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# Charge exchange effects in elastic scattering with radioactive beams

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The scattering of $^{13}$C on $^{13}$C and on its mirror nucleus $^{13}$N is considered in the framework of a four-body model with two (inert) $^{12}$C cores and a pair of weakly bound nucleons. Numerical calculations of the elastic angular distributions for the two systems show that the surface transparency of the core-core interaction and the weak binding of the proton in $^{13}$N will enable one to extract the effective neutron-proton interaction from analyses of the $^{13}\text{C}+^{13}\text{N}$ elastic scattering.

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In a recent paper [1], we had proposed the investigation of the elastic scattering of mirror nuclei and we demonstrated that it was reasonable to expect enhanced charge exchange effects. In particular, we had suggested the $^{13}\text{C}+^{13}\text{N}$ system for experimental and theoretical investigation where $^{13}$C has a neutron weakly bound to a $^{12}$C core and $^{13}$N has a weakly bound proton. This is of particular current interest since collisions of mirror nuclei involve at least one unstable nucleus and could be explored with new radioactive beam facilities.

The core-core interaction for $^{13}\text{C}+^{13}\text{N}$ at low energies is believed to be surface transparent and as a consequence, the elastic scattering of $^{13}$N by $^{13}$C would be expected to be sensitive to the interaction between the valence nucleons. The model we had proposed was a four-body model where the two $^{12}$C cores were treated as inert and the two nucleons were weakly bound. In this paper, we present the results of the analyses of such a model for the elastic scattering of two $^{13}$C ions as well as that of the scattering of $^{13}$N by $^{13}$C at energies close to the Coulomb barrier. We show here that, indeed, the elastic scattering is very sensitive to the effective $n$-$p$ interaction and affords a new tool to extract it from the data.

In the four-body model, the radial wave function of relative motion of the centers-of-mass of the two nuclei is a solution of the equation

$$[K_L+U(R)+V^D(R)+(-)^L V^E(R)]/\{[1+(-)^L S(R)]-E\}u_L(R)=0 \ , \tag{1}$$

where $K_L$ is the radial kinetic energy operator in the $L$th partial wave, $U(R)$ is the core-core potential, $V^D(R), V^E(R)$ are the direct and exchange potentials, and $S(R)$ is a nonorthogonality overlap. Explicit expressions for the direct and exchange potential and the nonorthogonality overlap have been given in Ref. [1]. The relative wave functions for both the $^{13}\text{C}+^{13}\text{C}$ and $^{13}\text{C}+^{13}\text{N}$ systems satisfy equations of the above type. In the former case, because of the identity of the projectile and target, the even and odd partial waves contribute to the cross section incoherently yielding an angular distribution symmetric around 90°. This symmetry does not exist for the $^{13}\text{C}+^{13}\text{N}$ collision. There exist experimental data on the angular distributions and excitation functions for the elastic scattering of two $^{13}$C nuclei. Helb *et al.* [2] concluded that the angular distributions at low energies (laboratory energies in the range 15–20 MeV) were described well by the method of molecular wave functions. Korotky *et al.* [3] concluded that the elastic excitation function for the same system is consistent with the occurrence of orbiting in the dynamical interaction. We reanalyzed the angular distribution and excitation function for the system of two $^{13}$C nuclei in the energy range of 7.5 to 10 MeV in the center of mass. The core-core potential is the one given by Reeves [4] (quoted by Michaud and Vogt [5]) and is a Woods-Saxon potential with $V=50$ MeV, $W=10$ MeV, $R=5.77$ fm, and $a=0.4$ fm. The real and imaginary potentials were assumed to have the same geometry. For the neutron-core potential, we used a Woods-Saxon potential with radius and diffuseness parameters given by $r_0=1.16$ fm, where $R=r_0 A^{1/3}$, and $a=0.705$ fm and a well depth adjusted to yield the experimental binding energy of 4.95 MeV. For the neutron-neutron interaction, we considered two options: firstly we used a delta function force with a strength of 300 MeV fm$^3$ and then we tried a M3Y interaction [6,7] given by

$$v(r)=7999(e^{-4r}/4r)-2134(e^{-2.5r}/2.5r) \tag{2}$$

with an exchange contribution approximated by a delta function force with strength 262 MeV fm$^3$. Both the zero range and finite range forms of the neutron-neutron interaction gave almost identical predictions for the angu-

