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Microscopic approach to pion-nucleus dynamics

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Elastic scattering of pions from finite nuclei is investigated utilizing a contemporary, momentum-space first-order optical potential combined with microscopic estimates of second-order corrections. The calculation of the first-order potential includes (1) full Fermi-averaging integration including the Δ propagation and the intrinsic nonlocalities in the π-N amplitude, (2) covariant kinematics, (3) invariant amplitudes, and (4) a finite-range off-shell pion-nucleon model which contains the nucleon-pole term. The Δ nucleus interaction is included via the mean spectral-energy approximation. This approach produces a convergent perturbation theory in which the Pauli corrections (treated as a second-order term) cancel remarkably against the pion true-absorption terms. Parameter-free results, including the Δ-nucleus shell-model potential, Pauli corrections, pion true absorption, and short-range correlations, are presented.

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Pion scattering measurements, in combination with phenomenological descriptions [1] of the propagation of the pion and the Δ in the nuclear medium, have proved useful for probing details of nuclear structure. The situation is not, however, entirely satisfactory because some of the parameters in these phenomenological descriptions have not been derived quantitatively, even though their physical origin is believed to be understood. Chief among these is a shift [2] in the energy of the two-body, pion-nucleon scattering amplitude, which is evaluated somewhat arbitrarily. Therefore, in this work we want to see how far we can go in describing the dynamics of the pion and the Δ starting from a purely microscopic approach in which the dynamics (including the energy at which the in-medium two-body amplitude is to be evaluated) are completely determined from theory. Such an understanding is needed before one can envision making a reliable extension of the theory to higher nuclear densities and high temperatures, where the propagation of the pion in the nuclear medium plays an important role in both heavy-ion reactions and in astrophysical problems.

From the results of such a microscopic approach we hope to learn the extent to which the existing phenomenologies are in quantitative agreement with the dynamics as understood in a variety of contexts, including what is known about the reactive content of the interaction (true absorption, quasielastic scattering, and correlation effects), delta-nucleus dynamics (the delta-nucleus interaction, delta propagation, and the Pauli principle), and the interplay of the reaction dynamics with nuclear structure effects. Although some calculations [3–7] of pion scattering do include higher-order terms coming from these effects, a modern, microscopic test of pion-nucleus dynamics that makes contact with all this information does not yet exist.

Such a test of pion-nucleus dynamics must deal carefully with several well appreciated but technically awkward aspects of the dynamics. One is Fermi averaging, which is expressed as a three-dimensional integral of the off-shell pion-nucleon scattering amplitude over the nuclear density matrix. The exact performance of this integration incorporates both the propagation of the delta and the intrinsic nonlocalities that are inherent to a two-body resonating amplitude. Another is the Lorentz-covariant kinematics. Finally, the pion-nucleon amplitude utilized should contain explicitly the nucleon pole. The neglect of this singularity in the two-body amplitude leads [8] to an artificially low momentum cutoff that produces a geometrical change in the effective radius [9] of the nucleus. We here make a test of pion-nucleus dynamics within the framework of the optical potential which incorporates all of these features. The isobar-hole model [7], which was a successful semimicroscopic approach to the dynamics, has served as a phenomenological tool to fit various pion- (and photon-) induced reactions, including the true-absorption and quasielastic channels. Much has been learned about pion and delta dynamics from this model. Even more has been learned from the abundance of high-precision data that have been taken at the meson factories during the ten years since the inception of the model. Our work relies on this progress to generate a parameter-free microscopic theory, which we will compare here to elastic scattering data from 80 to 226 MeV.

The improvements that we feel to be needed are naturally incorporated by working in momentum space. A technical advance which is particularly suited to momentum space is [10] the use of "relativistic, three-body, recoupling coefficients." These incorporate exactly Lorentz-covariant kinematics [11,12] (including Wigner spin precision), and they provide natural variables for performing the Fermi-averaging integration. The first-order optical potential is given by

