p^2/2m+\omega]S(\mathbf{p},\omega)$$

$$=2\int_{-\infty}^{+\infty}\frac{d^3p}{(2\pi\hbar)^3}[p^2/2m+\omega_h(\mathbf{p})]\rho(\mathbf{p}), \tag{20}$$

where $\omega_h(\mathbf{p})$ is the average energy of a hole with momentum p

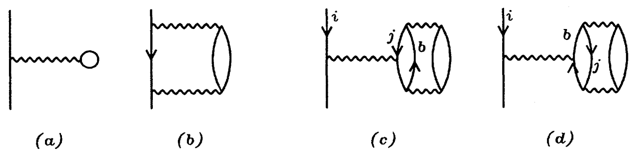
$$\omega_h(\mathbf{p})=\frac{1}{2\pi\hbar\rho(\mathbf{p})}\int_{-\infty}^{+\infty}\omega f(\omega)S(\mathbf{p},\omega)d\omega$$

$$=\omega_0+\frac{1}{2\pi\hbar\rho(\mathbf{p})}\int_{-\infty}^{+\infty}(\omega-\omega_0)f(\omega)S(\mathbf{p},\omega)d\omega. \tag{21}$$

The choice for ω_0 will be discussed later on (Sec. IV). The occupation numbers in the correlated medium are given by

$$\rho(\mathbf{p})=\frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty}f(\omega)S(\mathbf{p},\omega)d\omega. \tag{22}$$

In the path-ordered Green's-function method one defines a retarded effective interaction T^+ in a homogeneous medium by (see, e.g., Refs. [5,6])



(a) (b) (c) (d)

FIG. 2. Single-particle energy diagrams discussed in text. Horizontal lines are effective interactions.

$$\langle \mathbf{p},\mathbf{p}'|T^{+}(\Omega)|\mathbf{p},\mathbf{p}'\rangle=\langle \mathbf{p},\mathbf{p}'|V|\mathbf{p},\mathbf{p}'\rangle+(i\hbar)^2\sum_{\mathbf{p}'',\mathbf{p}''',\omega'',\omega'''}\langle \mathbf{p},\mathbf{p}'|V|\mathbf{p}'',\mathbf{p}'''\rangle \times\frac{g^{>}(\mathbf{p}'',\omega'')g^{>}(\mathbf{p}''',\omega''')-g^{<}(\mathbf{p}'',\omega'')g^{<}(\mathbf{p}''',\omega''')}{\Omega-\omega''-\omega'''+i\eta} \times\langle \mathbf{p}'',\mathbf{p}'''|T^{+}(\Omega)|\mathbf{p},\mathbf{p}'\rangle\ . \tag{23}$$

It is important to realize that the $+i\eta$ term is a consequence of the retarded character of T^{+}.

The retarded mean field Σ^{+} is given by

$$\Sigma^{+}(\mathbf{p},\omega)=-i\hbar\sum_{\mathbf{p}',\omega'}[\langle \mathbf{p},\mathbf{p}'|T^{+}(\omega+\omega')|\mathbf{p},\mathbf{p}'\rangle g^{<}(\mathbf{p}',\omega')+\langle \mathbf{p},\mathbf{p}'|T^{<}(\omega+\omega')|\mathbf{p},\mathbf{p}'\rangle g^{-}(\mathbf{p}',\omega')] \tag{24}$$

with $T^{<}$ given by

$$\langle \mathbf{p},\mathbf{p}'|T^{<}(\Omega)|\mathbf{p},\mathbf{p}'\rangle=i\hbar\sum_{\mathbf{p}'',\mathbf{p}''',\omega'',\omega'''}\langle \mathbf{p},\mathbf{p}'|T^{+}(\Omega)|\mathbf{p}'',\mathbf{p}'''\rangle g^{<}(\mathbf{p}'',\omega'')g^{<}(\mathbf{p}''',\omega''')\langle \mathbf{p}'',\mathbf{p}'''|T^{-}(\Omega)|\mathbf{p}',\mathbf{p}\rangle \tag{25}$$

and with the Green's functions g given by

$$g^{<}(\mathbf{p},\omega)=-(1/i\hbar)S(\mathbf{p},\omega)f(\omega)\ , \tag{26}$$

$$g^{>}(\mathbf{p},\omega)=(1/i\hbar)S(\mathbf{p},\omega)[1-f(\omega)]\ , \tag{27}$$

$$g^{\pm}(\mathbf{p},\omega)=\frac{1}{\omega-p^2/2m-\Sigma^{\pm}(\mathbf{p},\omega)\pm i\eta}\ . \tag{28}$$

Here $f(\omega)$ is a distribution function. For nuclear matter in thermal equilibrium we have

$$f(\omega)=1/[e^{\beta(\omega-\mu)}+1]\ . \tag{29}$$

The spectral function is given in terms of ω-dependent mean fields by

$$S(\mathbf{p},\omega)=\frac{-2\hbar\mathrm{Im}\Sigma^{+}(\mathbf{p},\omega)}{[\omega-p^2/2m-\mathrm{Re}\Sigma^{+}(\mathbf{p},\omega)]^2+[\mathrm{Im}\Sigma^{+}(\mathbf{p},\omega)]^2}\ . \tag{30}$$

It satisfies the sum rule

$$\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}S(\mathbf{p},\omega)d\omega=1\ . \tag{31}$$

IV. EXTENDED QUASIPARTICLE APPROXIMATION

The calculation of, e.g., the total energy requires a self-consistent (iterative) solution of Eqs. (23)–(30) which is a heroic task. The ω integrations in Eq. (23), etc., are, in general, especially complicated to be performed in practice. Some approximation of the spectral function is called for. The simplest is the quasiclassical defined in Sec. IV A. This is however too simplistic for most studies as will be shown, but is convenient as a first step in an iteration scheme also discussed below. An improved but still numerically manageable extended quasiparticle approximation (EQP) for the spectral function will be discussed in Sec. IV C.