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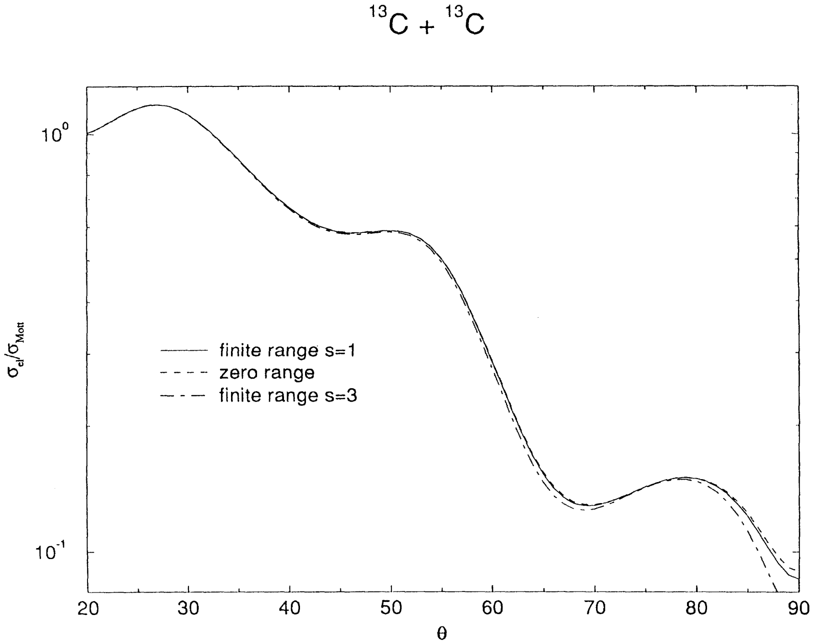


FIG. 1. The effect of the range of the $n$-$n$ interaction on the elastic cross section for two $^{13}$C nuclei at 10 MeV center-of-mass energy is shown. The dashed line corresponds to a delta function interaction while the full line curve is the prediction with a M3Y interaction [Eq. (2)] for the $n$-$n$ interaction.

lar distributions. In addition, changing the strength of the $n$-$n$ interaction did not affect the angular distribution. This is shown in Fig. 1 where we compare the predictions of our model for the elastic scattering of two $^{13}$C nuclei at a center-of-mass energy of 10 MeV with a delta function interaction with that of the M3Y interaction and also one where the strength of the M3Y interaction was scaled by a factor of 3. The figure shows that the details of the $n$-$n$ interaction have very little effect on the elastic cross section. The predicted angular distributions for center-of-mass energies of 7.5, 8, 9, and 10 MeV are shown along with the data of Helb *et al.* [2] in Fig. 2. Our predictions are very similar to those of Helb *et al.* with their molecular wave function method, which itself was found to be very similar to the predictions of the optical potential suggested by Korotky *et al.* [3].

Having the confidence that the core-core potential chosen yields a reasonable description for the scattering of two $^{13}$C nuclei, we adopted it for the $^{13}$C+$^{13}$N system. We assumed the nuclear part of the proton-core interaction to be the same as the neutron-core potential. Adding the proton-core Coulomb potential resulted in a proton binding of 1.91 MeV which is close to the experimental value. The neutron-proton interaction was assumed to be three times stronger than the neutron-neutron interaction [8]. The latter was taken to be a M3Y interaction [see Eq. (2)]. The predicted differential cross section for center-of-mass energies of 7.5, 8, 9, and 10 MeV are shown in Fig. 3. It can be seen from the figure that the differential cross section is structureless at low energies

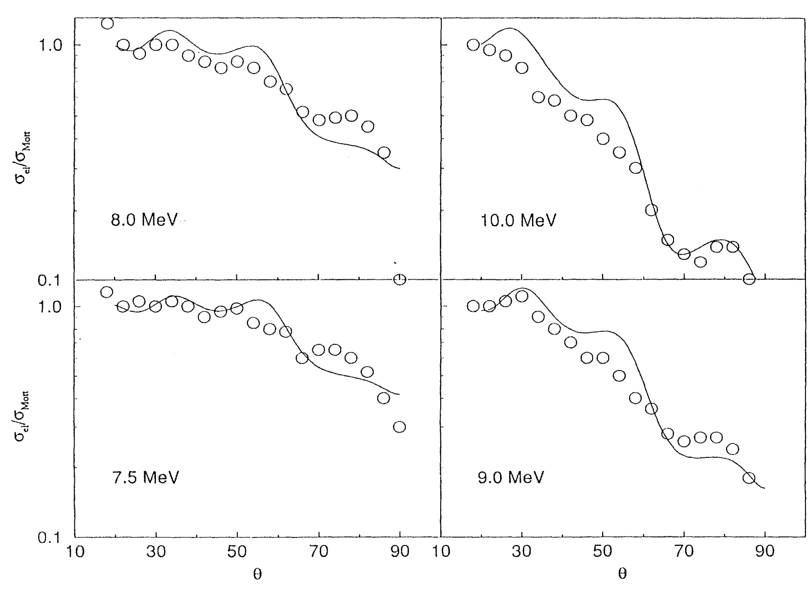


FIG. 2. The predicted angular distributions for the elastic scattering of two $^{13}$C nuclei are shown for center-of-mass energies of 7.5, 8, 9, and 10 MeV. The experimental data of Helb *et al.* [2] are also shown.