$$(\mathbf{k}'_\pi \mathbf{k}'_A | U | \mathbf{k}_\pi \mathbf{k}_A) = \sum_\alpha \int \frac{d^3k'_n}{2E'_n} \frac{d^3k_{A-1}}{2E_{A-1}} \frac{d^3k_n}{2E_n} \langle \psi^\alpha_{k_A} | \mathbf{k}'_n k'_{A-1} \rangle \langle \mathbf{k}'_\pi \mathbf{k}'_n | t(W_\alpha) | \mathbf{k}_\pi \mathbf{k}_n \rangle \langle \mathbf{k}_n \mathbf{k}_{A-1} | \psi^\alpha_{k_A} \rangle \ . \quad (1)$$

with $\langle \mathbf{k}_n \mathbf{k}_{A-1} | \psi^{\alpha}_{k_A} \rangle$ the target wave function (labeled by its eigenvalues α and proportional to a momentum-conserving delta function). The pion-nucleon amplitude $\langle \mathbf{k}'_\pi \mathbf{k}'_n | t(W) | \mathbf{k}_\pi \mathbf{k}_n \rangle$ also contains a momentum-conserving delta function. The three implicit momentum-conserving delta functions produce an overall momentum-conserving delta function and leave a three-dimensional integration (the Fermi averaging integration) to be performed numerically. The kinematics involved in Eq. (1) are those of a relativistic three-body problem with momenta $\mathbf{k}_\pi$, $\mathbf{k}_n$, and $\mathbf{k}_{A-1}$; the details of how relativistic recoupling coefficients allow one to calculate Eq. (1) can be found in Ref. [13]. We use invariant amplitudes [13,14] that are free of kinematic singularities and utilize invariantly normed wave functions; these introduce phase-space factors into the calculation which can only be treated exactly by working in momentum space.

In momentum-space the lowest-order optical potential as we formulate it can be evaluated without approximation. In this sense, our work improves not only the phenomenological optical model [1], but also the numerous aspects of the isobar-hole model [7], which were both expressed in coordinate space, where nonlocalities are not as easily handled. The propagation of the delta was fully incorporated in the isobar-hole model, but the integration over the nonlocalities associated with the two-body amplitude were approximated by factorization—an approximation that necessitates [15] a non-negligible correction, particularly for lighter nuclei. To deal with Lorentz-covariant kinematics, expansions and further factorizations of integrals were made. Also, the pole in the two-body amplitude was neglected, something we have avoided in order to eliminate the possibility of a spurious geometrical change in the effective radius [9] of the nucleus.

Given that we are able to calculate the first-order optical potential without approximation, there remains the question of how to organize many-body theory (in particular, choosing the energies of the nucleon and the delta in the medium) to optimize its rate of convergence. The role of the energy W_α in Eq. (1) is quite important in this regard because the half width of the delta resonance (55 MeV) is the same size as energies that characterize nuclei. Thus, the results of a calculation will be sensitive to how the energies that constitute W_α are chosen. W_α is defined covariantly as the energy available in the center-of-momentum frame of the pion-nucleon system, $W_\alpha^2 = W_{\pi n}^2 - (\mathbf{k}_\pi + \mathbf{k}_n)^2$, with $W_{\pi n}$ defined as the energy available to the πN pair in the pion-target center-of-momentum frame

$$W_{\pi n} = W_0 - \sqrt{(\mathbf{k}_\pi + \mathbf{k}_n)^2 + m_{A-1}^2}$$

and $W_0^2 = S$, the invariant square energy of the reaction. The mass of the $A-1$ system, m_{A-1}, differs from the mass of the A-body target, m_A, by a nucleon mass and a binding energy, $m_A = m_{A-1} + m_n + E_b$. In its nonrelativistic limit, this energy is known [16] as the "three-body energy denominator."

Utilizing the above definition of W_α produces needlessly large higher-order corrections [16,17] in the many-body expansion. This is because the delta-nucleus shell-model potential U_Δ, which is believed to be nearly equal to the potential energy of a nucleon in the nucleus, has not yet been included. Including the effects of U_Δ in the T matrix causes an effective downward shift in the position of the resonance that tends to cancel the upward shift caused by the nucleon binding energy. To incorporate this effect, we have proposed [17] a treatment of W_α that includes U_Δ in a first approximation via an energy-dependent and target-dependent energy shift. This shift, called the mean spectral energy $E_{\rm MS}$ is derived in Ref. [17] as