A. Quasiclassical approximation

In the limit of small absorption one can use the quasiclassical approximation S_{QC} for the spectral function with

$$S_{\mathrm{QC}}(\mathbf{p},\omega)=2\pi\hbar\delta(\omega-\omega_0)\ , \tag{32}$$

where ω_0 is defined by

$$\omega_0=p^2/2m+\mathrm{Re}\Sigma^{+}(\mathbf{p},\omega_0)\ . \tag{33}$$

One then finds from Sec. III

$$\rho_{\mathrm{QC}}(\mathbf{p})=f(\omega_0)\ , \tag{34}$$

$$\omega_h(\mathbf{p})=\omega_0\ , \tag{35}$$

$$E_{\mathrm{QC}}=2\int_{-\infty}^{+\infty}\frac{d^3p}{(2\pi\hbar)^3}(p^2/2m+\omega_0)f(\omega_0)\ . \tag{36}$$

We next use this approximation to evaluate the first term of Σ^{+} in Eq. (24) and find

$$\Sigma^{+}_{1,\mathrm{QC}}(\mathbf{p},\omega)=\sum_{\mathbf{p}'}\langle \mathbf{p},\mathbf{p}'|T^{+}_{\mathrm{QC}}(\omega+\omega_0')|\mathbf{p},\mathbf{p}'\rangle f(\omega_0')\ . \tag{37}$$

The second term of Σ^{+} in Eq. (24) becomes

$$\Sigma^{+}_{2,\mathrm{QC}}(\mathbf{p},\omega)=\sum_{\mathbf{p}',\mathbf{p}'',p'''}\frac{f(\omega_0'')f(\omega_0''')}{\omega+\omega_0'-\omega_0''-\omega_0'''+i\eta}|\langle \mathbf{p}'',\mathbf{p}'''|T^{+}_{\mathrm{QC}}(\omega_0''+\omega_0''')|\mathbf{p},\mathbf{p}'\rangle|^2\ . \tag{38}$$

The effective interaction T^{+}_{QC} is in the quasiclassical approximation Eq. (32) given by

$$\langle \mathbf{p},\mathbf{p}'|T^{+}_{\mathrm{QC}}(\Omega)|\mathbf{p},\mathbf{p}'\rangle=\langle \mathbf{p},\mathbf{p}'|V|\mathbf{p},\mathbf{p}'\rangle+\sum_{\mathbf{p}'',\mathbf{p}'''}\langle \mathbf{p},\mathbf{p}'|V|\mathbf{p}'',\mathbf{p}'''\rangle\frac{1-f(\omega_0'')-f(\omega_0''')}{\Omega-\omega_0''-\omega_0'''+i\eta}\langle \mathbf{p}'',\mathbf{p}'''|T^{+}_{\mathrm{QC}}(\Omega)|\mathbf{p},\mathbf{p}'\rangle\ . \tag{39}$$

B. Quasiparticle approximation

Generally accepted as a better approximation for the spectral function is the quasiparticle approximation

$$S_{\mathrm{QP}}(\mathbf{p},\omega)=2\pi\hbar\delta(\omega-\omega_0)Z(\mathbf{p}) \tag{40}$$

with the quasiparticle strength Z given by

$$Z(\mathbf{p})=\left[1-\left[\frac{\partial\,\mathrm{Re}\Sigma^+(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=\omega_0}\right]^{-1} \tag{41}$$

It should, however, be noted that this approximation for S does not satisfy the sum rule in Eq. (31). In the quasiparticle approximation hole-state occupations are renormalized by factors $Z(\mathbf{p})$. Thus

$$\rho_{\mathrm{QP}}(\mathbf{p})=Z(\mathbf{p})f(\omega_0)\;. \tag{42}$$

Note that Z is similar to but not equal to the strength Z_B in Brueckner theory given by Eq. (18). With the quasiparticle renormalization one further finds

$$E_{\mathrm{QP}}=2\int_{-\infty}^{+\infty}\frac{d^3p}{(2\pi\hbar)^3}(p^2/2m+\omega_0)Z(\mathbf{p})f(\omega_0)\;. \tag{43}$$

Furthermore,

$$\omega_h(\mathbf{p})=\omega_0 \tag{44}$$

just as in the quasiclassical approximation. We next use this approximation to evaluate the first term of Σ^+ in Eq. (24) and find

$$\Sigma^+_{1,\mathrm{QP}}(\mathbf{p},\omega)=\sum_{\mathbf{p}'}\langle\mathbf{p},\mathbf{p}'|T^+_{\mathrm{QC}}(\omega+\omega_0')|\mathbf{p},\mathbf{p}'\rangle Z(\mathbf{p}')f(\omega_0')\;. \tag{45}$$

The second term of Σ^+ in Eq. (24) becomes

$$\Sigma^+_{2,\mathrm{QP}}(\mathbf{p},\omega)=\sum_{\mathbf{p}',\mathbf{p}'',\mathbf{p}'''}\frac{Z(\mathbf{p}')Z(\mathbf{p}'')Z(\mathbf{p}''')f(\omega_0'')f(\omega_0''')}{\omega+\omega_0'-\omega_0''-\omega_0'''+i\eta}|\langle\mathbf{p}'',\mathbf{p}'''|T^+_{\mathrm{QP}}(\omega_0''+\omega_0''')|\mathbf{p},\mathbf{p}'\rangle|^2\;. \tag{46}$$