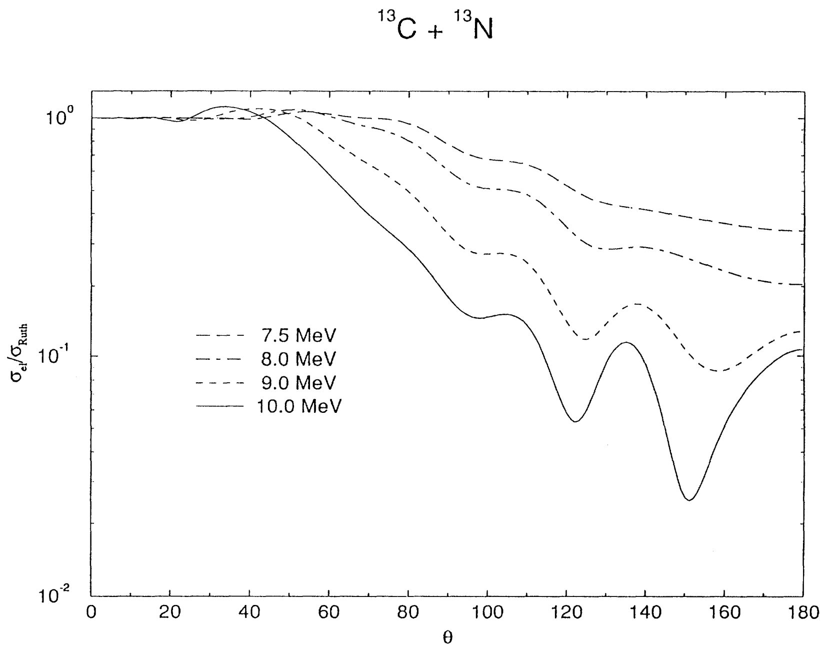


FIG. 3. The predicted angular distributions for the elastic scattering of $^{13}$N and $^{13}$C are shown for center-of-mass energies 7.5, 8, 9, and 10 MeV. In these calculations, the neutron was bound to the core by 4.95 MeV while the proton was bound to the core by 1.91 MeV.

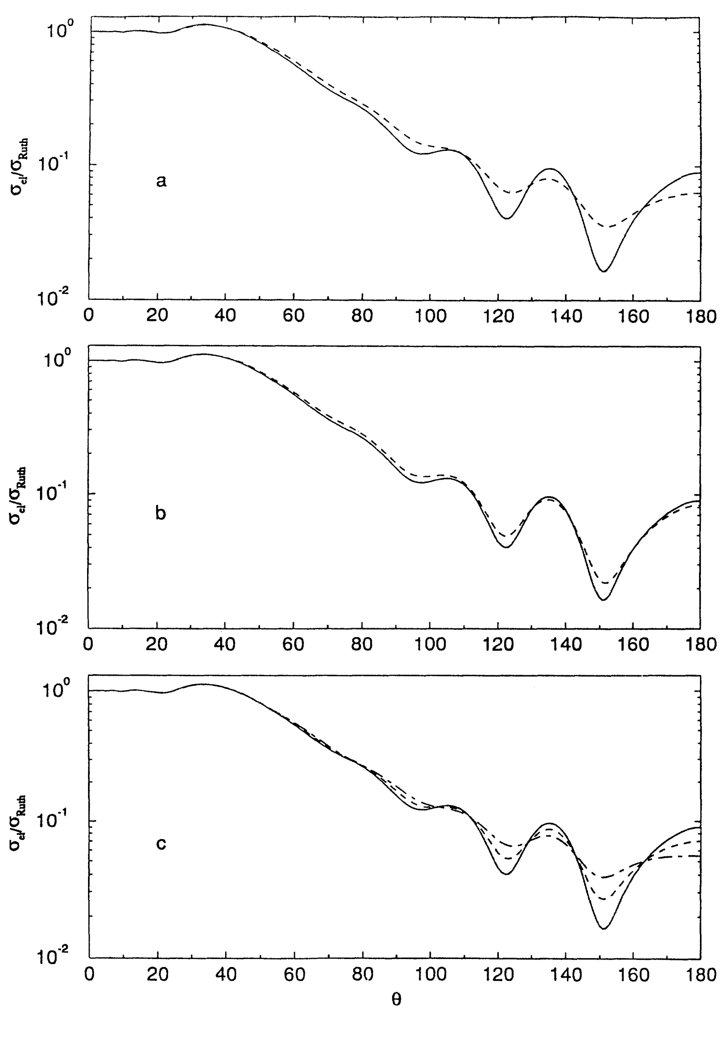


FIG. 4. (a) The dependence of the angular distribution for the elastic scattering of $^{13}$N and $^{13}$C at a center-of-mass energy of 10 MeV on the range of the $n$-$p$ interaction. The dashed line corresponds to the delta function interaction while the full line corresponds to the finite range interaction. (b) The effect of the proton binding energy on the elastic cross section is shown. The dashed line corresponds to setting the proton binding energy to be the same as the neutron (4.95 MeV) while the full line corresponds to the proton bound by 1.91 MeV. (c) The effect of the strength of the $n$-$p$ interaction on the elastic cross section is shown. The three curves are shown with the scale factors $S=1,2,3$ where $S$ represents the factor by which the original M3Y interaction is multiplied.