$$E_{\rm MS}(W_0) = \frac{\int d^3r\, \phi_\pi^{(-)*}(r)\phi_\pi^{(+)}(r)\rho(r)U_\Delta(r)}{\int d^3r\, \phi_\pi^{(-)*}(r)\phi_\pi^{(+)}(r)\rho(r)} \,, \qquad (2)$$

where $U_\Delta(r)$ is taken to be equal to the shell-model potential of a nucleon. In Fig. 1 we present results for π^+ elastic scattering from ^{12}C at 80, 100, 148, 162, and 226 MeV. The data are from Ref. [18]. The dashed line is the result of using the full lowest-order optical potential, including $E_{\rm MS}$. At all energies shown here we find that the inclusion of U_Δ is not only significant but moves the results remarkably close to the data.

At this point, the agreement of the theoretical results with the experimental data is surprising, because there remains much that has not been considered. We know that the pion true-absorption channel is about one-half [19] of the total reaction cross section. The Pauli principle [3,4,20] also should play a significant role in the scattering of the light-mass pion from the heavier nucleon. The p-wave character of the pion-nucleon interaction produces non-negligible correlation corrections [21] (the Ericson-Ericson-Lorentz-Lorenz correction). We will next include each of these higher-order terms. The results will provide a test of our understanding of each piece of the physics and the role that it plays in pion-nucleus dynamics. Details of our treatment of the second-order optical potential can be found in Ref. [22].

In order to utilize existing calculations of the second-order terms, we will make extensive use of the local density approximation. For the Pauli and true-absorption terms, we utilize the functional form of the second-order corrections as derived in Ref. [1]:

$$U^{(2)}(\mathbf{k}',\mathbf{k}) = \lambda_0^{(2)} \mathbf{k}\cdot\mathbf{k}' \rho^{(2)}(\mathbf{k}-\mathbf{k}') \,, \qquad (3)$$

where $\rho^{(2)}$ is the Fourier transform of the square of the target density. Microscopic calculations of higher-order terms yield a coefficient $\lambda_0^{(2)}$ which itself depends weakly on r. In the same spirit as the mean spectral energy calculations, we may define the radius R_2 at which the pion interacts in a finite nucleus by

$$R_2 = \frac{\int d^3r\, \phi_\pi^{(-)*}(r)\phi_\pi(r)\rho(r) r}{\int d^3r\, \phi_\pi^{(-)*}(r)\phi_\pi(r)\rho(r)} \,. \qquad (4)$$

In Table I we give the value of R_2 and the density $\rho(R_2)$ calculated for various pion energies for ^{12}C. We note that over this energy region (80 MeV $\le T_\pi \le 315$ MeV) the interaction is confined to the nuclear surface and low densi-

TABLE I. Parameters $\lambda_0^{(2)}$ [corresponding to the density in ^{12}C at the radius R_2 (fm)] for the Pauli and spreading interaction as a function of pion kinetic energy T_π (MeV). The units for $\lambda_0^{(2)}$ are fm^3.

T_π	R_2	$\rho(R_2)/\rho_0$	$\lambda_0^{(2)}$ (Pauli)	$\lambda_0^{(2)}$ (spread)	$\lambda_0^{(2)}$ (sum)
80	2.40	0.289	−0.40, −1.46	−0.93, 2.02	−1.33, 0.56
100	2.52	0.252	0.08, −1.88	−1.43, 2.09	−1.36, 0.21
148	2.80	0.175	2.50, −1.74	−3.29, 1.18	−0.80, −0.56
162	2.86	0.160	3.20, −0.90	−3.70, 0.35	−0.50, −0.55
230	2.90	0.151	054, 2.50	−1.02, −2.80	−0.47, −0.31
315	2.67	0.209	−0.60, 0.26	0.67, −0.68	0.07, −0.42

ties. Here $\rho_0 = 0.16$ fm^{-3} (nuclear matter density) and the pion distorted waves are taken from Ref. [1].