The effective interaction T^+_{QP} is evaluated similarly in the quasiparticle approximation Eq. (32) to give

$$\begin{aligned}\langle\mathbf{p},\mathbf{p}'|T^+_{\mathrm{QP}}(\Omega)|\mathbf{p},\mathbf{p}'\rangle&=\langle\mathbf{p},\mathbf{p}'|V|\mathbf{p}',\mathbf{p}'\rangle\\&\quad+\sum_{\mathbf{p}'',\mathbf{p}'''}Z(\mathbf{p}'')Z(\mathbf{p}''')\langle\mathbf{p},\mathbf{p}'|V|\mathbf{p}'',\mathbf{p}'''\rangle\frac{1-f(\omega_0'')-f(\omega_0''')}{\Omega-\omega_0''-\omega_0'''+i\eta}\langle\mathbf{p}'',\mathbf{p}'''|T^+_{\mathrm{QP}}(\Omega)|\mathbf{p},\mathbf{p}'\rangle\;.\end{aligned} \tag{47}$$

The zero-temperature limit of the quasiparticle approximation will be discussed in Sec. V and inconsistencies will be pointed out which are related to the fact that the sum rule is not satisfied. The strength of the quasiparticle peak is reduced by the factor Z but is not distributed over any other states.

C. Extended quasiparticle approximation

The quasiclassical and quasiparticle approximations discussed above imply great numerical simplifications and are therefore common but we have seen above that they are deficient in several respects. The width of the spectral function cannot be neglected in a strongly correlated medium like nuclear matter. A new approximation, the *extended quasiparticle* (EQP) approximation, will now be introduced. In Brueckner theory the occupation-numbers $\rho(\mathbf{p})$ in the correlated medium, to be distinguished from occupation numbers $n(\mathbf{p})$ in the uncorrelated medium, define the third-order rearrangement diagram $V_B^{(3)}$ by Eq. (19). In Green's-function theory $\rho(\mathbf{p})$ is given by Eq. (22), i.e., it is closely related to the spectral function. We shall therefore discuss the EQP approximation together with rearrangement energies and the occupation numbers in the correlated medium. When used in the Green's-function formalism at zero temperature it is found (in Sec. V) to give expressions practically equal to those of Brueckner theory and this approximation therefore establishes a link between the two theories.

In an expansion with $\mathrm{Im}\Sigma^+\ll\mathrm{Re}\Sigma^+$ and $\partial\,\mathrm{Re}\Sigma^+/\partial\omega\ll 1$ one finds [23]

$$S_{\mathrm{EQP}}(\mathbf{p},\omega)=2\pi\hbar\delta(\omega-\omega_0)Z'(\mathbf{p})-\mathcal{P}\frac{2\hbar\,\mathrm{Im}\Sigma^+(\mathbf{p},\omega)}{(\omega-\omega_0)^2}\;, \tag{48}$$

where

$$Z'(\mathbf{p})=1+\left[\frac{\partial\,\mathrm{Re}\Sigma^+(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=\omega_0} \tag{49}$$

and where $\mathcal{P}$ indicates that the principal value is to be taken when integrating over S. This approximation differs from the quasiparticle approximation, shown in Eq. (40) by the last term and also by the strength function Z which is now replaced by a function similar to the Z_B of Eq. (18). The last difference is not important because S_{EQP} is expanded to first order in the ω derivative anyway. More important is the last term in Eq. (48). As pointed out above $S_{\mathrm{QP}}(\mathbf{p},\omega)$ does not satisfy the sum rule, Eq. (31). This is because normally occupied states are depleted (due to the correlations) but in the quasiparticle approximation there is no corresponding occupation of normally unoccupied states. The last term of the *extended* quasiparticle approximation (48) corrects for this deficiency. To show this we use the dispersion relation

$$\mathrm{Re}\Sigma^{+}(\mathbf{p},\omega)=\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{\mathrm{Im}\Sigma^{+}(\mathbf{p},\omega')d\omega'}{\omega'-\omega} \tag{50}$$

to get

$$\left[\frac{\partial\,\mathrm{Re}\Sigma^{+}(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=\omega_0}=\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{\mathrm{Im}\Sigma^{+}(\mathbf{p},\omega')d\omega'}{(\omega'-\omega_0)^2} \tag{51}$$

and one finds

$$\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}S_{\mathrm{EQP}}(\mathbf{p},\omega)d\omega=1+\left[\frac{\partial\,\mathrm{Re}\Sigma^{+}(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=\omega_0}-\left[\frac{\partial\,\mathrm{Re}\Sigma^{+}(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=\omega_0} . \tag{52}$$

The two last terms cancel showing that S_{EQP} satisfies the sum rule, Eq. (31).

From Eq. (22) one obtains in the EQP approximation for the average energy of a hole

$$\omega_h(\mathbf{p})=\omega_0-\frac{1}{\pi\rho(\mathbf{p})}\mathcal{P}\int_{-\infty}^{+\infty}\frac{\mathrm{Im}\Sigma^{+}(\mathbf{p},\omega)f(\omega)}{\omega-\omega_0}d\omega . \tag{53}$$

In the quasiparticle (as well as the quasiclassical) approximation $\omega_h\equiv\omega_0$ but this is not the case here. We find the mean energy of a hole to be shifted relative to the quasiparticle energy ω_0. This is a consequence of the nonsymmetry (skewing) of the spectral function. We shall find below that the shift is large and negative at zero and low temperatures.