and as the energy of the projectile is increased, one begins to observe oscillations at large angles. The oscillations are a consequence of the interference between the direct and charge exchange contributions to the elastic scattering. The charge exchange contribution increases with the projectile energy and shows up through the oscillations in the different cross section.

To test the sensitivity of the charge exchange contribution to the input parameters, we repeated the calculation for different sets of values of these parameters. Firstly, we considered the effect of the range of the $n$-$p$ interaction on the cross section. In Fig. 4(a), we show the difference in the predictions of the elastic scattering between a delta function force for the interaction and the M3Y interaction used in the calculations of Fig. 3, for a projectile energy of 10 MeV. One observes enhanced oscillations for the case of the finite range M3Y interaction in contrast to the delta function interaction. The charge exchange contribution is considerably enhanced by the range of the $n$-$p$ interaction. This was in contrast to the case of the scattering of two $^{13}$C ions where the range of the $n$-$n$ interaction had little effect on the cross section. Next we tested the effect of the binding energy of the proton on the elastic cross section. Figure 4(b) shows the difference between a choice of the experimental binding energy of 1.91 MeV for the proton and a choice of 4.95 MeV (same as the neutron) for the proton in $^{13}$N. One observes a much smaller effect of the binding energy on the elastic cross section. Finally, in Fig. 4(c) we show the effect of the strength of the $n$-$p$ interaction on the elastic cross section. The effect is observed in the oscillations in the cross section and the results suggest that the charge exchange cross section is scaled by the $n$-$p$ interaction.

To summarize the above results, we found the following effects in the differential cross sections.

(i) In the case of the scattering of the two $^{13}$C nuclei, there was little effect due to the last neutron. This is mainly due to the symmetry of the two nuclei whereby the even and odd partial waves contribute incoherently.

(ii) In contrast to the above, the differential cross section in the collision of $^{13}$N by $^{13}$C was found to be very sensitive to the neutron-proton interaction. This is a consequence of the lack of symmetry in this system which allows the even and odd partial waves to contribute coherently to the cross section. This provides a more complete description of the charge exchange amplitude and its phase relative to the direct amplitude.

The sensitivity to the effective $n$-$p$ interaction in the large angle $^{13}\mathrm{C}+^{13}\mathrm{N}$ scattering is reflected in the magnitude of the oscillations in $\sigma_{\mathrm{el}}/\sigma_{\mathrm{Ruth}}$ but not in the overall strength of $\sigma_{\mathrm{el}}/\sigma_{\mathrm{Ruth}}$. Will it be possible to calculate the theory with sufficient precision to lead to a meaningful extraction of the effective $n$-$p$ interaction? Figure 2 is illustrative for this purpose since it shows a closely related comparison between theory and experiment where the valence nucleons' interaction played essentially no role. There, the level of discrepancies ranged up to a maximum of about 20%. On the other hand, the differences in theoretical results arising from varying separately the range and strength of the effective $n$-$p$ interaction in the $^{13}\mathrm{C}+^{13}\mathrm{N}$ scattering reached more than a factor of 2 at back angles. Clearly, it will be desirable to have both $^{13}\mathrm{C}+^{13}\mathrm{C}$ and $^{13}\mathrm{C}+^{13}\mathrm{N}$ data to large angles to meaningfully extract the effective $n$-$p$ interaction.

The results presented here demonstrate the potential for extracting the effective neutron-proton interaction from a careful analysis of the collision of $^{13}$N by $^{13}$C. Preliminary data on the elastic scattering of $^{13}$N by $^{13}$C at a center-of-mass energy of 8.15 MeV by Vervier *et al.* [9] show a differential cross section which has a minimum at around 120° and which rises to a value of between 6 to 10 times the Rutherford value at 180°. Our calculations do not predict the large increase at large angles. If the data are confirmed, it would lead to an interesting speculation that one is observing an enhanced effect due to meson exchange between the valence nucleons. This would motivate a more complete treatment than the one we have presented and may require considering additional processes that play a selective role at higher momentum transfers. For example, we can speculate that $\Delta$ excitations, effective three-body forces, and/or a more complicated medium-modified meson dynamics may then be important. A consistent treatment of these effects is well beyond the scope of our present work.

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