The Pauli exchange term is taken directly from Ref. [3] evaluated at the density $\rho(R_2)$. We extend the term by including ρ-meson propagation in the intermediate state, following Ref. [22]. We omit pion distortions for the intermediate pion to avoid including multiple reflection corrections in the Pauli term. The $\lambda_0^{(2)}$ coefficients are given in Table I. The dotted curve in Fig. 1 gives differential cross sections resulting from adding the second-order Pauli correction to the lowest-order calculation. We see that the Pauli correction (1) is large and (2) completely destroys the nearly quantitative agreement of the dashed curve.

We will include true absorption and other sources of

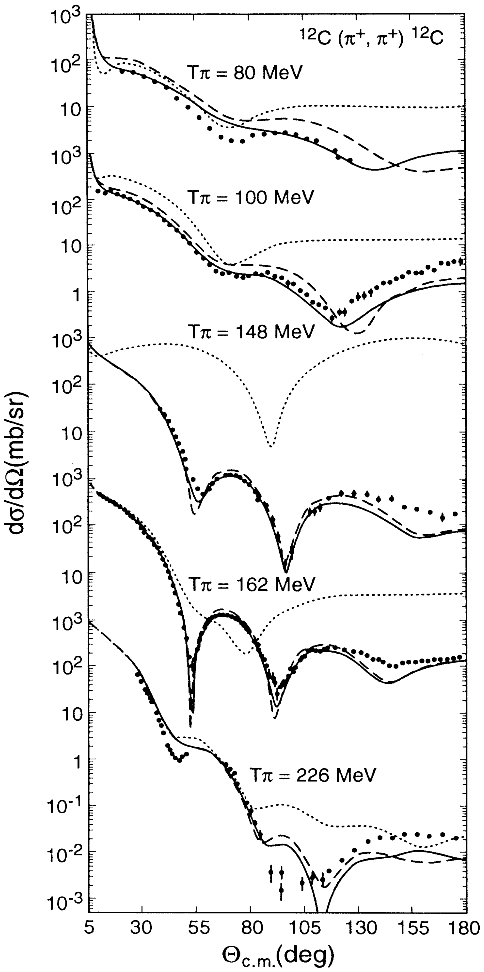

FIG. 1. The differential cross section for elastic scattering of π^+ from ^{12}C at the energies indicated. The dashed curves are a lowest-order optical model calculation including the delta-nucleus interaction through the mean spectral approximation; dotted curves are from the lowest-order optical potential with the second-order Pauli corrections included; solid curves are with the lowest-order optical potential, and second-order Pauli corrections and the second-order spreading potential all included.

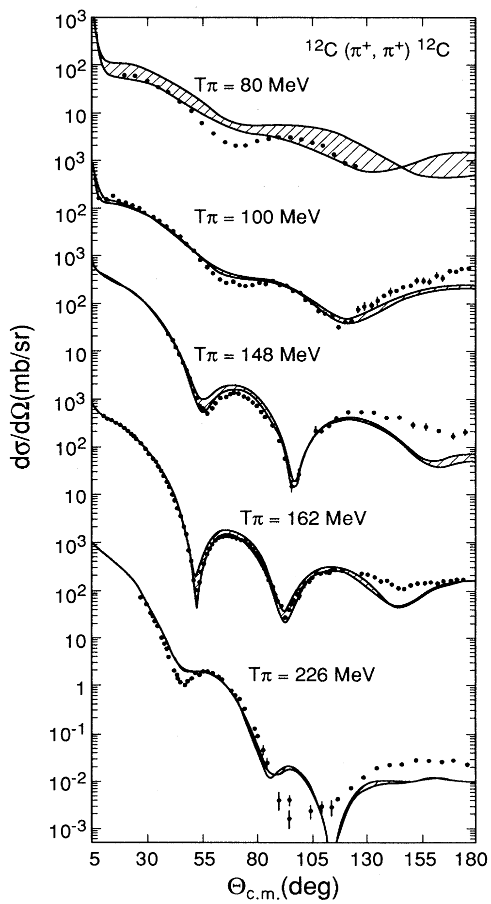

FIG. 2. The same as Fig. 1, except the shaded area includes the full lowest-order optical potential, Pauli and spreading corrections, and the LLEE correlation corrections. The two curves forming the boundary result from the LLEE parameter ξ given by 0.08 (the lowest curve in the forward direction) and 0.23.