We next use this approximation to evaluate the first term of Σ^{+} in Eq. (24). Consistent with the linear expansion of $\mathrm{Re}\Sigma^{+}$ with respect to ω we expand T^{+} similarly and find

$$\Sigma_1^{+}(\mathbf{p},\omega)=\sum_{\mathbf{p}'}\langle\mathbf{p},\mathbf{p}'|T^{+}(\omega+\omega_h(\mathbf{p}'))|\mathbf{p},\mathbf{p}'\rangle\rho(\mathbf{p}') . \tag{54}$$

This is an interesting result. It shows, for example, that the mean field at an energy ω_0 should be evaluated with a T^{+} matrix defined with $\Omega=\omega_0+\omega_h$ but not at $\Omega=\omega_0+\omega_0'$ as in the quasiclassical approximation in Sec. III. We shall find in Sec. V C that the difference between the two energies is essentially the Brueckner second-order rearrangement energy which for hole states is ~20 MeV or more [20]. It also shows that the mean field becomes "renormalized" with a factor ρ which corresponds to the inclusion of a Brueckner third-order rearrangement energy [19]. The occupation-number ρ in the correlated medium is now given by

$$\rho(\mathbf{p})=Z'(\mathbf{p})f(\omega_0)-\frac{\mathcal{P}}{\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{Im}\Sigma^{+}(\mathbf{p},\omega)f(\omega)}{(\omega-\omega_0)^2}d\omega . \tag{55}$$

The principal value part of S_{EQP} [the second term in Eq. (48)] was an important part of the expressions above. In evaluating the second term of the mean field in Eq. (24) and the effective interaction T^{+} in Eq. (23) this part is, however, neglected below. This will be partly justified in Sec. V C. The second term of the retarded mean field Σ^{+} in Eq. (24) then becomes exactly like $\Sigma_{2,\mathrm{QP}}^{+}$ in Sec. IV B except with Z replaced by Z' and similarly for T^{+}.

The effect of the broadening of the spectral function is only to reduce the strength with no shift in energies ω from the quasi-particle value ω_0. At zero temperature the contribution from the skewness of the spectral function expressed by the second term of the EQP approximation is of a higher-order derivative of Σ^{+} with respect to ω as will be shown in Sec. V.

V. ZERO-TEMPERATURE LIMIT

The Green's-function formalism presented above is valid for nonzero as well as zero temperatures. In the zero-temperature limit it is of importance to compare it with more established theories. In turns out to be especially interesting to do so with Brueckner theory.

A. Quasiclassical approximation

The quasiclassical approximation for the spectral function is given by Eq. (32) with quasiparticle energies ω_0 defined by Eq. (33). At zero temperature one finds

$$E_{\mathrm{QC}}=4\int_0^{p_F}\frac{d^3p}{(2\pi\hbar)^3}[p^2/2m+\tfrac{1}{2}\Sigma^{+}(\mathbf{p},\omega_0)] . \tag{56}$$

The SPE's ω_0 differ from the Brueckner energies e_p defined in Eq. (2). For a comparison with the mean fields in Brueckner formalism we shall therefore replace the ω_0 with e_p in Eq. (40). This change will be discussed again below. For T^{+} one gets from Eq. (39) at zero temperature

$$\langle\mathbf{p},\mathbf{p}'|T_{\mathrm{QC}}^{+}(\Omega)|\mathbf{p},\mathbf{p}'\rangle=\langle\mathbf{p},\mathbf{p}'|V|\mathbf{p},\mathbf{p}'\rangle+\sum_{\mathbf{p}'',\mathbf{p}'''}\langle\mathbf{p},\mathbf{p}'|V|\mathbf{p}'',\mathbf{p}'''\rangle\frac{[1-n(\mathbf{p}'')][1-n(\mathbf{p}''')]-n(\mathbf{p}'')n(\mathbf{p}''')}{\Omega-e_p''-e_p'''+i\eta}\langle\mathbf{p}'',\mathbf{p}'''|T_{\mathrm{QC}}^{+}(\Omega)|\mathbf{p},\mathbf{p}'\rangle . \tag{57}$$

The effective interaction T_{QC}^{+} differs from K defined in Eq. (1) by the hole-hole propagation.

The mean field becomes, from (37) and (38),

$$\Sigma_{\mathrm{QC}}^{+}(\mathbf{p},\omega)=\sum_{\mathbf{p}'}\langle\mathbf{p},\mathbf{p}'|T_{\mathrm{QC}}^{+}(\omega+e_p')|\mathbf{p},\mathbf{p}'\rangle n(\mathbf{p}')+\sum_{\mathbf{p}',\mathbf{p}'',\mathbf{p}'''}\frac{n(\mathbf{p}'')n(\mathbf{p}''')}{\omega+e_{p'}-e_p''-e_p'''+i\eta}|\langle\mathbf{p},\mathbf{p}'|T_{\mathrm{QC}}^{+}(e_p''+e_p''')|\mathbf{p}'',\mathbf{p}'''\rangle|^2 . \tag{58}$$

The last term is of second order in T^{+}_{QC} but is different from the Brueckner second-order term $V_B^{(2)}$ given by Eq. (7). The sign of the $i\eta$ term is opposite and we shall return to this below. More important for the present discussion is that $V_B^{(2)}$ has an extra factor $1-n(\mathbf{p}')$. We note, however, that while T^{+}_{QC} and K are different they are related by

$$\langle \mathbf{p},\mathbf{p}'|T^{+}(\Omega)|\mathbf{p},\mathbf{p}'\rangle=\langle \mathbf{p},\mathbf{p}'|K(\Omega)|\mathbf{p},\mathbf{p}'\rangle-\sum_{\mathbf{p}'',\mathbf{p}'''}\langle \mathbf{p},\mathbf{p}'|K(\Omega)|\mathbf{p}'',\mathbf{p}'''\rangle\frac{n(\mathbf{p}'')n(\mathbf{p}''')}{\Omega-e_{p''}-e_{p'''}+i\eta}\langle \mathbf{p}'',\mathbf{p}'''|T^{+}(\Omega)|\mathbf{p},\mathbf{p}'\rangle\ . \qquad (59)$$

Inserting this expression for T^{+} into the expression (58) for Σ^{+} one finds to second order in K

$$\begin{aligned}\Sigma^{+}(\mathbf{p},\omega)&\sim\sum_{\mathbf{p}'}\langle \mathbf{p},\mathbf{p}'|K(\omega+e_{p'})|\mathbf{p},\mathbf{p}'\rangle n(\mathbf{p}')+\sum_{\mathbf{p}',\mathbf{p}'',\mathbf{p}'''}\left[\frac{n(\mathbf{p}'')n(\mathbf{p}''')}{\omega+e_{p'}-e_{p''}-e_{p'''}+i\eta}|\langle \mathbf{p},\mathbf{p}'|K(e_{p''}+e_{p'''})|\mathbf{p}'',\mathbf{p}'''\rangle|^2\right.\\&\qquad\left.-\frac{n(\mathbf{p}')n(\mathbf{p}'')n(\mathbf{p}''')}{\omega+e_{p'}-e_{p''}-e_{p'''}+i\eta}|\langle \mathbf{p},\mathbf{p}'|K(\omega+e_{p'})|\mathbf{p}'',\mathbf{p}'''\rangle|^2\right]\\&\sim\sum_{\mathbf{p}'}\langle \mathbf{p},\mathbf{p}'|K(\omega+e_{p'})|\mathbf{p},\mathbf{p}'\rangle n(\mathbf{p}')+\sum_{\mathbf{p}',\mathbf{p}'',\mathbf{p}'''}\frac{n(\mathbf{p}'')n(\mathbf{p}''')[1-n(\mathbf{p}')]}{\omega+e_{p'}-e_{p''}-e_{p'''}+i\eta}|\langle \mathbf{p},\mathbf{p}'|K(e_{p''}+e_{p'''})|\mathbf{p}'',\mathbf{p}'''\rangle|^2\ .\end{aligned}\qquad (60)$$

In the last step $K(\omega+e_{p'})$ was approximated by $K(e_{p''}+e_{p'''})$. This would not be a good approximation for short-range correlations but the hole-hole scatterings are mostly forward scatterings from the long-ranged part of the interaction and it is therefore expected to be satisfactory in this case [10]. In this approximation the terms of second order in K now agree, except for the signs of the imaginary terms, with the Brueckner second-order rearrangement term in (7). This sign difference is, however, familiar from the difference between retarded and chronological formalisms [24,25]. In this respect it should be noted that when calculating, e.g., the total energy in the chronological formalism, one has to incorporate a converging factor to eliminate the semicircle contribution in the omega-integration. This is equivalent to directly using the appropriate retarded quantities in the total energy expression. Apart from the sign we conclude that there should be rather small difference between the Green's-function retarded mean field Σ^{+} calculated in the quasiclassical approximation and the Brueckner mean field, i.e.,

$$\begin{aligned}\mathrm{Re}\Sigma^{+}&\sim\mathrm{Re}V_B^{(1)}+\mathrm{Re}V_B^{(2)}\ ,\\ \mathrm{Im}\Sigma^{+}&\sim\mathrm{Im}V_B^{(1)}-\mathrm{Im}V_B^{(2)}\ .\end{aligned}\qquad (61)$$

This has been verified by actual calculation [19].

It must be emphasized however that our conclusion was based on assuming that the SPE's are defined equally in both formalisms. In actual applications this is usually not so. The definition e_p by Eq. (2) is (most commonly) used in Brueckner calculations. This energy is at (or near) the average energy of a hole, averaged over the spectral function [19] while in Green's-function calculations ω_0 defined by Eq. (33) is at the peak of the spectral function. We shall return to this point in Sec. V C.

The main conclusion from the results above is that at the quasiclassical level of approximation the Green's-function theory contains a term quite similar to the Brueckner second-order rearrangement energy. However, there are no terms that correspond to the Brueckner *third*-order rearrangement terms. A better approximation is needed for the spectral function in order for the Green's-function theory to be on a par with Brueckner theory.

B. Quasiparticle approximation

The quasiparticle approximation defined by Eq. (40) differs from the quasiclassical by the renormalization-factors $Z(\mathbf{p})$ given by Eq. (41). In this approximation the total energy becomes, at zero temperature,

$$E_{\mathrm{QP}}=4\int_0^{p_F}\frac{d^3p}{(2\pi\hbar)^3}[p^2/2m+\tfrac{1}{2}\Sigma^{+}(\mathbf{p},\omega_0)]Z(\mathbf{p})\ . \qquad (62)$$

The occupation numbers and hole renormalizations of the mean field are in Brueckner theory shown by Eqs. (17) and (19) while in the quasiparticle approximation one has from Eq. (42) $\rho(\mathbf{p})=Z(\mathbf{p})n(\mathbf{p})$ and therefore $\rho(\mathbf{p})=0$ for $p>p_F$ which differs from Brueckner theory. Another difference from Brueckner theory is that the full field Σ^{+} rather than just $V_B^{(1)}$ defines the renormalization factor. Although the quasiparticle approximation thus generates a term similar to the Brueckner third-order rearrangement term $V_B^{(3a)}$ represented by the diagram in Fig. 2(c). It is numerically different. In addition there is no term corresponding to $V_B^{(3b)}$ represented by the diagram in Fig. 2(d). This is related to the fact that the sum rule (31) for the spectral function is not satisfied in the quasiparticle approximation. In conclusion, the quasiparticle approximation is an improvement over the quasiclassical but it also leads to serious inconsistencies. The *extended* quasiparticle approximation discussed at zero temperature in the next section will remedy this situation.