resonance broadening by introducing a $\lambda_0^{(2)}$ parameter determined phenomenologically from the spreading potential of the delta-hole model [23]. These two terms cannot be equated directly because the spreading potential occurs in the denominator of the delta propagator. We can make the correspondence by first isolating the P_{33} partial wave contribution to the lowest-order optical potential and expressing it in a resonant form. The difference between this potential evaluated once with width $\Gamma_0+\mathrm{Im}W_{\mathrm{sp}}$ and then with width Γ_0 (the free width) is a true-absorption potential that can be expanded at low density to give a $\lambda_0^{(2)}$ independent of r. Rather than expanding, however, we determine $\lambda_0^{(2)}$ by matching this difference to Eq. (3) at the radius R_2. The resulting values of $\lambda_0^{(2)}$ are given in Table I. We see that at all energies there is a large cancellation between $\lambda_0^{(2)}$ (Pauli) and $\lambda_0^{(2)}$ (spreading), yielding a small total second-order correction. The solid curve in Fig. 1 gives the differential cross sections obtained when E_{MS}, $\lambda_0^{(2)}$ (Pauli), and $\lambda_0^{(2)}$ (spreading) are all included. The cancellation of the Pauli and spreading terms is evident.

The small size of the second-order terms is pleasing. It indicates that an optical model approach is a practical representation of the pion-nucleus interaction. This is an important result, because much of the nuclear-structure studies with pions are made beginning with the optical-potential description. The fact that the higher-order terms are small for the optical potential does not guarantee, however, that they will be small for transition operators in inelastic or charge-exchange scattering.

Finally, we also include the correlation [or Lorentz-Lorenz-Ericson-Ericson (LLEE)] corrections. It has been shown [25] that the LLEE effect can be included in the delta self-energy by a modification

$$\delta E_{\mathrm{MS}}=\frac{4\xi}{27}\left[\frac{f_{\pi N\Delta}}{m_\pi}\right]^2\rho_0\ , \tag{5}$$

where ξ is the usual Lorentz-Lorenz parameter. The value of ξ depends on the range of the short-range repulsive correlations between nucleons, the range of the pion-nucleon form factor, and the strength of the delta-nucleon interaction. We will allow for some uncertainty in ξ. The minimum value that is reasonable is about $\xi/3=0.08$, which results from a pion-nucleon monopole cutoff of 800 MeV/c and no Δ-N interaction. The maximum value of $\xi/3$ is 0.23, which arises from a cutoff of 990 MeV/c and includes a ΔN interaction contribution. This value gives a real part of the delta-hole spreading interaction that corresponds to $\delta E_{\mathrm{MS}}=23$ MeV. The final result of this work is given by the shaded area between the solid curves in Fig. 2 (corresponding to the range $0.08\le\xi/3\le 23$). These results combine the first-order potential in which the delta-nucleus potential is included via the mean spectral energy the Pauli, true-absorption, and correlation corrections.

The agreement with the data shown in Fig. 2 is not exact, but it is remarkably good for a parameter-free calculation. Discrepancies could be due to the fact that our treatment of the second-order corrections is neither exact nor totally consistent (we have made certain approximations to the isobar spreading width [23]). For these reasons, it is probably unwarranted to conclude that the smaller value of $\xi/3=0.08$ is preferred, even though this result is everywhere closer to the data. Firm conclusions should await a more thorough, internally consistent treatment [22] of all the higher-order terms. We are motivated to pursue this because our present calculation is intriguingly close to the data.

We have for the first time combined a contemporary momentum-space calculation of the first-order optical potential with microscopic predictions of the effects of the delta-nucleus interaction, Pauli corrections, pion true absorption, and short-range correlations. We have seen that convergence of the expansions is enhanced throughout the resonance region by (1) collecting U_Δ (via the mean spectral energy approximation) together with binding corrections into the first-order optical potential, and (2) collecting the Pauli and true-absorption terms together. This result supports our perturbative approach [22] to calculating the optical potential. Finally, the good results that we find from 80 to 226 MeV with no adjustable parameters suggest that pursuing calculations of greater accuracy for the second-order terms might yield a definitive determination of the short-range correlations (i.e., the parameter ξ) and the delta-nucleus interaction U_Δ.

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