C. Extended quasiparticle approximation

The EQP approximation introduced in Sec. IV C is here applied to zero-temperature nuclear matter. In order to simplify the result and for comparison with Brueckner theory we first note that for $T=0$, $\mathrm{Im}\Sigma^{+}(\mathbf{p},\mu)=0$. We furthermore define $\mathrm{Im}V_0$ and $\mathrm{Im}V_2$ by

$$\begin{aligned}
&\mathrm{Im}V_0(\mathbf{p},\omega)=0, \quad \omega<\mu \ , \\
&\mathrm{Im}V_0(\mathbf{p},\omega)=\mathrm{Im}\Sigma^+(\mathbf{p},\omega), \quad \omega>\mu \ , \\
&\mathrm{Im}V_2(\mathbf{p},\omega)=0, \quad \omega>\mu \ , \\
&-\mathrm{Im}V_2(\mathbf{p},\omega)=\mathrm{Im}\Sigma^+(\mathbf{p},\omega), \quad \omega<\mu \ .
\end{aligned} \tag{63}$$

It was shown in Sec. V A that V_2 is then (essentially) the Brueckner second-order rearrangement energy and obeys a dispersion relation. As a consequence then, ω_h given by Eq. (53) becomes

$$\begin{aligned}
\omega_h(\mathbf{p})&=\omega_0-\frac{\mathrm{Re}V_2(\mathbf{p},\omega_0)}{\rho(\mathbf{p})} \\
&\sim p^2/2m+\mathrm{Re}V_0(\mathbf{p},\omega_0) \ .
\end{aligned} \tag{64}$$

The last approximation is obtained with $\rho\sim 1$ and applies only to states below the Fermi surface, $p<p_F$. It illustrates the often used statement that the Brueckner second-order rearrangement energy contributes not to the removal energy (average energy of a hole state) but only to its width [18,19,26] which comes from the imaginary part.

It is at this point also of interest to observe that as a consequence of $\mathrm{Re}V_0(\mathbf{p},\omega_0)\sim\mathrm{Re}V_B^{(1)}(\mathbf{p},e_p)$ we also find from Eq. (2) that $\omega_h\sim e_p$. A main reason that $\mathrm{Re}V_0$ and $\mathrm{Re}V_B^{(1)}$ are only approximately equal is that the energies ω_0 and e_p differ, essentially by a second-order rearrangement energy. Using $S_{\mathrm{EQP}}(\mathbf{p},\omega)$ to calculate the occupation numbers by Eq. (22) we find for states below the Fermi surface $(\omega_0<\mu)$

$$\begin{aligned}
\rho(\mathbf{p})=1+&\left[\frac{\partial\,\mathrm{Re}\Sigma^+(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=\omega_0} \\
&-\frac{1}{\pi}P\int_{-\infty}^{\mu}\frac{\mathrm{Im}\Sigma^+(\mathbf{p},\omega)d\omega}{(\omega-\omega_0)^2} \ .
\end{aligned} \tag{65}$$

But using the approximate relation in Eq. (61) and the fact that $\mathrm{Im}V_B^{(1)}(\mathbf{p},\omega)=0$ for $\omega<\mu$ we can substitute the last Σ^+ in Eq. (65) by $V_B^{(2)}$ and because $V_B^{(2)}(\mathbf{p},\omega)=0$ for $\omega>\mu$ we can extend the integration to $+\infty$. Using the dispersion relation we then find that Eq. (65) reduces to

$$\rho(\mathbf{p})=1+\left[\frac{\partial\,\mathrm{Re}V_B^{(1)}(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=\omega_0} \tag{66}$$

and this is exactly the result obtained from Brueckner theory given in Eq. (17) for particles below the Fermi surface.

For $\omega_0>\mu$, i.e., for states not occupied in the uncorrelated medium the δ function, the first term in S_{EQP}, does not contribute to the integral over ω. Again using the above relations and properties of the mean field we find the contribution from the second term to give

$$\rho(\mathbf{p})=\left[\frac{\partial\,\mathrm{Re}V_B^{(2)}(\mathbf{p},\omega)}{\partial\omega}\right]_{\omega=\omega_0} \tag{67}$$

that also agrees exactly with the Brueckner result, Eq. (17). The question of whether this approximation for $\rho(\mathbf{p})$ is number conserving or not has been discussed at length by Mahaux and Sartor [20]. If the two potentials $V_B^{(1)}$ and $V_B^{(2)}$ are calculated from the same effective interactions, it will be.

The total energy given by Eq. (20) becomes in the approximation (64) for ω_h

$$E_{\mathrm{EQP}}=4\int_{-\infty}^{+\infty}\frac{d^3p}{(2\pi\hbar)^3}[p^2/2m+\tfrac{1}{2}V_0(\mathbf{p},\omega_0)]\rho(\mathbf{p}) \ . \tag{68}$$

This is a very interesting result because it shows that the second-order rearrangement energy does not contribute to the total energy. Note that the Brueckner approximation for the energy E_B, given by Eq. (3) is just this expression except that $\rho(\mathbf{p})$ is replaced by $n(\mathbf{p})$ so that the integration over momentum is only over normally occupied states when calculating E_B.

The result for E_{EQP} at zero temperature in Eq. (3) should also be compared with the quasiclassical approximation E_{QC} in Eq. (56). The difference between these two approximations for the total energy is essentially an averaged Brueckner second-order rearrangement energy which is ~ 10 MeV [20].

The quasiparticle approximation E_{QP}, on the other hand, simulates that for E_{EQP} except that the momentum-integration extends only to p_F. Some estimates of these various expressions for the energy are found in Sec. VI.

In Brueckner theory the third-order rearrangement energy was expressed in terms of occupation numbers $\rho(\mathbf{p})$ as shown by Eq. (19). Exactly the same expression is obtained in Green's-function theory if in Eq. (54) (i) the $\omega_h(\mathbf{p}')$ in T^+ is replaced by $e_{p'}$ which we already established after Eq. (64) to be a pair approximation and (ii) T^+ is expanded to first order in K by Eq. (59).

Occupation numbers $\rho(\mathbf{p})$ take into account the correlations among nucleons in the medium. We have found that the occupation-numbers derived by the Brueckner formalism above are consistent with the use of the approximate spectral function S_{EQP} in the Green's-function method. We can therefore conclude that this approximation establishes a link between the two theories. In Brueckner theory improvements consist of considering higher-order diagrams, higher-order insertions in hole and/or particle lines, three-body and ring diagrams, etc. Improvements in Green's-function methods consist of using improved spectral functions in Eqs. (23) and (24) thereby including higher-order correlations.

What we have established is that at zero-temperature the expressions obtained in the EQP approximation for the spectral function yield results that have a structure very similar to that of the Brueckner theory. There are some differences. A difference is that in Brueckner theory the mean field is (usually) calculated at a prechosen self-consistent energy e_p defined by Eq. (2) differing from the choice ω_0 which is dictated by the spectral function. The difference appears essentially in the second-order rearrangement energy.

The contribution to T^+ and Σ_2^+ from the principal value part of S_{EQP} was neglected above in Sec. IV C. At zero temperature one is better able to estimate these terms. One is, in general, faced with performing energy integrals of two types:

$$I_1=\frac{\mathcal{P}}{\pi}\int\frac{[1-f(\omega)]\mathrm{Im}\Sigma^+(\mathbf{p},\omega)}{(a-\omega)(\omega-\omega_0)^2}d\omega \tag{69}$$

and

$$I_2=\frac{\mathcal{P}}{\pi}\int\frac{f(\omega)\mathrm{Im}\Sigma^+(\mathbf{p},\omega)}{(a-\omega)(\omega-\omega_0)^2}d\omega\ . \tag{70}$$

At zero temperature one finds

$$I_1=\frac{\mathcal{P}}{\pi}\int_{\mu}^{\infty}\frac{\mathrm{Im}\Sigma^+(\mathbf{p},\omega)}{(a-\omega)(\omega-\omega_0)^2}d\omega \tag{71}$$

or

$$I_1=\frac{\mathcal{P}}{\pi}\frac{\partial}{\partial\omega_0}\int_{-\infty}^{\infty}\frac{\mathrm{Im}V_0(\mathbf{p},\omega)}{(a-\omega)(\omega-\omega_0)}d\omega\ , \tag{72}$$

where in the last step the integration was extended using Eq. (63). This integral can now be performed using the dispersion relation for V_0 to give

$$\begin{aligned}I_1&=\frac{\partial}{\partial\omega_0}\frac{1}{a-\omega_0}[-\mathrm{Re}V_0(\mathbf{p},a)+\mathrm{Re}V_0(\mathbf{p},\omega_0)]\\&=\frac{1}{(a-\omega_0)^2}[-\mathrm{Re}V_0(\mathbf{p},a)+\mathrm{Re}V_0(\mathbf{p},\omega_0)]\\&\quad+\frac{1}{a-\omega_0}\left[\frac{\partial}{\partial\omega}\mathrm{Re}V_0(\mathbf{p},\omega)\right]_{\omega=\omega_0}\ .\end{aligned} \tag{73}$$

Results of calculations of $V_0(\mathbf{p},\omega)$ show a near linear ω dependence. From the last expression for I_1 one can therefore argue that it should be small. Although this was derived for zero temperature we assumed I_1 to also be small at the nonzero temperatures considered in Sec. IV.

The integral I_2 will lead to similar results at zero temperature as Eq. (73) but with V_0 replaced by V_2. It is found in Refs. [12,19] that the ω dependence of V_2 certainly has a nonzero second derivative so that I_2 may have to be included by explicitly evaluating Eq. (73) (with V_0 replaced by V_2).

It deserves to be pointed out that the EQP approximation for the spectral function implies an expansion of the imaginary part of the mean field and its ω dependence to first order and that the validity of such an approximation may be questioned especially for a nonequilibrium system at high temperature or excitation. For ground-state properties for which Brueckner theory is designed this seems, however, a valid assumption. On the other hand, it can be concluded that in order for the Green's-function methods to be on a par with Brueckner theory one needs a spectral function at the very least as good as the extended quasiparticle approximation or EQP.

VI. DISCUSSION AND SUMMARY

Although Brueckner theory is basically a static theory it provides a connection with dynamic theory through the imaginary part of the self-energy and this has been an important contribution of this theory for discussing and understanding H.I. collisions [8]. Path-ordered Green's-function methods provides a more complete theory of the dynamics and we have here referred mostly to recent work by Botermans and Malfliet [6] developed specifically for nuclear transport phenomena. Brueckner theory has been applied to both nuclear matter and the nuclear Hartree-Fock problem for finite nuclei with considerable success. Using the (Goldstone) diagrammatic techniques usually associated with Brueckner theory, diagrams of arbitrary order can, in principle, be calculated. Considerable effort has been used in calculating such contributions especially for nuclear matter and considerable experience in their relative importance has been gained. In order to explain basic properties of nuclei related to correlations, the minimal requirement is that the terms and diagrams considered in Sec. II and Figs. 1 and 2 are considered. These include the second- and third-order rearrangement terms.

As an example of the difference between the different approximations that we have discussed the following examples are illustrative. Using the extended quasiparticle approximation the total binding energy is obtained from Eq. (68) to give

$$E_{\mathrm{EQP}}=17.4\ \mathrm{MeV/nucleon}$$

as shown in Ref. [19]. This calculation involves an integration over both normally occupied *and* unoccupied states. The latter are very strongly bound. In contrast, in the quasiparticle approximation there is no contribution from $p>p_F$ as shown by Eq. (43). Using the data from Ref. [19] one approximately finds

$$E_{\mathrm{QP}}=15.9\ \mathrm{MeV/nucleon}\ .$$

Brueckner theory meanwhile gives [19]

$$E_B=16.7\ \mathrm{MeV/nucleon}\ .$$

However, with $V_B^{(1)}(\mathbf{p},e_p)$ replaced by $V_B^{(1)}(\mathbf{p},\omega_0)$ we found in Ref. [19] a binding

$$E_B=20.0\ \mathrm{MeV/nucleon}\ .$$

The quasiclassical approximation would likewise yield

$$E_{\mathrm{QC}}=10.5\ \mathrm{MeV/nucleon}\ .$$

The difference is due to the repulsive second-order rearrangement energy. (The last results above are partly estimates. Each approximation is not carried through self-consistently.) This serves to illustrate the relative numerical accuracy of these various approximations.

We have found that in order to bring the Green's-function method up to a similar degree of approximation as Brueckner theory the minimal requirement is that with mean fields calculated from Eq. (24) the spectral function should be given by the extended quasiparticle approximation of Eq. (48). In other words, we can state that doing a Green's-function calculation in the EQP approximation in many ways is equivalent to doing a Brueckner calculation.

We have above stressed the similarities between the two formalisms and the next step is to explore the differences. The Green's-function formalism contains corrections associated with the spreading of the spectral function. We have seen above that the Brueckner theory in fact does contain some of this spreading although implicitly. We did, in fact, show that in the limit of small absorption the occupation numbers are identical but we like here to emphasize that this may be a poor approximation for high-energy heavy-ion collisions for which the spreading of the spectral function can be expected to affect the dynamics in an important way.

The question of choice of single particle energies e_p in the definition of the K and T^+ matrices was brought up above. In most Brueckner calculations one chooses a spectrum defined by Eq. (2) while the Green's-function expression is given by Eq. (33) which is closely related to the spectral function behavior. For detailed calculations the difference can be important, as already illustrated by the calculation of binding energies above.

To go beyond the approximations in this paper, numerical work is necessary. Some such calculations have already been done and a main conclusion of this paper, namely, the close agreement between Brueckner and Green's-function theory in the EQP approximation has been verified numerically [19]. Recently a complete calculation of the equation of state, spectral function, and occupational probabilities of nuclear matter has been performed using a relativistic version of the Green's-function formalism [27].

Our approximation for the spectral function can be modified slightly while maintaining its simplicity and renormalization property. As an example, preliminary work indicates that a slightly better approximation is given by

$$S_{\mathrm{EQP}}(\mathbf{p},\omega)=Z(\mathbf{p})\left[2\pi\hbar\delta(\omega-\omega_0)-\mathcal{P}\frac{2\hbar\,\mathrm{Im}\Sigma^+(\mathbf{p},\omega)}{(\omega-\omega_0)^2}\right].$$

The solution of the many-body problem by the Green's-function formalism of Sec. III requires, in principle, a repeated iteration of the spectral function. This would be very tedious to carry through numerically. Some approximation is necessary. The simplest approximation of the spectral function is the quasiclassical where a δ function at the quasiparticle energy is chosen. It is, however, known that this would be a very poor approximation. In Brueckner theory it would, for example, correspond to neglecting third-order rearrangement energies and all single-particle energies would be quasiparticle energies. The extended quasiparticle approximation corresponds more closely to the Brueckner theory and we believe it is practical to implement numerically. The spectral function S_{EQP} contains the mean field and an iteration scheme is still necessary. The suggested scheme is to start by using the quasiclassical approximation in the first iteration. Quasiparticle energies ω_0 and mean fields $\Sigma^+(\mathbf{p},\omega_0)$ as well as derivatives of Σ^+ with respect to ω are determined. This requires an additional iteration for self-consistent mean fields (Brueckner self-consistency). A first approximation for the extended quasiparticle spectral function is then given by Eq. (48). The strength function $Z'(\mathbf{p})$ [or $Z(\mathbf{p})$], the occupation numbers $\rho(\mathbf{p})$, and hole energies $\omega_h(\mathbf{p})$ are obtained from Eqs. (49), (55), and (53), respectively. The total energy can now be calculated from Eq. (20) and this is an improvement over the quasiclassical approximation. New interactions T^+ and mean fields Σ^+ can now be calculated by the expressions in Sec. IV C. The iteration of the spectral function can then be continued until convergence. Convergence of these iterations can only be tested by actual calculation but it is expected that one full iteration is sufficient.

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